

# The X Factor for Business Taxation

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**X fac·tor** /'ɛks ,fæktər/

*noun*

- 1 a noteworthy special talent or quality
- 2 a variable in a given situation that could have the most significant impact on the outcome



# What is the X Factor for Business Taxation?

- ▶ Intangible investment:
  - R&D, software, data in the very large corporations
  - Customer bases, workforce, goodwill in smaller businesses
  
- ▶ Why this *X* is the *X factor*:
  - Important source of business income and value
  - But notably hard to measure
  - And likely significant for studying impacts of business taxation



## Noteworthy Qualities of Multinationals' Intangible Assets

- ▶ Mobile across tax jurisdictions
  - ▶ Uncertain useful lives
  - ▶ Limited secondary markets
  - ▶ Possible knowledge spillovers
- ⇒ Implications for transfer pricing, expensing, tax credits



## Noteworthy Qualities of Intangibles in Smaller Businesses

- ▶ Created by owners
- ▶ Mostly with their time
- ▶ Not recorded on the balance sheet

⇒ Implications for capital reallocation and private business taxation



Recent Work with Anmol Bhandari and Paolo Martellini  
“Capital Reallocation and Private Firm Dynamics”



# Why Focus on Private Businesses?

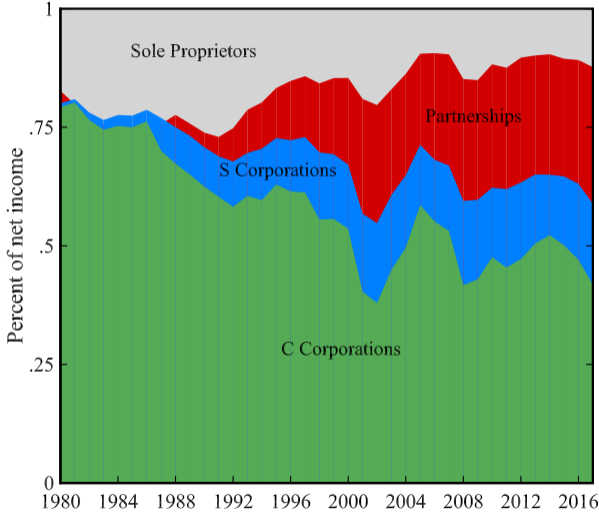


## Why Focus on Private Businesses?

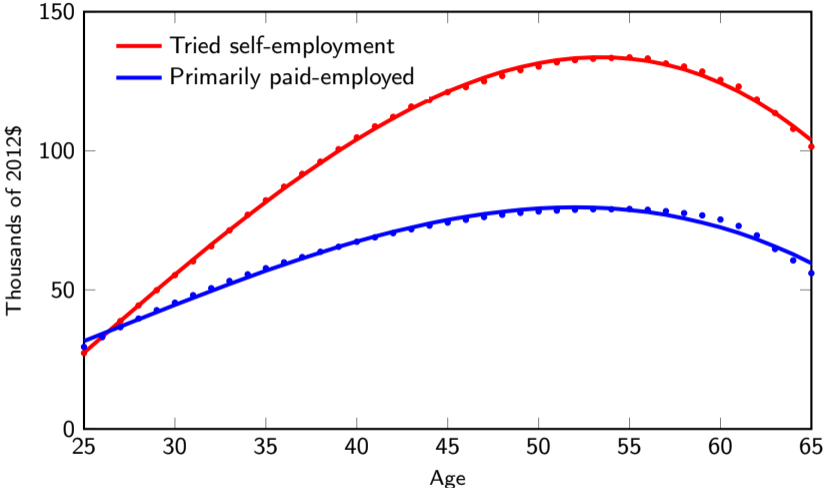
- ▶ Account for more than 1/2 of US business net income
- ▶ Dominate discussions on growth, wealth inequality, tax policy
- ▶ But pose challenge for
  - o Theory: technology of capital accumulation and reallocation
  - o Measurement: lack of reliable data on capital used in private businesses



# Private Business Share is Growing

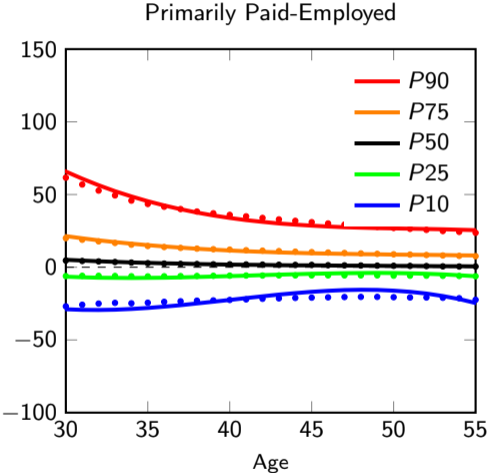
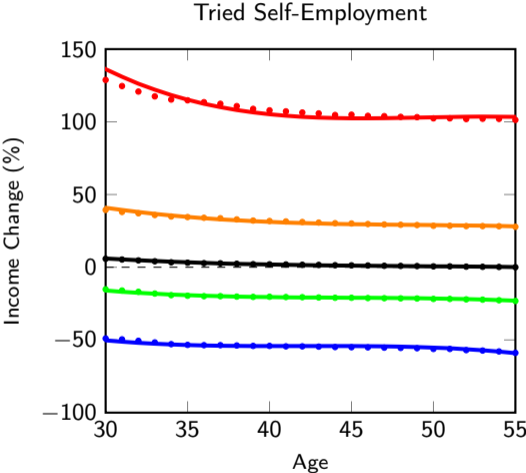


# Owners are Earning Higher Average Incomes



Source: Bhandari et al. (2025), "On the Nature of Entrepreneurship"

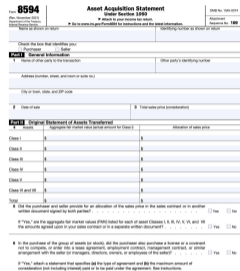
# Owners Take on More Risk



Source: Bhandari et al. (2025), "On the Nature of Entrepreneurship"

# Assets Generating Income/Wealth Not Observed Until Transferred

- ▶ Transferred assets are primarily **intangible** (from Form 8594  $\approx 70\%$ )
  - o Customer bases and client lists, non-compete covenants
  - o Trademarks and tradenames
  - o Licenses and permits
  - o Workforce in place
  - o Goodwill and on-going concern value
- ▶ Assets are **sold as a group**
- ▶ Sale **requires time** to find buyers/negotiate (typical from brokered data  $\approx 7$  months)



Form **8594** **Asset Acquisition Statement** (Under Section 338) (OMB No. 1545-0047)

Buyer (Name, EIN, and address)  Seller (Name, EIN, and address)

Check the box that identifies your transaction:

**1** General Substantive: Name of other party to the transaction: \_\_\_\_\_ Other party's identifying number: \_\_\_\_\_  
Address (number, street, and room or suite no.): \_\_\_\_\_  
City or town, state, and ZIP code: \_\_\_\_\_

**2** Cash sale  **3** Transfer price determination

**Part II - Organize Statement of Assets Transferred**

Class	Aggregate fair market value (FMV) of assets (grouped as to Class)	Allocation of sales price
Class I	\$ _____	\$ _____
Class II	\$ _____	\$ _____
Class III	\$ _____	\$ _____
Class IV	\$ _____	\$ _____
Class V	\$ _____	\$ _____
Class VI and VII	\$ _____	\$ _____
Total	\$ _____	\$ _____

**4** Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document agreed by both parties?  Yes  No

**5** "Yes," are the aggregate fair market values (FMV) listed for each of asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document?  Yes  No

**6** In the purchase of the group of assets (or assets), did the purchaser also purchase a license or a contract not to compete, or enter into a sales agreement, management contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)?  Yes  No

**7** "Yes," attach an statement that specifies (a) the type of agreement and (b) the maximum amount of consideration paid resulting therefrom, paid or to be paid under the agreement. See instructions.

← Cash/securities  
← Inventories  
← Fixed assets  
← Sec. 197 intangibles



## What We Do

- ▶ Develop new theory of private firm dynamics
  - o Incorporate properties of capital currently used in private business
    - Capital not observable (until transacted), requires time to trade, sold as a group
  - o Contrast with “traditional” theory of firm dynamics (Lucas-Hopenhayn)
    - Capital is observable, rented and traded divisibly
- ▶ Parameterize model with IRS data
  - o Novel data on business transfers (Form 8594)
  - o Panel data on business incomes and payroll
- ▶ Compare tax policy instruments (income vs. capital vs. gains)



## Model Environment

- ▶ Infinite horizon with continuous time
- ▶ Demographics
  - Unit mass of individuals
  - Birth/death at constant rate  $\psi_e$
- ▶ Individuals choose
  - Paid-employment, supplying labor inelastically or
  - Self-employment, running a private business
- ▶ Preferences risk neutral, discount rate  $\rho$



## Production Technology

- ▶ Final goods technology:  $y = zk^\alpha b^\beta n^\gamma$
- ▶ Non-transferable productivity,  $z$ 
  - Evolves in business:  $dz = \mu(z)dt + \sigma(z)dW$
- ▶ Transferable business capital,  $k$ 
  - Built through investment:  $dk = (x - \delta_k k) dt$ , convex cost  $c(x) = Ax^{1+\chi}/(1 + \chi)$
- ▶ Rentable factors: labor,  $n$ , and buildings and equipment,  $b$ 
  - Investment in other fixed assets:  $db = (x_b - \delta_b b) dt$



## Entry and Exit Technology

Let  $s = (z, k)$  be the state of an individual

- ▶ Birth/death:
  - Birth and death at rate  $\psi_e$
  - Draw  $s \sim G(s)$ , where  $s = (z, k = 0)$
  
- ▶ Occupation choice:
  - Option to exit at rate  $\psi_o$
  - Cost of starting a business  $c_e$



# Market for Transferable Business Capital

- ▶ Businesses access market at Poisson rate  $\eta$
- ▶ Bilateral trades for pair  $s, \tilde{s}$ :
  - o Feasible allocations:  $k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\} \Rightarrow$  indivisibility  
(extension w/ costly divisibility)
  - o Transfers (prices):  $p^m(s, \tilde{s})$ , negative if selling  
(extension w/ financing constraints:  $p^m(s, \tilde{s}) \leq \xi y(s, n, b)$ )



## Owner's Value Solves HJB

$$\begin{aligned}(\rho + \psi_e)V(s) = & \underbrace{\max_{n,b} y(s, n, b) - wn - rb}_{\text{production}} \\ & + \underbrace{\max_x \partial_k V(s)(x - \delta_k k) - c(x)}_{\text{investment}} + \underbrace{\max_{\lambda} \eta V^{\text{trade}}(s; \lambda)}_{\text{trade}} \\ & + \underbrace{\mu(z)\partial_z V(s) + \frac{1}{2}\sigma(z)^2\partial_{zz} V(s)}_{\text{evolution of productivity}} + \underbrace{\psi_o \max\{W - V(s), 0\}^+}_{\text{endogenous exit}}\end{aligned}$$

where

$$V^{\text{trade}}(s; \lambda) = \int [V(z, k^m(s, \tilde{s})) - V(z, k) - p^m(s, \tilde{s})] \lambda(s, \tilde{s}) ds$$

$$\int \lambda(s, \tilde{s}) d\tilde{s} + \lambda(s, 0) = 1$$



## Definition of Recursive Equilibrium

A *stationary recursive equilibrium* is a set of value functions  $V(s)$ ,  $W$ , policy functions for investment  $x(s)$ , occupation choice  $\iota^{\text{in}}(s)$ ,  $\iota^{\text{out}}(s)$ , trade  $\lambda(s, \tilde{s})$ , terms of trade  $(k^m(s, \tilde{s}), p^m(s, \tilde{s}))$ , wage  $w$ , rental rate  $r$ , distribution  $\phi(s)$ , and mass of firms  $m$  that satisfy:

- ▶ Owners' and workers' optimality
- ▶ Labor market clearing
- ▶ Trading arrangements that are:
  - Mutually consistent:  $\lambda(s, \tilde{s})\phi(s) = \lambda(\tilde{s}, s)\phi(\tilde{s})$
  - Feasible:  $k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\}$
  - $\nexists (s, \tilde{s})$  and feasible trade that makes the pair strictly better off
- ▶ Stationary measure over owner types

Details



## An Intuitive Example



## Who Trades with Whom?

- ▶ Intuitive example:
  - Simple production function:  $y = zk$
  - Productivity types: 20 with  $z_H = 1$ , 10 with  $z_L = 0$
  - Capital pre-trade: all have  $k = 1$
  
- ▶ Efficient allocation implies:
  - 10 low types sell to 10 of the high types



## How are Terms of Trade Determined?

- ▶ Intuitive example:
  - Simple production function:  $y = zk$
  - Productivity types: 20 with  $z_H = 1$ , 10 with  $z_L = 0$
  - Capital pre-trade: all have  $k = 1$
  
- ▶ Price leaves high types indifferent between:
  - Trading, with  $k = 2$  post-trade
  - Not trading, with  $k = 1$  post-trade



## Equilibrium Policy Functions

- ▶ Intuitive example:
  - Simple production function:  $y = zk$
  - Productivity types: 20 with  $z_H = 1$ , 10 with  $z_L = 0$
  - Capital pre-trade: all have  $k = 1$
  
- ▶ Equilibrium in this example:
  - Capital allocations:  $k^m(s_H, s_L) = 2$ ,  $k^m(s_L, s_H) = 0$
  - Prices:  $p^m(s_H, s_L) = 1$ ,  $p^m(s_L, s_H) = -1$
  - Choice probabilities:  $\lambda(s_L, s_H) = 1$ ,  $\lambda(s_H, s_L) = 1/2$ ,  $\lambda_o(s_L) = 0$ ,  $\lambda_o(s_H) = 1/2$



## Solution More Generally

Details

- ▶ Given  $(\phi, V)$ , solve standard linear programming problem
  - Maximize social surplus over all possible trading pairs
  - Use solution to derive equilibrium objects:
    - Prices:  $p^m$
    - Allocations:  $\lambda, k^m$
    - Gains from trade:  $V^{\text{trade}}(s)$
- ▶ Update:  $(\phi, V) \rightarrow (\lambda, p^m, k^m) \rightarrow (\phi', V')$
- ▶ Easy to extend to non-transferable utility



## Properties of the Equilibrium

- ▶ Prices only depend on seller's  $k$ :

$$p^m(s, \tilde{s}) = \mathcal{P}(k(\tilde{s}))$$

- o If  $z$  non-transferable, then equivalence between:
    - Our trading protocol for capital and
    - Competitive equilibrium
      - + unit demand over differentiated products (indexed by  $k$ )
      - + intermittent market access
  - o Can extend to allow consulting contracts  $\Rightarrow z$  impacts price
- 
- ▶ Allocations solve planner's problem starting at  $\phi(s, t = 0) = \phi^{ss}(s)$



## Next Up

- ▶ Calibration with IRS data on
  - Tax returns
  - Business transfers
- ▶ Using the model
  - Predictions of dispersion in MPKs, returns, wealth
  - Analysis of business taxation



# Measurement



# IRS Data

- ▶ Sample period: 1996 to 2022
- ▶ An observation is firm-year pair with
  - o Subchapter S corporation filing (Form 1120S)
  - o Wage bill  $> 10K$
  - o At least 3 years of data
- ▶ Two samples:
  - o *Full* sample: all firm-year pairs meeting above criteria
  - o *Trading* sample: seller-buyer pairs with
    - Seller: meeting above criteria and counterparty on Form 8594
    - Buyer: counterparty on Form 8594 with wage bill  $> 10K$
- ▶ Key moments: distributions of age, growth; sales price; buyer/seller relative size

Details

Details



## Identification Strategy

- ▶ From *traditional firm dynamics* moments (life-cycle growth, mass and age distribution):
  - Entry distribution and entry cost
  - Stochastic process for productivity
  - Rentable input shares
  
- ▶ From *novel trading* moments (sale price, relative size of buyers/sellers):
  - Capital elasticity of output: larger  $\alpha \Rightarrow$  smaller relative size
  - Investment cost: larger  $A \Rightarrow$  higher valuation for capital
  - Speed of transaction  $\eta$

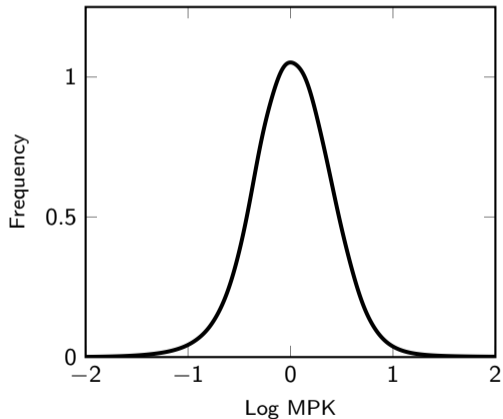


## Model Predictions



## Dispersion in MPK

- ▶ Idiosyncratic change in productivity  $\rightarrow$  input reallocation toward higher MPK
- ▶ Dispersion in marginal product of capital induced by
  - Decentralized trading
  - Indivisibility of asset sold
- ▶ Standard deviation of log-MPK: 40%

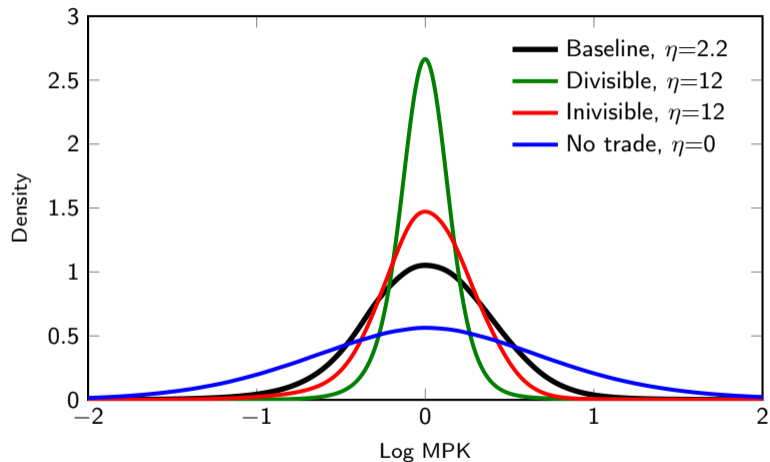


## Dispersion in MPK: Role of Trading Frequency and Indivisibility

- ▶ Consider 3 values of  $\eta$ 
    - $\eta = 0$ : no trade
    - $\eta = 2.2$ : trading opportunity every 7 months
    - $\eta = 12$ : trading opportunity every month
  
  - ▶ Allow any amount of capital for sale (“divisible trade”)
- ⇒ Standard deviation of log-MPK
- Ranging from 30% to 77%



## Dispersion in MPK: Role of Trading Frequency and Indivisibility



## Business Wealth: Two Measures

- ▶ Finance textbook: Present value of owner's dividend
  - Model counterpart:  $V(s)$
- ▶ SCF respondent: Answer to the survey question—"What could you sell it for?"
  - Model counterpart:  $\mathcal{P}(k)$



## Business Wealth: Two Measures

- ▶ Finance textbook: Present value of owner's dividend
  - Model counterpart:  $V(s)$
- ▶ SCF respondent: Answer to the survey question—"What could you sell it for?"
  - Model counterpart:  $\mathcal{P}(k)$
- ▶ Theory and measurement:
  - Both are inputs to analyses of business capital and wealth
  - Neither are on private business balance sheets (until  $\mathcal{P}(k)$  amortized)



## Model Predictions for Business Wealth (in %)

► Model generates heterogeneity in

- Business returns
- Transferable shares

► Implications:

- Capitalization method won't work
- Surveys not capturing all value
- Aggregate values:

$$\int \mathcal{P}(k(s)) ds = 60\% \times \text{output}$$

$$\int V(s) ds = 266\% \times \text{output}$$

%TILE	INCOME YIELD	TRANSFERABLE SHARE
5 <sup>th</sup>	2.1	6.8
50 <sup>th</sup>	5.4	14.1
95 <sup>th</sup>	14.8	39.1



# Tax Analysis



# Business Taxation

- ▶ Recent debate on business taxation
- ▶ What to tax?
  - o Flows: business income
  - o Stocks: business capital (Guvenen et al. 2023)
  - o Transfers: capital gains (Sarin et al. 2022, Agersnap and Zidar 2021)
- ▶ Our model can speak to all three forms of taxation



# Tax Instruments

- ▶ Compare instruments:
  - Business income:  $\tau_b(y - wn - rb)$
  - Business capital:  $\tau_k \mathcal{P}(k)$  [capital ownership]
  - Capital gains:  $\tau_c \mathcal{P}(k)$  [capital transfer]
  
- ▶ Compare outcomes for a given level of revenue



## Business Taxation

- ▶ Revenue raised: 1.2% of baseline output
  - Business income tax:  $\tau_b = 4.1\%$
  - Capital gains tax:  $\tau_c = 20\%$
  - Capital tax:  $\tau_k = 2.4\%$
  
- ▶ Outcomes: % change from baseline



## Main Results: % Changes from Baseline

▶ Taxing income dominates taxing capital

▶ Taxing capital gains

○ Distorts capital reallocation

○ Decreases owner investment

○ Decreases entry

○ Lowers welfare

▶ Taxing capital value in between

STATISTIC	BUSINESS INCOME	CAPITAL VALUE	CAPITAL GAINS
Mass of firms	1.8	-2.0	-6.5
Fraction traded	0.7	-1.3	-48.1
Avg. investment	0.3	0.3	-3.2
MPK dispersion	-0.4	-0.4	22.0
Per-unit price	0.2	1.0	7.7
Wage	-0.4	-2.9	-3.0

⇒ Findings contrary to recent literature



## Recap

- ▶ Need for new theory and measurement to study private firms
- ▶ We make progress on both fronts using
  - Model that captures features of market for private businesses
  - Novel data combining value of intangible capital with firm dynamics
- ▶ Business income (capital gains) taxes are least (most) distortionary



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⇒ What are current estimates for optimal tax rate on business income?



Current Work with Anmol Bhandari, David Evans, Yuki Yao  
“Income Taxation and Entrepreneurship”



## What is the Optimal Tax Rate on Business Income?

- ▶ Wide range of estimates in literature
  - ▶ 0% or less if financial constraints bind or capital is misallocated
  - ▶ 60% or more if greater insurance and redistribution needed
- ▶ Abstracting from the X factor



# What is the Optimal Tax Rate on Business Income?

- ▶ To answer this
  - Introduce the X factor into the analysis
  - Calibrate the model using NIPA and IRS income data
  - Compute transitional firm dynamics
  - Compare results to current estimates in literature and US
  
- ▶ What do preliminary results show?

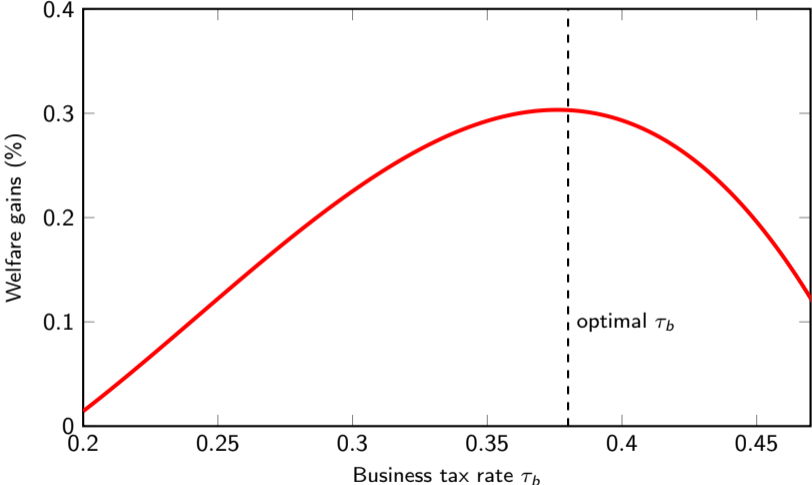


## What Preliminary Results Show

- ▶ Optimal rate close to what owners should pay *if compliant*
  - Current US tax rate is 20% because 1/2 of income misreported
  - Our estimate is about 38% which is about they owe to IRS
  
- ▶ Elasticities are critical (always!) and depend on
  - Introducing sweat capital
  - Matching IRS income data (above) for business owners
  - Matching data on actual borrowing levels and capital rents



# Welfare Gain Estimates



## The X Factor in Business Taxation

*Not everything that can be counted counts, and not everything that counts can be counted.*

— William Bruce Cameron



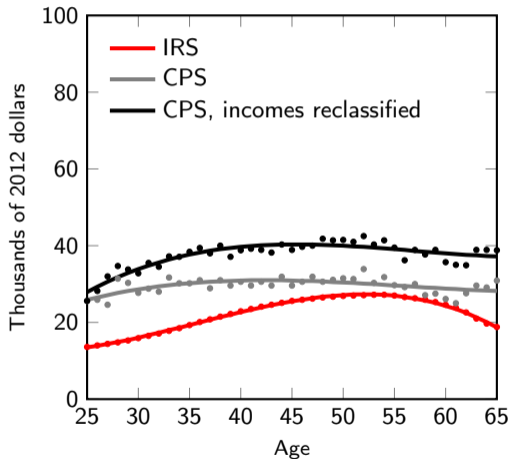
# Appendix



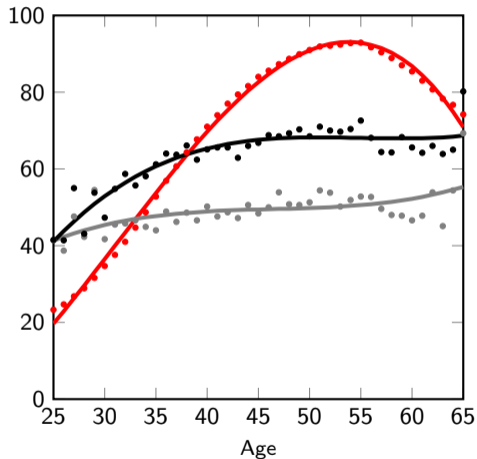
# CPS Misses Both Tails

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### Self-Employed Median Income



### Self-Employed Mean Income



# Business Transfers are Taxable Events

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- ▶ Buyers and sellers both report sale
  - o Seller has to pay capital gains
  - o Buyer has to report depreciable assets
- ▶ Price allocated across asset types
  - o Seller wants to allocate to long-term
  - o Buyer wants to allocate to short-term

⇒ Conflict of interest and thus consistent reporting



Define gains from trade between  $s, \tilde{s}$ :

$$X(s, \tilde{s}) = \max_{k^m \in \{k(s) + k(\tilde{s}), 0\}} \{V(z(s), k^m) + V(z(\tilde{s}), k(s) + k(\tilde{s}) - k^m)\} - (V(s) + V(\tilde{s}))$$

$$Q(\phi, V) = \max_{\pi \geq 0} \sum_{s, \tilde{s}} X(s, \tilde{s}) \pi(s, \tilde{s})$$

$$\text{s.t.} \quad \sum_{\tilde{s}} \pi(s, \tilde{s}) + \pi(s, 0) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^a(s)]$$

$$\sum_{\tilde{s}} \pi(\tilde{s}, s) + \pi(0, s) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^b(s)]$$



► **Lemma**

○  $V^{\text{trade}}(s) = \frac{\partial Q}{\partial \phi(s)} = \frac{\mu^a(s) + \mu^b(s)}{2} \equiv \mu(s)$

○  $\lambda(s, \tilde{s}) = \frac{2\pi(s, \tilde{s})}{\phi(s)}$

○  $k^m(s, \tilde{s}) = \arg \max X(s, \tilde{s}) \quad p^m(s, \tilde{s}) = V(z, k^m(s, \tilde{s})) - V(z, k) - V^{\text{trade}}(s)$

► Multipliers capture gains from trade,  $\mu = \nabla_{\phi} Q$

► Prices implement gains from trade,  $p^m(s, \tilde{s}) = V(z(s), k^m(s, \tilde{s})) - \mu(s)$



- ▶ From minimax theorem, solution of primal problem = solution of dual
- ▶ Multipliers in primal = choice variables in dual, and vice versa

$$Q(\phi) = \min_{\mu^a \geq 0, \mu^b \geq 0} \sum_s \left( \mu^a(s) + \mu^b(s) \right) \frac{\phi(s)}{2}$$
$$s.t. \quad \mu^a(s) + \mu^b(\tilde{s}) \geq X(s, \tilde{s}) \quad \forall s, \tilde{s} \quad [\pi(s, \tilde{s})]$$



- ▶ Under cap on paid price equal to  $\xi y(s, n, b)$

$$v_b(s; \hat{k}) = \begin{cases} V(z, k(s) + \hat{k}) - p(\hat{k}) & \text{if } p(\hat{k}) \leq \xi y(s, n, b) \\ -\infty & \text{otherwise} \end{cases}$$

$$v_s(s) = V(\tilde{s}, 0) + p(k(s))$$

- ▶ Under capital gain tax  $\tau$ ,

$$v_b(s; \hat{k}) = V(z, k(s) + \hat{k}) - p(\hat{k})$$

$$v_s(s) = V(\tilde{s}, 0) + (1 - \tau)p(k(s))$$



- ▶ After-trade values for buyers ( $v_b$ ) and sellers ( $v_s$ )
  - ▶  $v_b(s, \hat{k}; p)$ : value from buying  $\hat{k}$
  - ▶  $v_s(s, 0; p)$ : value from selling  $k(s)$
- ▶ Matching probability

$$\lambda(s, \hat{k}; p) = \exp\left(\frac{v_b(s, \hat{k}; p) - W(s)}{\sigma}\right)$$

$$\lambda(s, 0; p) = \exp\left(\frac{v_s(s, 0; p) - W(s)}{\sigma}\right)$$

where  $W(s) = \mathbb{E} \max\{v_b(s, \hat{k}; p), v_s(s, 0; p)\}$

- ▶ Find  $\{p(s)\}$  such that  $\forall \hat{k}$

$$\underbrace{\int \lambda(s, \hat{k}; p)}_{\text{demand}} = \underbrace{\int \lambda(s, 0; p) \mathbb{I}\{k(s) = \hat{k}\}}_{\text{supply}}$$



BUSINESS SUBSAMPLES	COUNTS
S corporation population	3,167,266
S corporation sellers	105,162
Sales to S corporations	46,708
to Partnerships	33,462
to C corporations	35,792
Seller-buyer pairs	51,286
S Corporation–S Corporation	28,078
–Partnership	14,040
–C Corporation	9,168



## Intangible Intensities of US S Corporation Trading Sample

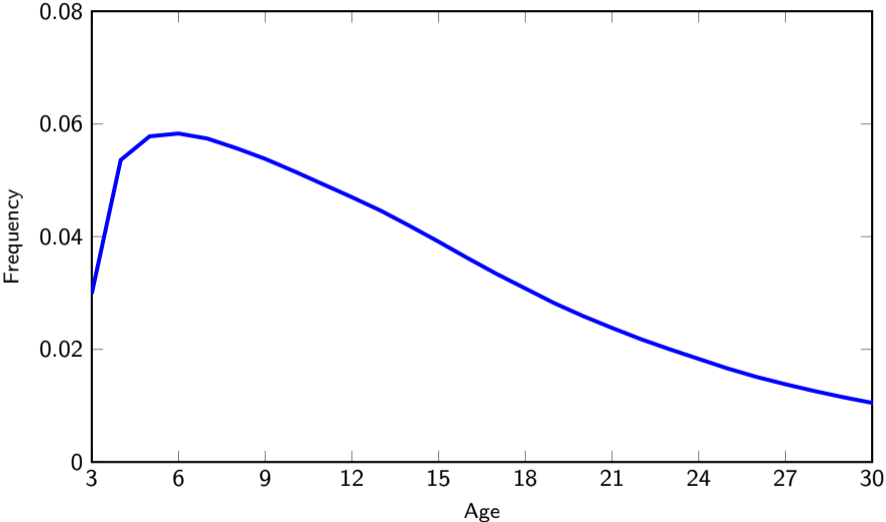
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INTANGIBLE INTENSITIES	PERCENTILES		
	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>
Sales to S corporations	29.2	66.8	87.0
Partnerships	33.3	71.0	90.9
C corporations	39.3	73.4	92.7
All sales	32.1	69.3	89.3



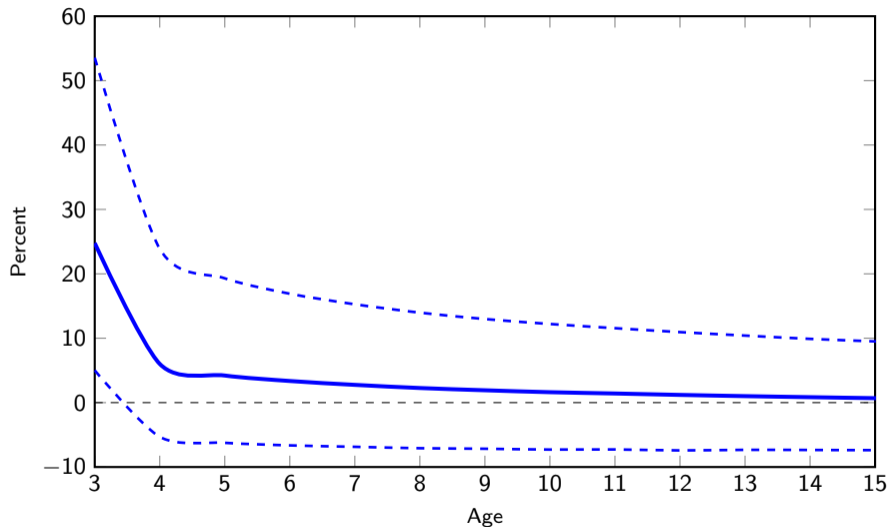
# Age Distribution: Sample of U.S. S Corporations

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# Distribution of Annualized 3-Year Growth by Age: Sample of U.S. S Corporations

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STATISTIC	PERCENTILES		
	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>
Business Age	8.0	13.0	21.0
Wage Growth	-6.8	1.4	11.6
Log Wage Bill: Entrants	11.0	11.7	12.5
Population	11.1	11.9	12.8



STATISTIC	PERCENTILES		
	25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>
Valuation Ratios			
Sales to S corporations	1.0	2.4	5.2
C corporations	1.4	3.5	8.6
Partnerships	1.5	4.0	9.9
All sales	1.2	2.9	6.7
Relative Wage Bill Sizes			
Sales to S corporations	0.7	1.4	5.6
C corporations	1.0	2.8	17.4
Partnerships	2.2	14.9	130.7
All sales	0.9	2.1	13.5



# Buyer and Seller Wage Bills By Seller Size

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► Let

- o  $\phi$  = measure over owner types  $s$
- o  $m$  = be the mass of firms
- o  $\iota^{\text{in}}(s) \equiv \{V(s) - W - c_e w > 0\}$  = entry policy
- o  $\iota^{\text{out}}(s) \equiv \{W - V(s) > 0\}$  = exit policy

► Then

- o  $\dot{\phi} = \Gamma(\phi; x, \lambda)$
- o  $\dot{m} = \psi_e \int \iota^{\text{in}}(s) dG(s) - m(\psi_e + \psi_o \int (1 - \iota^{\text{out}}(s)) \phi(s))$



► Production

$$y(s, n, b) = z(s)k(s)^\alpha n^\gamma b^\beta, \quad \alpha = 0.125, \beta = \gamma = 0.35$$

► Investment

$$\text{Cost } c(x) = Ax^{1+\chi}/(1+\chi), \quad A = 1833, \chi = 1.6$$

$$\text{Depreciation, } \delta_k = \delta_b = 0.1$$

► Productivity

$$\text{Entry distribution } G(z) = LN(\mu_0, \sigma_0) \text{ on } 1 + \log z, \quad \mu_0 = -0.5, \sigma_0 = 0.5$$

$$\text{Post-entry process } d \log z = \theta(1 - z)dt + \sigma dW, \quad \theta = \sigma = 0.1$$

► Rates

$$\text{Discounting, } \rho = 0.05$$

$$\text{Trading, } \eta = 2.2$$

$$\text{Entry/exit, } \psi_e = 1/40, c_e = 4.0, \psi_o = 1.0$$



MOMENT	MODEL	DATA
Fraction of owners	0.16	0.18
Business age, median	8.9	10.0
mean	11.8	12.0
Growth, age 3, median	0.27	0.25
age 3, IQR	0.48	0.49
age 10, median	0.03	0.02
age 10, IQR	0.27	0.20
Log wagebill, population, IQR	1.60	1.70
entrants, IQR	1.14	1.50
Valuation ratio, 25 <sup>th</sup>	3.86	1.27
50 <sup>th</sup>	4.25	3.13
75 <sup>th</sup>	4.75	7.47
Relative size, 25 <sup>th</sup>	1.84	0.90
50 <sup>th</sup>	2.68	2.33
75 <sup>th</sup>	3.84	13.5

