

2025 Simpson Lecture

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Thanks to Co-Authors

- ▶ R. Aiyagari (Optimum Quantity of Debt)
- ▶ E. Prescott (Unmeasured Investment and the Puzzling US Boom in the 1990s)
- ▶ E. Prescott (Taxes, Regulations, and the Value of US and UK Corporations)
- ▶ A. Bhandari (Sweat Equity in US Private Business)
- ▶ A. Bhandari, T. Kass, T. May, E. Schulz (On the Nature of Entrepreneurship)
- ▶ A. Bhandari, P. Martellini ([Capital Reallocation and Private Firm Dynamics](#))
- ▶ A. Bhandari, D. Evans (Approximating Transition Dynamics with Discrete Choice)
- ▶ A. Bhandari, D. Evans, Y. Yao (Business Income Underreporting and Public Finance)



Disclaimer

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The Macroeconomics of US Public Finance

- ▶ Net government saving: ≈ -1 Trillion (in 2018\$)
 - Current receipts: 5.6T
 - Current expenditures: 6.7T



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 - Current receipts: 5.6T
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- ▶ Untaxed business income ≈ 1 Trillion
 - Income reported to IRS: 3.2T
 - Estimate of true: 4.2T



The Macroeconomics of US Public Finance

- ▶ Net government saving: ≈ -1 Trillion
 - Current receipts: 5.6T
 - Current expenditures: 6.7T

- ▶ Untaxed pass-through business income ≈ 700 Billion
 - Income reported to IRS: 1.3T
 - Estimate of true: 2T



The Macroeconomics of US Public Finance

- ▶ Net government saving: $\approx -5.4\%$
 - Current receipts: 27%
 - Current expenditures: 33%

- ▶ Untaxed pass-through business income $\approx 3.4\%$
 - Income reported to IRS: 6.4%
 - Estimate of true: 9.8%

- ▶ Pass-through business income underreporting is largest source of US tax gap



Challenges for Tax Policy and Administration

- ▶ Privately-owned businesses (which are mostly pass-through entities)
 - Account for more than 1/2 of US business net income
 - Dominate discussions of growth, wealth inequality, and fiscal policy

- ▶ But pose challenges for measurement and theory:
 - Lack of reliable data on capital
 - Lack of appropriate theory for capital accumulation and allocation

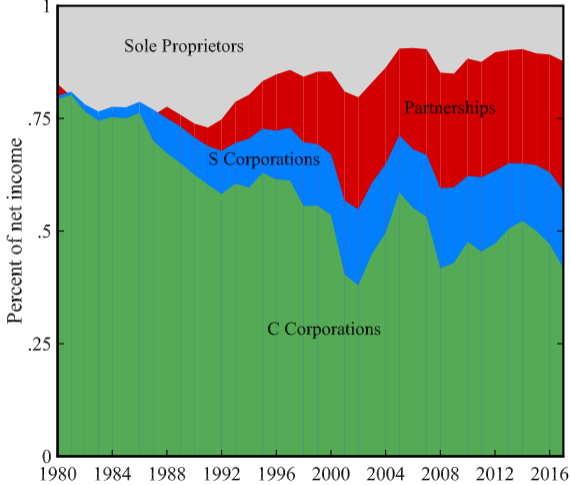


Significant Rise in Private Business

- ▶ Major shift in US legal forms of organization after TRA86
 - From C corporations that pay corporate income tax
 - To pass-through entities taxed at individual level
 - S corporations
 - Partnerships
 - Sole proprietorships
- ⇒ Major shift to private firms with no SEC reporting



Significant Rise in Private Business Net Incomes



New Evidence on Business Investment from Tax Filings

IRS v. CPS

- ▶ Indirect evidence from individual income tax returns
 - Form 1120S, Schedule K-1, S-corporation owner share of income
 - Form 1065, Schedule K-1, Partner share of income
 - Form 1040, Schedule C, Sole Proprietor profit/loss
 - Form W2, Wage and Tax statement for employees

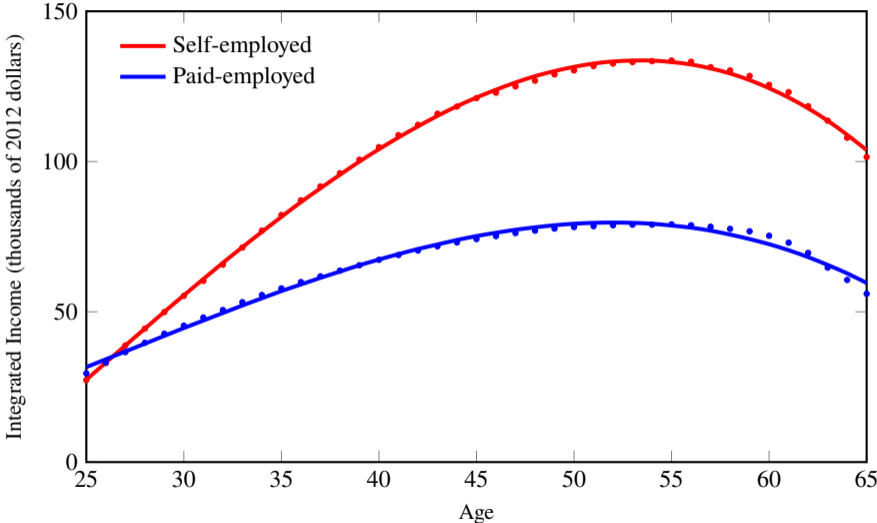
⇒ Steeper income profiles for owners relative to paid peers suggests payments to investments

- ▶ Direct evidence from business sales
 - Form 8594, Asset acquisition statement

⇒ Allocation of sale price shows assets are primarily intangible



Estimated Income Profiles for Self- and Paid-Employed



Direct Evidence from Business Sales

- ▶ When businesses are sold
 - Business assets are sold as a group
 - Total price must be allocated across asset classes
 - Allocation must be reported by both buyer and seller
 - Allocation establishes:
 - Asset bases for the buyer
 - Capital gains/losses for the seller

- ▶ Information is recorded on IRS Form 8594



Sales of Businesses: Tax Filings

Form 8594 <small>(Rev. November 2021) Department of the Treasury Internal Revenue Service</small>	Asset Acquisition Statement Under Section 1060 <small>▶ Attach to your income tax return. ▶ Go to www.irs.gov/Form8594 for instructions and the latest information.</small>	<small>OMB No. 1545-0074</small> Attachment Sequence No. 169
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Name as shown on return	Identifying number as shown on return
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Check the box that identifies you:
 Purchaser Seller

Part I General Information

1 Name of other party to the transaction	Other party's identifying number
Address (number, street, and room or suite no.)	
City or town, state, and ZIP code	

2 Date of sale	3 Total sales price (consideration)
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Part II Original Statement of Assets Transferred

4 Assets	Aggregate fair market value (actual amount for Class I)	Allocation of sales price
Class I	\$	\$
Class II	\$	\$
Class III	\$	\$
Class IV	\$	\$
Class V	\$	\$
Class VI and VII	\$	\$
Total	\$	\$

← Cash/securities
 ← Inventories
 ← Fixed assets
 ← Sec. 197 intangibles

5 Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document signed by both parties? Yes No

If "Yes," are the aggregate fair market values (FMV) listed for each of asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document? Yes No

6 In the purchase of the group of assets (or stock), did the purchaser also purchase a license or a covenant not to compete, or enter into a lease agreement, employment contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)? Yes No

If "Yes," attach a statement that specifies (a) the type of agreement and (b) the maximum amount of consideration (not including interest) paid or to be paid under the agreement. See instructions.



Transferred Assets are Primarily Intangible

- ▶ Roughly 70% are *Section 197 intangibles*:
 - Customer bases and client lists
 - Supplier-based intangibles and contracts
 - Trademarks, tradenames, and franchises
 - Patents, copyrights, formulae, processes, designs, etc.
 - Business books, records, operating systems
 - Licenses and permits
 - Non-compete covenants
 - Workforce in place, including composition and contracts
 - Goodwill and on-going concern value
 - ▶ Most intangible values only known when business sells
- ⇒ Total capital stock harder to measure with rise in private business



What About BEA Measure of IPP?

- ▶ Intellectual property products:
 - Introduced in NIPA in 2013
 - Accounts for roughly half of nonresidential investment

- ▶ Includes expenditures on:
 - R&D
 - Software and databases
 - Entertainment and artistic originals

- ▶ Excludes most intangible investment in private firms

IPP share



Want Operator

- ▶ A theory of private firm dynamics
 - Consistent with IRS evidence on private business capital
 - Suitable for fiscal policy analysis

- ▶ That departs from the “traditional” Lucas-Hopenhayn framework
 - Where capital modeled as divisible, rentable, observable
 - Instead model capital as lumpy, non-rentable, and non-observable until business is sold



Properties of Capital Used in Private Business

- ▶ Assets are **sold as a group**
- ▶ Transferred assets are **primarily intangible**
- ▶ Intangible assets are generally **non-rentable**
- ▶ Sale **requires time** to find buyers/negotiate (from brokered data \approx 290 days)



Model Environment

- ▶ Infinite horizon with continuous time
- ▶ Demographics
 - Unit mass of individuals
 - Birth/death at constant rate ψ
- ▶ Individuals choose
 - Paid-employment, supplying labor inelastically or
 - Self-employment, running a private business
- ▶ Preferences risk neutral, discount rate ρ



Technology

- ▶ Goods technology: $y(z, k, n, b) = zk^\alpha n^\gamma b^\beta$
- ▶ Non-transferable factor: productivity, z
 - evolves exogenously when running a business: $dz = \mu(z)dt + \sigma(z)\sqrt{dt}dW$
- ▶ Transferable factor: business capital, k
 - built through investment: $dk = (\theta - \delta_k k) dt$, convex cost $C(\theta)$
- ▶ Rentable factors: labor n , other fixed assets and equipments b
 - investment in other fixed assets: $db = (x - \delta_b b) dt$



Entry and Exit Technology

Let $s = (z, k)$ be the state of a business

- ▶ Entry:
 - draw $s \sim G(s)$, where $s = (z, k = 0)$
 - decide whether to run a firm or be a worker

- ▶ Exit:
 - option to exit at rate δ
 - exit due to death at rate ψ



Spot Markets for Labor and Buildings

- ▶ Spot market for labor n
 - supply: occupation choice
 - demand: incumbent firms

- ▶ Spot rental market for other fixed assets b
 - supply: competitive “mutual fund”
 - demand: incumbent firms



Market for Transferable Business Capital

- ▶ Businesses access market at rate η
- ▶ Bilateral trades: For a pair s, \tilde{s}
 - Feasible allocations: buy all, sell all, no trade \Rightarrow indivisibility
 - Transfers (prices): positive if buying, negative if selling
- ▶ Minimal restrictions on outcomes
 - anyone can trade with anyone
 - any feasible allocation



Market for Transferable Business Capital

- ▶ Businesses access market at rate η
- ▶ Bilateral trades: For a pair s, \tilde{s}
 - Feasible allocations: $k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\} \Rightarrow$ indivisibility (extension w/ costly divisibility)
 - Transfers (prices): $p^m(s, \tilde{s})$, negative if selling (extension w/ financing constraints: $p^m(s, \tilde{s}) \leq \xi y(s, n)$)
- ▶ Minimal restrictions on outcomes
 - anyone can trade with anyone
 - any feasible allocation s.t. $p^m(s, \tilde{s}) + p^m(\tilde{s}, s) \geq 0$



Owner's Value Solves HJB

- ▶ Existing firms

$$\begin{aligned}(\rho + \psi)V(s) = & \underbrace{\max_{n,b} y(s, n, b) - wn - rb}_{\text{production}} \\ & + \underbrace{\max_{\theta} \partial_k V(s)(\theta - \delta_k k) - C(\theta)}_{\text{investment}} + \underbrace{\max_{\lambda} \eta W(s; \lambda)}_{\text{trade}} \\ & + \underbrace{\mu(z)\partial_z V(s) + \frac{1}{2}\sigma(z)^2\partial_{zz} V(s)}_{\text{evolution of productivity}} + \underbrace{\delta \max \left\{ \frac{w}{\rho + \psi} - V(s), 0 \right\}}_{\text{endogenous exit}}\end{aligned}$$

where

$$W(s; \lambda) = \int [V(z, k^m(s, \tilde{s})) - V(z, k) - p^m(s, \tilde{s})] \lambda(s, \tilde{s}) ds$$

$$\int \lambda(s, \tilde{s}) d\tilde{s} + \lambda(s, 0) = 1$$



Law of Motion

- ▶ Let ϕ = distribution over states s and m be the mass of firms
 - ▶ ϕ evolves with exogenous shocks: $\mu(z)$, $\sigma(z)$, δ , ψ and choices
 - Investment: θ
 - Trade: λ
 - Entry/Exit: $\iota_{\text{entry}}(s)$, $\iota_{\text{exit}}(s)$
- ▶ Law of motion for ϕ and m given by:

$$d\phi = \Gamma(\phi; \theta, \lambda, \iota_{\text{entry}}, \iota_{\text{exit}})dt$$

$$dm = \psi \int \iota_{\text{entry}}(s)dG(s) - m \left(\psi + \delta \int \iota_{\text{exit}}(s)\phi(s) \right) dt$$



Definition of Recursive Equilibrium

A (stationary) equilibrium is a set of value functions $V(s)$, policy functions for investment $\theta(s)$, entry $\iota_{entry}(s)$, exit $\iota_{exit}(s)$, and trade $\lambda(s, \tilde{s})$, terms of trade $(k^m(s, \tilde{s}), p^m(s, \tilde{s}))$, wage w , rental rates r , and distribution over the state space $\phi(s)$ and a mass of firms m that satisfy

- ▶ business owners' optimality
- ▶ indifference in occupational choice
- ▶ market clearing
- ▶ consistency and stationarity of measures and mass
 - Consistency: $\lambda(s, \tilde{s})\phi(s) = \lambda(\tilde{s}, s)\phi(\tilde{s})$
 - Distribution: $d\phi = 0$
 - Mass: $dm = 0$



Properties of the Equilibrium: Limiting case

- ▶ To better understand properties
- ▶ First consider the simplest model with
 - Businesses that can be instantaneously traded
 - Capital that is perfectly divisible and rentable



Properties of the Equilibrium: Limiting case

- ▶ If:
 - Businesses can be instantaneously traded
 - Capital is perfectly divisible and rentable

- ▶ Theory predicts:
 - Capital perfectly allocated across firms
 - Valuation ratios, marginal product of capital, return on business investment same for all firms
 - ▶ Tobin's Q : $p = C'(\theta)$
 - ▶ Marginal product of capital (MPK): $\alpha y/k$
 - ▶ Returns: $\rho + \delta_k$



Properties of the Equilibrium: Our Baseline

- ▶ If:
 - Businesses cannot be instantaneously traded
 - Capital is not perfectly divisible

- ▶ Theory predicts:
 - Gradual capital reallocation toward higher MPK firms
 - Transferable share of capital below 100%
 - Dispersion in Tobin's Q, MPKs, net returns

Intuitive example



General Solution

- ▶ Given (ϕ, V) , solve linear programming problem:
 - Maximize static social surplus
 - Subject to adding up constraints

- ▶ Which delivers:
 - Equilibrium allocations: λ, k^m
 - Prices: p^m
 - Gains from trade: $W(s)$

- ▶ Then update: $(\phi, V) \rightarrow (\lambda, p^m, k^m) \rightarrow (\phi', V')$

- ▶ Easy to extend if utility non-transferable

Details



Properties of the Equilibrium

- ▶ **Competitive allocation** solves the planner's problem starting at $\phi(s, t = 0) = \phi^{ss}(s)$
- ▶ **Competitive prices** are independent of seller's z

$$p^m(s, \tilde{s}) = \mathcal{P}(k(\tilde{s}))$$

Intuition: competitive nature of the equilibrium \Rightarrow same good sold at same price



Using the Model

- ▶ Calibration using data on
 - Tax returns
 - Business transfers

- ▶ Model deliverables
 - Dispersion in marginal product of capital (MPK)
 - Business price and value

- ▶ Tax policy analysis



Measurement



Sample

- ▶ Sample Period: 1996 to 2022
- ▶ An observation is firm-year pair with
 - ▶ Subchapter S corporation filing (Form 1120S)
 - ▶ Wage bill > 10K
 - ▶ At least 3 years of data
- ▶ Two samples:
 - ▶ *Full* sample: all firm-year pairs meeting above criteria
 - ▶ *Trading* sample: seller-buyer pairs with
 - ▶ Seller: meeting above criteria and counterparty on Form 8594
 - ▶ Buyer: counterparty on Form 8594 with wage bill > 10K



Variable Definitions

- ▶ Business Age: Number of years from Date of Incorporation (1120S, Box E)
- ▶ Business Size (Wage Bill): Sum of all Wages, tips, other compensation (W2, Box 1)
- ▶ Business Valuation: Total consideration (8594, line 3)
- ▶ Relative Size: Wage bill of buyer $t+1$ / wage bill of seller $t-1$ (8594, 1120S)



Data Samples

BUSINESS SUBSAMPLES	COUNTS
S corporation population	3,167,266
S corporation sellers	105,162
Sales to S corporations	46,708
to Partnerships	33,462
to C corporations	35,792
Seller-buyer pairs	51,286
S Corporation–S Corporation	28,078
–Partnership	14,040
–C Corporation	9,168



Identification Strategy

- ▶ Life-cycle firm dynamics

- Productivity process
- Rentable input share
- Exit rates
- Size distributions

Data

- ▶ Transaction data

- Trading rate
- Investment cost
- Output elasticity wrt business capital

Data



Transaction Data and Key Parameters

Key parameters

- Trading rate η
- Investment cost $C(\theta) = \frac{A}{1+\chi} \theta^{1+\chi}$
- Output elasticity wrt k ,
 $y(z, k, n, b) = zk^\alpha n^\gamma b^\beta$

Key moments from data

- ▶ brokered sales: time to sell
- ▶ IRS filings
 - Relative size of buyer/seller
 - Sale price/wage bill



Identification of Investment Cost (A) and Output Elasticity (α)

- ▶ Vary A and α and trace out
 - Relative size of buyer and seller
 - Sale price relative to business capital (proxied by price to wagebill ratio)

- ▶ Same relative size if
 - $\alpha \uparrow$: more linear technology
 - $A \uparrow$: more costly for buyer to scale up

- ▶ Same sale price to capital if
 - $\alpha \uparrow$: buyer willing to pay more
 - $A \downarrow$: buyer less willing to pay



Functional Forms and Parameters

▶ Production

- $y(s, n, b) = z(s)k(s)^\alpha n^\beta b^\gamma, \quad \alpha = 0.15, \beta = \gamma = 0.35$

▶ Investment

- cost $C(\theta) = A\theta^{1+\chi}/(1+\chi), \quad A = 30, \chi = 2$

- depreciation, $\delta_k = \delta_m = 0.1$

▶ Productivity

- entry distribution $G(z) \propto z^{-2}$

- post entry process $d \log z = \mu dt + \sigma d\mathcal{W}, \mu = -0.02, \sigma = 0.1$

▶ Rates

- discounting, $\rho = 0.05$

- trading, $\eta = 1.0$

- exiting, $\psi = 1/40, \delta \rightarrow \infty$

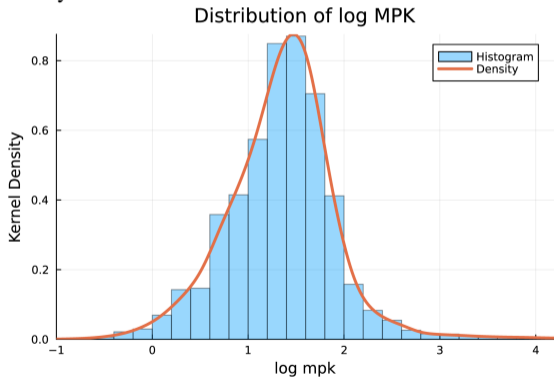


Results



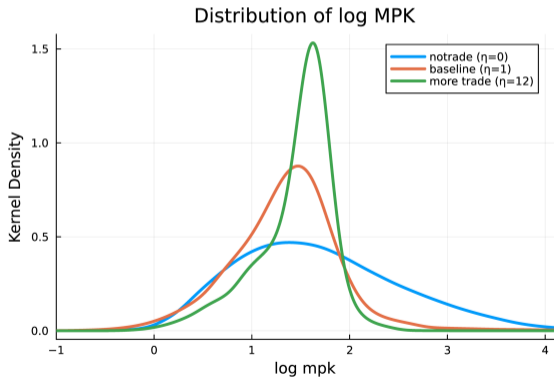
Dispersion in MPK

- ▶ Idiosyncratic change in productivity → input reallocation toward higher MPK
- ▶ Dispersion in marginal product of capital induced by
 - Decentralized trading
 - Indivisibility of asset sold
- ▶ Standard deviation of log-MPK: 57%



Dispersion in MPK: Role of η

- ▶ Consider 3 values of η
 - ▶ $\eta = 0$: No trade
 - ▶ $\eta = 1$: trading opportunity every year
 - ▶ $\eta = 12$: trading opportunity every month
- ▶ Standard deviation of log-MPK
 - Ranges from 40% to 90%



Business Wealth

- ▶ Finance textbook: Present value of owner's dividend
 - Model counterpart: $V(s)$

 - ▶ SCF respondent: Answer to the survey question—"What could you sell it for?"
 - Model counterpart: $\mathcal{P}(k)$
- ⇒ Both are inputs to analyses of business capital and wealth



Model Predictions for Business Wealth (in %)

STATISTIC	INCOME YIELD	TRANSFERABLE SHARE
5 th percentile	2.6	9.8
10 th	3.1	11.0
25 th	4.6	12.3
50 th	7.5	21.3
75 th	12.7	30.3
90 th	16.5	38.1
95 th	17.4	43.6
Average	8.7	23.1
Aggregate	13.6	27.0



Business Taxation

- ▶ Recent debate on business taxation
- ▶ What to tax
 - Flows: business income
 - Stocks: business capital (Güvener et al. 2022)
 - Transfers: capital gains (Sarin et al 2022, Agersnap and Zidar 2021)
- ▶ Our model can speak to all three forms of taxation



Tax Instruments

- ▶ Compare instruments:
 - **Capital gains:** $\tau_c \mathcal{P}(k)$
 - **Business income:** $\tau_b (y - wn - rb)$
 - **Business capital:** $\tau_k \mathcal{P}(k)$ [capital ownership]

- ▶ Compare several outcomes for a given level of revenue R



Business Taxation

- ▶ Revenue raised 1.5% of baseline output
 - ▶ Business income tax: $\tau_b = 4.8\%$
 - ▶ Capital gains tax: $\tau_c = 50\%$
 - ▶ Capital tax: $\tau_k = 2.7\%$
- ▶ Outcomes % change from baseline



Business Taxation: Outcomes

PREDICTED TAX POLICY CHANGES

STATISTIC	BUSINESS INCOME	CAPITAL STOCK	CAPITAL GAINS
Mass of firms	0.8	-7.1	-32.3
Fraction traded	3.8	-5.8	-64.6
Average investment	0.3	-0.9	-1.6
Dispersion in MPK	3.5	-2.5	2.2
Wage	-0.5	-1.3	-6.0



Main Result

- ▶ Taxing income dominates taxing gains
- ▶ Taxing capital gains
 - ▶ distorts capital reallocation across businesses
 - ▶ decreases entry
 - ▶ decreases investment by owners
 - ▶ lowers welfare
- ▶ Taxing business capital is between the two

Next: Discuss intuition by comparing taxing income versus gains



Main Result: Intuition

The main differences are driven by extensive margin of entry

- ▶ Incidence of taxes
 - business income: high productivity firms earn profits
 - capital gains: low productivity firms typically sell
- ▶ Distortionary effects on entry
 - ▶ business income: small because high productivity firms enter anyways
 - ▶ capital gains: large because low productivity firms are marginal entrants
- ▶ Distortionary effects on investment
 - ▶ business income: small because tax base is wider
 - ▶ capital gains: small because higher prices make productive firms substitute toward investment



Related Work in Progress

- ▶ Making the analysis “NIPA-ready”
- ▶ Adding features of noncompliance disciplined by IRS audit data
- ▶ Computing transitional dynamics following changes in tax policy/administration



The Macroeconomics of US Public Finance

- ▶ Concerns the levels and composition of
 - Public spending
 - Taxation
 - Debt

- ▶ I focused today on the taxation of private businesses

- ▶ But many open questions remain ...



Epilogue: A Pressing Open Question



What is the Optimum Quantity of Debt?

- ▶ Aiyagari-McGrattan (1998) assessed welfare impacts using model with
 - Idiosyncratic shocks to income
 - Borrowing constraints
- ▶ Quantified positive and negative roles of debt
 - Positive: enhancing liquidity by effectively loosening borrowing constraints
 - Negative:
 - Crowding out private capital
 - Distorting labor by requiring higher distortive taxes
- ▶ Found
 - Optimum debt-to-GDP roughly $2/3$
 - Small welfare impacts

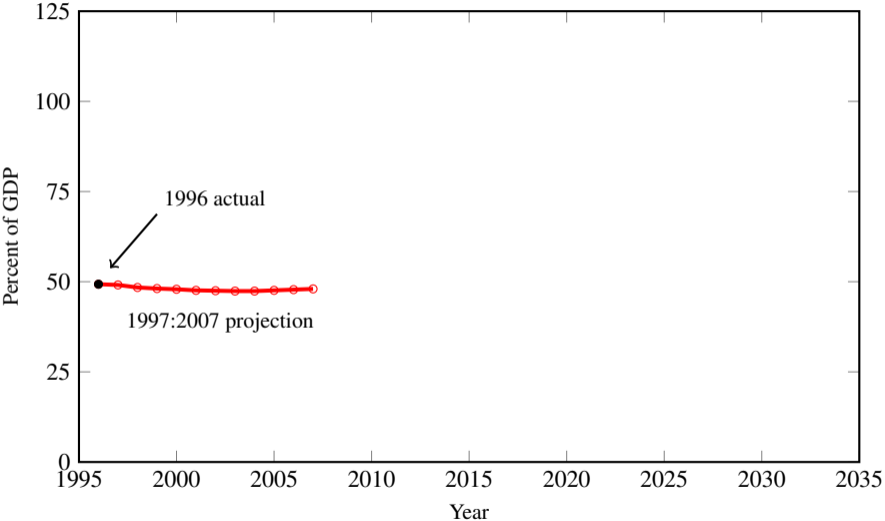


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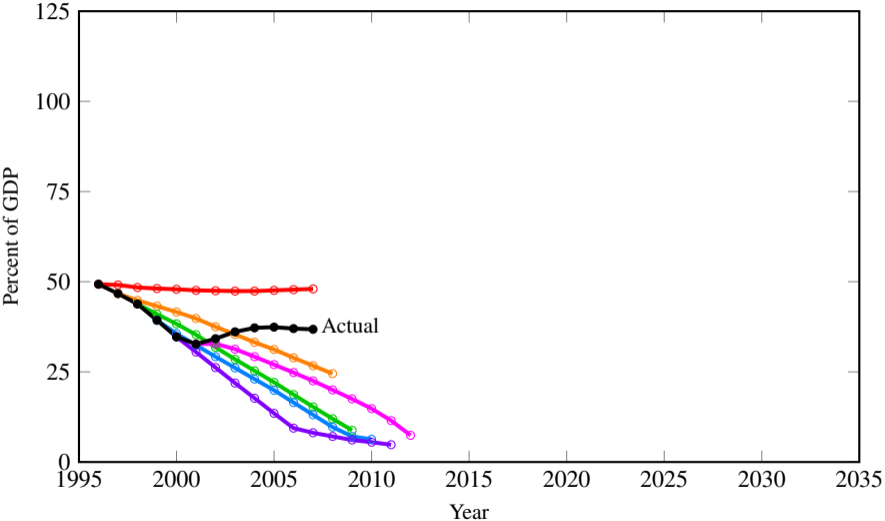
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 - Crowding out private capital
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- ▶ Found
 - Optimum debt-to-GDP roughly $2/3$
 - Small welfare impacts
- ▶ With updated theory and data, what would we conclude today?



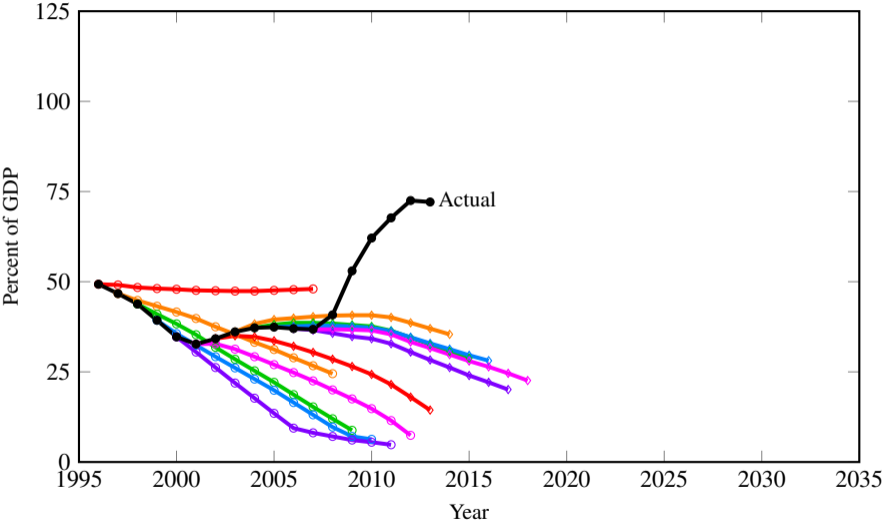
CBO 10-Year Debt-to-GDP Projections



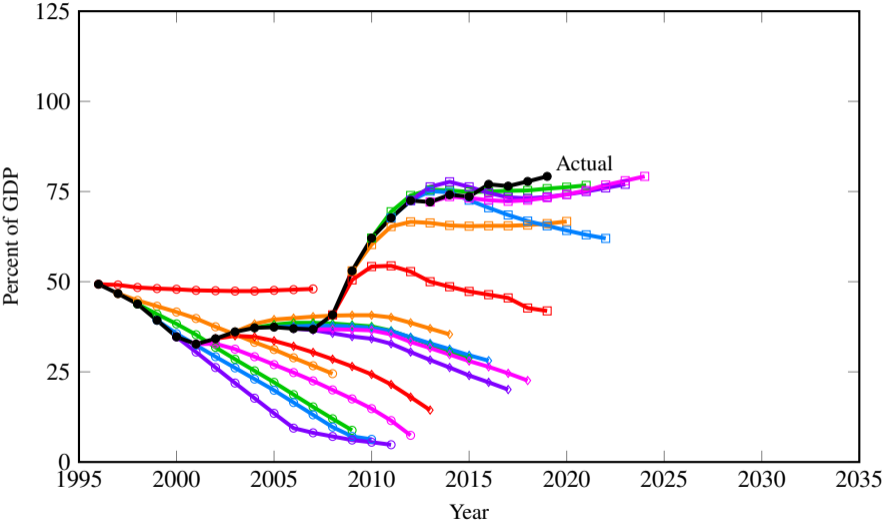
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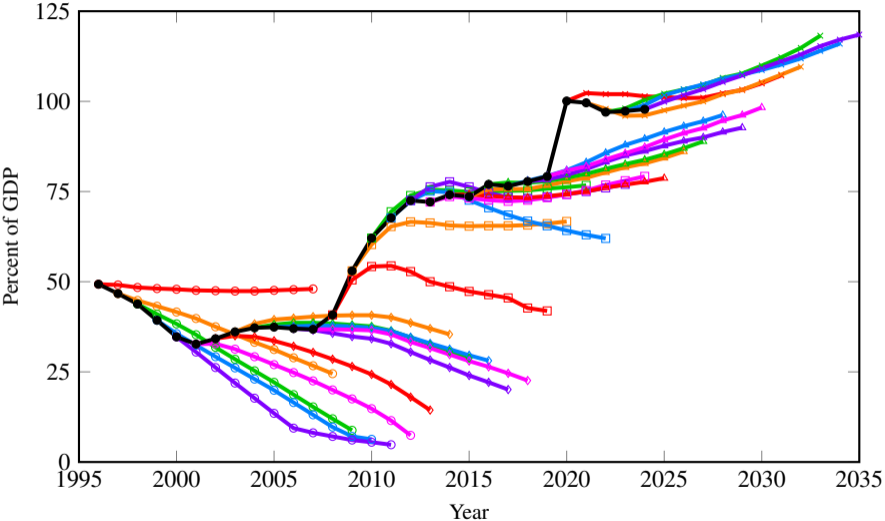
CBO 10-Year Debt-to-GDP Projections



CBO 10-Year Debt-to-GDP Projections



CBO 10-Year Debt-to-GDP Projections



Appendix



IPP Roughly Doubles NIPA Nonresidential Investment

► *Measured* investments (2023 \$, trillions):

Private structures	0.83 (20%)
Private equipment	1.41 (34%)
Private IPP	1.52 (37%)
Government IPP	0.35 (8%)
Total	4.11 (100%)

Continue

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Updated National Accounts

NIPA INCOME	NIPA PRODUCT
Capital consumption	Personal consumption
Taxes on production	Government consumption
Compensation less “sweat”	Government investment
Profits less “expenses”	Private tangible investment
Net interest	Net exports
+Sweat+expenses	+Intangible investments
= GDI	= GDP



IRS v. CPS Comparison

- ▶ IRS and CPS moments close for paid-employed
- ▶ CPS misses key moments for self-employed, especially
 - High mid-career earners in key industries (prof services, health)
 - Low early-career earners with business losses

Income Cutoff	CPS Share	IRS Share
< \$8,500	0.2	-8.1
> \$100,000	53.0	80.2

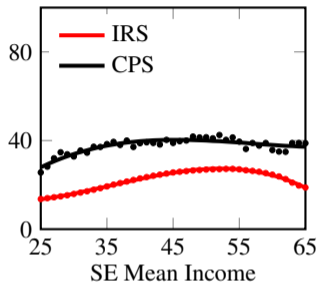
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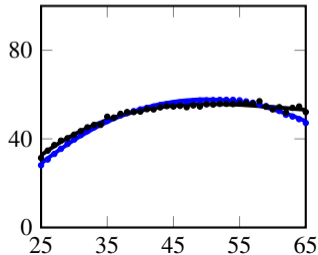
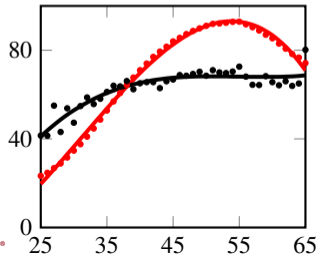
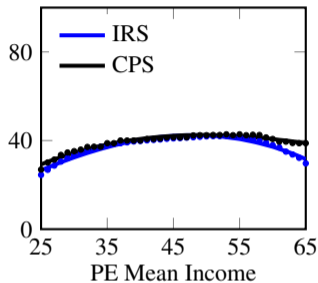


IRS v. CPS (Th. 2012\$)

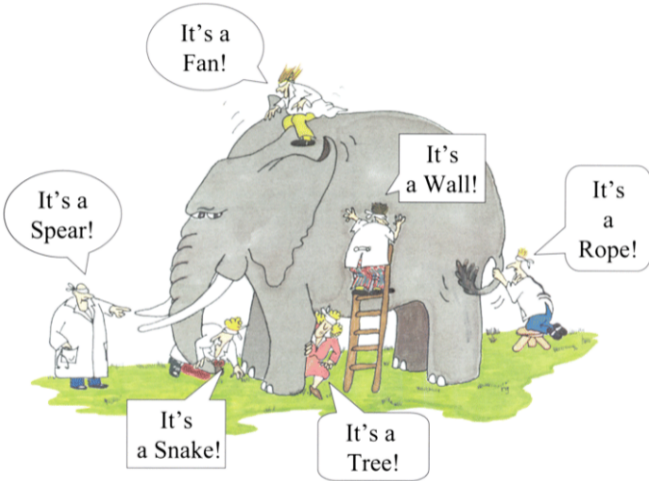
SE Median Income



PE Median Income



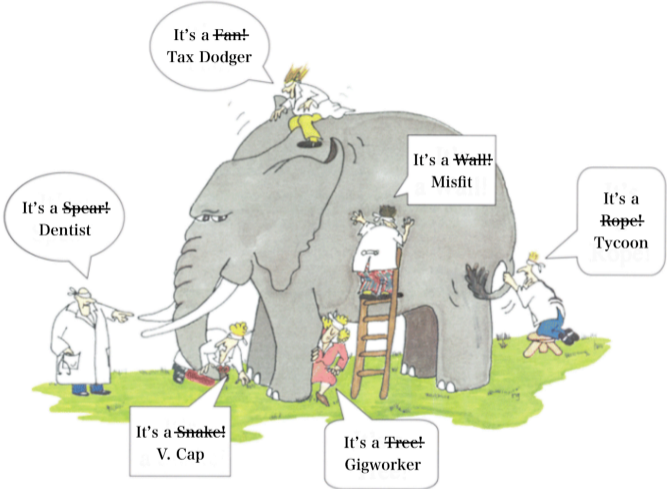
On the Nature of Entrepreneurship



Six Blind Men and an Elephant



On the Nature of Entrepreneurship



Six Blind Men and an Elephant Surveys and Entrepreneurship



Who Trades with Whom?

- ▶ Intuitive example:
 - Simple production function: $y = zk$
 - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
 - Capital pre-trade: all have $k = 1$

- ▶ Efficient allocation implies:
 - 10 low types sell to 10 of the high types



How are Terms of Trade Determined?

- ▶ Intuitive example:
 - Simple production function: $y = zk$
 - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
 - Capital pre-trade: all have $k = 1$

- ▶ Price leaves high types indifferent between:
 - Trading with $k = 2$ post-trade
 - Not trading with $k = 1$ post-trade



Equilibrium Policy Functions

► Intuitive example:

- Simple production function: $y = zk$
- Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
- Capital pre-trade: all have $k = 1$

► Equilibrium:

- Capital allocations: $k^m(s_H, s_L) = 2$, $k^m(s_L, s_H) = 0$
- Prices: $p^m(s_H, s_L) = 1$, $p^m(s_L, s_H) = -1$
- Choice probabilities: $\lambda(s_H | s_L) = 1$, $\lambda(s_L | s_H) = 1/2$, $\lambda_o(s_H) = 1/2$, $\lambda_o(s_L) = 0$



Auxiliary Problem: Static Planner

Define gains from trade between s, \tilde{s} :

$$X(s, \tilde{s}) = \max_{k^m \in \{k(s) + k(\tilde{s}), 0\}} \{V(z(s), k^m) + V(z(\tilde{s}), k(s) + k(\tilde{s}) - k^m)\} - (V(s) + V(\tilde{s}))$$

$$Q(\phi, V) = \max_{\pi \geq 0} \sum_{s, \tilde{s}} X(s, \tilde{s}) \pi(s, \tilde{s})$$

$$s.t. \quad \sum_{\tilde{s}} \pi(s, \tilde{s}) + \pi(s, 0) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^a(s)]$$

$$\sum_{\tilde{s}} \pi(\tilde{s}, s) + \pi(0, s) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^b(s)]$$

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Auxiliary Problem: Static Planner

► Mapping to equilibrium objects:

- $W(s) = \frac{\partial Q}{\partial \phi(s)} = \frac{\mu^a(s) + \mu^b(s)}{2} \equiv \mu(s)$
- $\lambda(s, \tilde{s}) = \frac{2\pi(s, \tilde{s})}{\phi(s)}$
- $k^m(s, \tilde{s}) = \arg \max X(s, \tilde{s})$
- $p^m(s, \tilde{s}) = V(z, k^m(s, \tilde{s})) - V(z, k) - W(s)$

► Multipliers $\mu = \mu^a = \mu^b$ capture gains from trade: $\mu = \nabla_{\phi} Q$

► Prices implement gains from trade: $p^m(s, \tilde{s}) = V(z(s), k^m(s, \tilde{s})) - \mu(s)$



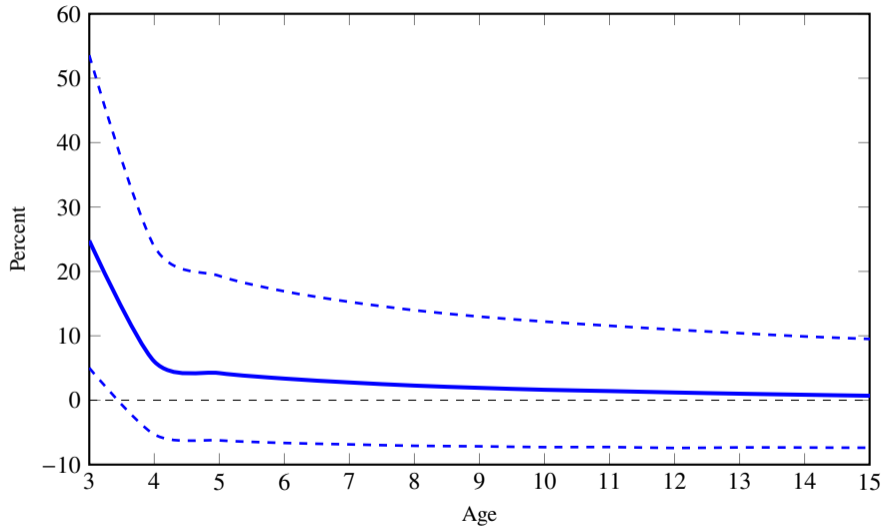
Dual Problem

- ▶ From the minimax thm, the solution of the primal problem is equal to the solution of the dual
- ▶ The multipliers in the primal are equal to the choice variable in the dual, and vice versa

$$Q(\phi) = \min_{\mu^a \geq 0, \mu^b \geq 0} \sum_s \left(\mu^a(s) + \mu^b(s) \right) \frac{\phi(s)}{2}$$
$$s.t. \quad \mu^a(s) + \mu^b(\tilde{s}) \geq X(s, \tilde{s}) \quad \forall s, \tilde{s} \quad [\pi(s, \tilde{s})]$$



Distribution of Annualized 3-Year Growth for S Corporations by Age



Summary Statistics for S Corporations

STATISTIC	PERCENTILES		
	25 th	50 th	75 th
Business Age	8.0	13.0	21.0
Wage Growth	-6.8	1.4	11.6
Log Wage Bill: Entrants	11.0	11.7	12.5
Population	11.1	11.9	12.8

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Summary Statistics for S Corporation Sellers

STATISTIC	PERCENTILES		
	25 th	50 th	75 th
Valuation Ratios			
Sales to S corporations	1.0	2.4	5.2
Partnerships	1.4	3.5	8.6
C corporations	1.5	4.0	9.9
All sales	1.7	2.9	6.7
Relative Wage Bill Sizes			
Sales to S corporations	0.7	1.4	5.6
Partnerships	1.0	2.8	17.4
C corporations	2.2	14.9	130.7
All sales	0.9	2.1	13.5



Buyer and Seller Wage Bills By Seller Size

