Macroeconomic Effects of Financial Shocks

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We document the cyclical properties of US firms’ financial flows and show that equity payout is procyclical and debt payout is countercyclical. We then develop a model with debt and equity financing to explore how the dynamics of real and financial variables are affected by “financial shocks.” We find that financial shocks contributed significantly to the observed dynamics of real and financial variables. The recent events in the financial sector show up as a tightening of firms’ financing conditions which contributed to the 2008–2009 recession. The downturns in 1990–1991 and 2001 were also influenced by changes in credit conditions. (JEL E23, E32, E44, G01, G32)

Recent economic events starting with the subprime crisis in the summer of 2007 suggest that the financial sector plays an important role as a source of business cycle fluctuations. While there is a long tradition in macroeconomics to model financial frictions, most of the literature has focused on the role played by the financial sector in propagating shocks that originate in other sectors of the economy—for example, in the propagation of productivity and monetary shocks. Instead, the importance of financial shocks—that is, perturbations that originate directly in the financial sector—has started to be explored only recently. Moreover, most of the previous studies have not tried to replicate simultaneously real aggregate variables and aggregate flows of financing, in particular, debt and equity. In this paper we attempt to make some progress along these dimensions.

We start by documenting the cyclical properties of firms’ equity and debt flows at an aggregate level. We then build a business cycle model with explicit roles for firms’ debt and equity financing that is capable of capturing the empirical cyclical properties of the financial flows. The central feature of our model is the pecking order in the financial decision of firms between equity and debt. Debt is preferred to equity, but the firms’ ability to borrow is limited by an enforcement constraint which is subject to random disturbances. Since these disturbances affect the firms’ ability to borrow, we refer to them as “financial shocks.”

To examine the macroeconomic effects of financial shocks quantitatively, we use two methodological approaches. Our first approach is new in the study of models with financial frictions. It is based on the construction of time series for the financial

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† To view additional materials, visit the article page at http://dx.doi.org/10.1257/aer.102.1.238.
shocks from the model’s enforcement constraint. Using empirical data for debt, capital, and output, we construct the shock series as the residuals in the enforcement constraint. This method parallels the standard approach for measuring productivity shocks as Solow residuals from the production function using empirical measurements for output, capital, and labor. Since the shock series constructed this way are independent of how many shocks we add to the model, we use a parsimonious model with only two shocks: productivity and financial shocks.

Using the constructed series, we show that financial shocks are important not only for capturing the dynamics of financial flows but also for the dynamics of the real business cycle quantities, especially labor. In particular, the simulation of the model shows a worsening of firms’ ability to borrow in 2008–2009 with a sharp economic downturn. This is in line with the standard interpretation of the economic events that started in the summer of 2007 and further deteriorated in the fall of 2008. The simulation also shows that the economic downturns in 1990–91 and 2001 were strongly influenced by changes in credit conditions.

The second method we use to assess the macroeconomic effects of financial shocks is based on the structural estimation of the model with Bayesian maximum likelihood methods. Since the structural estimation provides an assessment of the contribution of financial shocks “relative” to other shocks, the estimation is conducted using a richer model with many more shocks and frictions. The richer model has the same features of the model estimated by Smets and Wouters (2007) but with the addition of financial frictions and financial shocks. Through variance decomposition we find that financial shocks contribute to almost half of the volatility of output and about 30 percent to the volatility of working hours. Despite the differences in methodology—calibration versus estimation—the dynamics induced by financial shocks using the two approaches are similar.

The financial frictions of our model share some similarities with models studied in Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke, Gertler, and Gilchrist (1999); Mendoza and Smith (2006); and Mendoza (2010). Our model, however, differs in two important dimensions. First, the equity financing of the firm is not limited to reinvesting profits. We allow firms to have negative equity payouts, which can be interpreted as new equity issues. Second, we consider shocks that affect directly the financial sector of the economy. Therefore, the financial sector acts as a source of the business cycle, in addition to affecting the propagation of shocks that originate in other sectors of the economy. In this respect the article is related to Benk, Gillman, and Kejak (2005), who also consider shocks affecting the financial sector although the nature of the shocks and the structure of the model are different. Recent contributions by Christiano, Motto, and Rostagno (2008); Del Negro et al. (2010); and Kiyotaki and Moore (2008) have also considered shocks that originate in the financial sector and suggest that these shocks could play an important role as a source of macroeconomic fluctuations. Another contribution in this direction but with an explicit modeling of frictions in the financial intermediation sector is Gertler and Karadi (2011).

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1 Examples of other studies that allow for equity issuance over the business cycle are Choe, Masulis, and Nanda (1993); Covas and den Haan (2010); Leary and Roberts (2005); and Levy and Hennessy (2007).
The article is structured as follows. Section I presents empirical evidence on the financial cycle in the US economy. Section II proposes a relatively parsimonious model with financial frictions and financial shocks, and Section III studies the quantitative properties. Section IV extends the model by introducing additional frictions and shocks and studies the importance of financial shocks through a structural estimation. Section V concludes.

I. Financial Cycle in the US Economy

Figure 1 plots the net payments to equity holders and the net debt repurchases in the nonfinancial business sector (corporate and noncorporate). Financial data is from the Flow of Funds Accounts of the Federal Reserve Board. Equity payout is defined as dividends and share repurchases minus equity issues of nonfinancial corporate businesses, minus net proprietor’s investment in noncorporate businesses. This captures the net payments to business owners (shareholders of corporations and noncorporate business owners). Debt is defined as “Credit Market Instruments,” which include only liabilities that are directly related to credit markets transactions. Debt repurchases are simply the reduction in outstanding debt (or increase if negative). Both variables are expressed as a fraction of business GDP. See the online Appendix for a more detailed description.
Two patterns are clearly visible in the figure, very strongly so for the second half of the sample period. First, equity payouts are negatively correlated with debt repurchases. This suggests that there is some substitutability between equity and debt financing. Second, while equity payouts tend to increase in booms, debt repurchases increase during or around recessions. This suggests that recessions lead firms to restructure their financial positions by cutting the growth rate of debt and reducing the payments to shareholders.

The properties of financial cycles are further characterized in Table 1. The table reports the standard deviations and correlations with GDP for equity payouts and debt repurchases in the nonfinancial business sector. As for the series reported in Figure 1, these two variables are normalized by business GDP. We do not take logs because some observations are negative. The statistics are computed after detrending with a band-pass filter that preserves cycles of 1.5–8 years (Baxter and King 1999).

We focus on the period that starts in 1984 for two reasons. First, it has been widely documented in relation with the so-called Great Moderation that 1984 corresponds to a break in the volatility in many business cycle variables. Second, as documented by Jermann and Quadrini (2006), this time period also saw major changes in US financial markets compared to the previous period. In particular, spurred by regulatory changes, share repurchases had become more common, and this seems to have had a major impact on firms’ payout policies and financial flexibility. This is apparent in Figure 1, where the volatility of the financial flows changes after the early 1980s. Therefore, by concentrating on the period that starts in 1984 we do not have to address the causes of the structural break that arose in the early 1980s.

The correlations of equity payouts and debt repurchases with GDP confirm the properties highlighted in Figure 1, that is, the procyclicality of equity payouts and the countercyclicality of debt repurchases. These properties also hold if we exclude the noncorporate business and for alternative detrending methods. They are also consistent with the findings of Covas and den Haan (2011) based on the aggregation of Compustat data.2

### II. A Model with Financial Frictions and Financial Shocks

We introduce financial frictions and financial shocks to the standard real business cycle model. We start with the description of the environment in which an individual

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2 Covas and den Haan find a procyclical pattern for debt issuance (consistent with the countercyclicality of our debt repurchases) but they do not find a clear cyclical pattern for the “aggregate” measure of equity issuance. However, their primary measure of equity issuance does not include share repurchases, which is an important component of the equity payout series we use in our analysis. With the inclusion of share repurchases, Covas and den Haan also find a negative correlation with GDP. See their Table 5.
firm operates, as this is where our model diverges from the standard model. We then present the household sector and define the general equilibrium.

A. Firms Sector

There is a continuum of firms, in the $[0, 1]$ interval, with a gross revenue function $F(z_t, k_t, n_t) = z_t k_t^{\theta} n_t^{1-\theta}$. The variable $z_t$ is the stochastic level of productivity, common to all firms, $k_t$ the input of capital, $n_t$ the input of labor. Consistent with the typical timing convention, $k_t$ is chosen at time $t - 1$ and predetermined at time $t$. Instead, the input of labor $n_t$ can be flexibly changed at time $t$.

Capital evolves according to $k_{t+1} = (1 - \delta)k_t + i_t$, where $i_t$ is investment and $\delta$ is the depreciation rate. Later we will also consider capital adjustment costs. As we will see, adjustment costs improve the asset price properties of the model but do not affect the key results of the paper.

Firms use equity and debt. Debt, denoted by $b_t$, is preferred to equity (pecking order) because of its tax advantage. This is also the assumption made in Hennessy and Whited (2005). Given $r_t$ the interest rate, the effective gross interest rate for the firm is $R_t = 1 + r_t(1 - \tau)$, where $\tau$ represents the tax benefit.

In addition to the intertemporal debt, $b_t$, firms raise funds with an intraperiod loan, $l_t$, to finance working capital. Working capital is required to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of revenues. The intraperiod loan is repaid at the end of the period, and there is no interest.

Firms start the period with intertemporal liabilities $b_t$. Before producing they choose labor, $n_t$, investment, $i_t = k_{t+1} - (1 - \delta)k_t$, equity payout, $d_t$, and the new intertemporal debt, $b_{t+1}$. Since the payments to workers, suppliers of investments, shareholders and bondholders are made before the realization of revenues, the intraperiod loan contracted by the firm is

$$l_t = w_t n_t + i_t + d_t + b_t - b_{t+1}/R_t.$$

Using the firm’s budget constraint,

$$b_t + w_t n_t + k_{t+1} + d_t = (1 - \delta)k_t + F(z_t, k_t, n_t) + b_{t+1}/R_t,$$

we can verify that the intraperiod loan is equal to the firm’s revenues, i.e.,

$$l_t = F(z_t, k_t, n_t).$$

A key feature of this formulation is that working capital and the intraperiod loan are related to the production scale, especially labor. There are different ways of formalizing this, and in the sensitivity analysis of Section III we will consider some alternatives.

The ability to borrow (intra- and intertemporally) is bounded by the limited enforceability of debt contracts as firms can default on their obligations. The decision to default arises after the realization of revenues but before repaying the intraperiod loan. At this stage the total liabilities are $l_t + b_{t+1}/(1 + r_t)$, that is, the intraperiod
loan plus the new intertemporal debt. The firm also holds liquidity \( l_t = F(z_t, k_t, n_t) \). Since the liquidity can be easily diverted, the lender will be unable to recover these funds in case of default. Therefore, the only asset available for liquidation is the physical capital \( k_{t+1} \).

Suppose that at the moment of contracting the loan the liquidation value of physical capital is uncertain. With probability \( \xi_t \), the lender can recover the full value \( k_{t+1} \), but with probability \( 1 - \xi_t \), the recovery value is zero. The Appendix describes the renegotiation process between the firm and the lender in the event of default. Based on the predicted outcomes of the renegotiation, the firm will be subject to the enforcement constraint

\[
(2) \quad \xi_t \left( k_{t+1} - \frac{b_{t+1}}{1 + r_t} \right) \geq l_t.
\]

Higher debt, either intertemporal or intratemporal, makes the enforcement constraint tighter. On the other hand, a higher stock of capital relaxes the enforcement constraint. These properties are shared by most of the enforcement or collateral constraints used in the literature. The probability \( \xi_t \) is stochastic and depends on (unspecified) markets conditions. Because this variable affects the tightness of the enforcement constraint and, therefore, the borrowing capacity of the firm, we refer to its stochastic innovations as “financial shocks.” Notice that \( \xi_t \) is the same for all firms. Therefore, we have two sources of aggregate uncertainty: productivity, \( z_t \), and financial, \( \xi_t \). Since there are no idiosyncratic shocks, we will concentrate on the symmetric equilibrium where all firms are alike (representative firm).

To see more clearly how \( \xi_t \) affects the financing and production decisions of firms, we rewrite the enforcement constraint (2) in a slightly modified fashion. For simplicity consider the case in which \( \tau = 0 \) so that \( R = 1 + r \). Using the budget constraint (1) to eliminate \( k_{t+1} - b_{t+1}/(1 + r_t) \) and remembering that the intraperiod loan is equal to the revenues, \( l_t = F(z_t, k_t, n_t) \), the enforcement constraint can be rewritten as

\[
\left( \frac{\xi_t}{1 - \xi_t} \right) \left[ (1 - \delta)k_t - b_t - w_t n_t - d_t \right] \geq F(z_t, k_t, n_t).
\]

At the beginning of the period \( k_t \) and \( b_t \) are given. The only variables that are under the control of the firm are the input of labor, \( n_t \), and the equity payout, \( d_t \). Therefore, if we start from a preshock state in which the enforcement constraint is binding and the firm wishes to keep the production plan unchanged, a negative financial shock (lower \( \xi_t \)) requires a reduction in equity payout \( d_t \). In other words, the firm is forced to increase its equity and reduce the new intertemporal debt. However, if the firm cannot reduce \( d_t \), it has to cut employment. Thus, whether the financial shock affects

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3This could result from the assumption that the sale of the firm’s capital requires the search for a buyer. The variable \( \xi_t \) is then interpreted as the probability to find the buyer. Alternatively, we could assume that the sale price is bargained on a take-it-or-leave-it offer and \( \xi_t \) is the probability that the offer is made by the lender (seller). The probability of finding a buyer and/or making the offer increases when the market conditions improve. This is one way of thinking about the “liquidity” of the firm’s assets. Eisfeldt and Rampini (2006) provide some evidence about the cyclical properties of \( \xi_t \). They impute the liquidity of capital from a business cycle model and find that it must be procyclical to match the amount of capital reallocation.
employment depends on the flexibility with which the firm can change its financial structure, i.e., the composition of debt and equity.

To formalize the rigidities affecting the substitution between debt and equity, we assume that the firm’s payout is subject to a quadratic cost. Given \( d_t \) the equity payout, the actual cost for the firm is

\[
\varphi(d_t) = d_t + \kappa \cdot (d_t - \bar{d})^2,
\]

where \( \kappa \geq 0 \), and \( \bar{d} \) is a coefficient equal to the long-run payout target (steady state).

The equity payout cost should not be interpreted necessarily as a pecuniary cost. It is a simple way of modeling the speed with which firms can change the source of funds when the financial conditions change. Of course, the possible pecuniary costs associated with share repurchases and equity issuance can also be incorporated in the function \( \varphi(d_t) \). The convexity assumption would then be consistent with the work of Hansen and Torregrosa (1992) and Altinkilic and Hansen (2000), showing that underwriting fees display increasing marginal cost in the size of the offering.

Another way of thinking about the adjustment cost is that it captures the preferences of managers for dividend smoothing. Lintner (1956) showed that managers are concerned about smoothing dividends over time, a fact further confirmed by subsequent studies. This could derive from agency problems. The explicit modeling of the agency problems, however, is beyond the scope of this article.\(^4\)

The parameter \( \kappa \) is key for determining the impact of financial shocks. As we will see, when \( \kappa = 0 \) the economy is almost equivalent to a frictionless economy. In this case, debt adjustments triggered by financial shocks can be quickly accommodated through changes in firm equity. When \( \kappa > 0 \), the substitution between debt and equity becomes costly and firms readjust the sources of funds slowly. As a result, financial shocks will have nonnegligible short-term effects on the production decision of firms.

Recursion Formulation of the Firm’s Problem.—The individual states are the capital stock, \( k \), and the debt, \( b \). The aggregate states, specified below, are denoted by \( s \). The optimization problem is

\[
(3) \quad V(s;k,b) = \max_{d,n,k',b'} \left\{ d + Em'V(s';k',b') \right\}
\]

subject to

\[
(1 - \delta)k + F(z,k,n) - wn + \frac{b'}{R} = b + \varphi(d) + k'
\]

\[
\xi \left( k' - \frac{b'}{1 + r} \right) \geq F(z,k,n).
\]

\(^4\)As an alternative to the adjustment cost on equity payouts, we could use a quadratic cost on the change of debt, which would lead to similar properties. Therefore, our model can be interpreted more broadly as capturing the rigidities in the adjustment of all sources of funds, not only equity.
The function $V(s; k, b)$ is the cum-dividend market value of the firm, and $m'$ is the stochastic discount factor. The variables $w$ and $r$ are the wage rate and the interest rate, and $R = 1 + r(1 - \tau)$ is the effective gross interest rate for the firm. The stochastic discount factor, the wage and interest rate are determined in the general equilibrium and are taken as given by an individual firm.

Denoting by $\mu$ the Lagrange multiplier associated with the enforcement constraint, the first-order conditions for $n$, $k'$, and $b'$ are

\begin{equation}
F_n(z, k, n) = w \cdot \left( \frac{1}{1 - \mu \varphi_d(d)} \right),
\end{equation}

\begin{equation}
Em' \cdot \left( \frac{\varphi_d(d)}{\varphi_d(d')} \right) \left[ 1 - \delta + (1 - \mu' \varphi_d(d'))F_k(z', k', n') \right] + \xi \mu \varphi_d(d) = 1,
\end{equation}

\begin{equation}
REm' \cdot \left( \frac{\varphi_d(d)}{\varphi_d(d')} \right) + \xi \mu \varphi_d(d) \left( \frac{R}{1 + r} \right) = 1,
\end{equation}

where the detailed derivation is provided in the online Appendix.

Especially important is the optimality condition for labor, equation (4). As usual, the marginal productivity of labor is equalized to the marginal cost. The marginal cost is the wage rate augmented by a wedge that depends on the “effective” tightness of the enforcement constraint, that is, $\mu \varphi_d(d)$. A tighter constraint increases the effective cost of labor and reduces its demand. Therefore, the main channel through which financial shocks are transmitted to the real sector of the economy is through the demand of labor.

To get further insights, it will be convenient to consider the special case in which the cost of equity payout is zero, that is, $\kappa = 0$. In this case $\varphi_d(d) = \varphi_d(d') = 1$ and condition (6) becomes $REm' + \xi \mu R/(1 + r) = 1$. Taking as given the aggregate prices $R$, $r$, and $Em'$, this implies that there is a negative relation between $\xi$ and the multiplier $\mu$. In other words, lower liquidation values of the firm’s capital make the enforcement constraint tighter. Then from condition (4) we see that a higher $\mu$ implies a lower demand for labor.

This mechanism is reinforced when $\kappa > 0$. In this case it will be costly to re-adjust the financial structure, and the change in $\xi$ induces a larger movement in $\mu$. Of course, the change in the policies of all firms also affect prices, with some feedbacks on individual policies. These feedbacks will be considered when we characterize the general equilibrium.

**B. Households Sector and General Equilibrium**

There is a continuum of homogeneous households maximizing the expected lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$, where $c_t$ is consumption, $n_t$ is labor and $\beta$ is the discount factor. Households are the owners (shareholders) of firms. In addition to equity shares, they hold noncontingent bonds issued by firms.

The household’s budget constraint is

$$w_t n_t + b_t + s_t(d_t + p_t) = \frac{b_{t+1}}{1 + r_t} + s_{t+1} p_t + c_t + T_t,$$
where \( w_t \) and \( r_t \) are the wage and interest rates, \( b_t \) is the one-period bond, \( s_t \) the equity shares, \( d_t \) the equity payout received from owning shares, \( p_t \) is the market price of shares, and \( T_t = B_{t+1}/[1 + r_t(1 - \tau)] - B_{t+1}/(1 + r_t) \) are lump-sum taxes financing the tax benefit of debt for firms. The first-order conditions with respect to \( n_t, b_{t+1} \), and \( s_{t+1} \) are

(7) \[ w_t U_c(c_t, n_t) + U_n(c_t, n_t) = 0, \]

(8) \[ U_c(c_t, n_t) - \beta(1 + r_t)E U_c(c_{t+1}, n_{t+1}) = 0, \]

(9) \[ U_c(c_t, n_t)p_t - \beta E(d_{t+1} + p_{t+1})U_c(c_{t+1}, n_{t+1}) = 0. \]

The first two conditions determine the supply of labor and the interest rate. The last condition determines the price of shares. Using forward substitution we derive

\[
p_t = E_t \sum_{j=1}^{\infty} \left( \frac{\beta^j \cdot U_c(c_{t+j}, n_{t+j})}{U_c(c_t, n_t)} \right) d_{t+j}.
\]

Firms’ optimization is consistent with households’ optimization. Therefore, the stochastic discount factor is \( m_{t+i} = \beta^i U_c(c_{t+i}, n_{t+i})/U_c(c_t, n_t) \).

We can now provide the definition of a general equilibrium. The aggregate states \( s \) are the productivity \( z \), the variable \( \xi \), the aggregate capital \( K \), and the aggregate bonds \( B \).

**DEFINITION 1:** A recursive competitive equilibrium is defined as a set of functions for (i) households’ policies \( c^h(s) \), \( n^h(s) \), and \( b^h(s) \); (ii) firms’ policies \( d(s; k, b) \), \( n(s; k, b) \), \( k(s; k, b) \), and \( b(s; k, b) \); (iii) firms’ value \( V(s; k, b) \); (iv) aggregate prices \( w(s) \), \( r(s) \), and \( m(s, s') \); (v) law of motion for the aggregate states \( s' = \Psi(s) \). Such that: (i) household’s policies satisfy conditions (7)–(8); (ii) firms’ policies are optimal and \( V(s; k, b) \) satisfies the Bellman’s equation (3); (iii) the wage and interest rates clear the labor and bond markets and \( m(s, s') = \beta U_c(c', n')/U_c(c, n) \); (iv) the law of motion \( \Psi(s) \) is consistent with individual decisions and the stochastic processes for \( z \) and \( \xi \).

**C. Some Characterization of the Equilibrium**

To illustrate some of the properties of the model, it will be convenient to look at two special cases in which some features of the equilibrium can be characterized analytically. First, we show that for a deterministic steady state with constant \( z \) and \( \xi \), the default constraint is always binding. Second, if \( \tau = 0 \) and \( \kappa = 0 \), changes in \( \xi \) have no effect on the real sector of the economy.
PROPOSITION 1: If $\tau > 0$ the enforcement constraint binds in a steady state.

PROOF 1:
In a deterministic steady state $m = 1/(1 + r)$ and $\varphi_d(d) = \varphi_d(d') = 1$. Therefore, the first-order condition for debt, equation (6), simplifies to $Rm + \bar{\xi} \mu R/(1 + r) = 1$ ($\bar{\xi}$ is the average value). Substituting the above expression for $m$, we get $R/(1 + r) + \bar{\xi} \mu R/(1 + r) = 1$. Because $R = 1 + r(1 - \tau)$, this condition implies that $\mu > 0$ if $\tau > 0$.

Therefore, as long as there is a tax benefit of debt, the enforcement constraint is binding in a steady state. With uncertainty, however, the constraint may not be always binding because firms could reduce their borrowing in anticipation of future shocks. The constraint is always binding if $\tau$ is sufficiently large and the shocks are sufficiently small. This will be the case in the quantitative exercises conducted in the next section.

Let’s consider now the stochastic economy focusing on the case with $\tau = 0$ and $\kappa = 0$.

PROPOSITION 2: With $\tau = 0$ and $\kappa = 0$, changes in $\xi$ have no effect on employment $n$ and next period capital $k'$.

PROOF 2:
When $\kappa = 0$ we have $\varphi_d(d) = \varphi_d(d') = 1$. Thus, the first-order condition (6) can be written as $REm' + \xi \mu R/(1 + r) = 1$. From the household’s first-order condition (8) we have $(1 + r)Em' = 1$. Combining these two conditions we get $(1 + \xi \mu) R/(1 + r) = 1$, which implies that $\xi \mu = 0$ since $R = 1 + r$ when $\tau = 0$. Therefore, $\mu$ is always zero and, assuming that the aggregate prices do not change, $n$ and $k'$ will not be affected by the change in $\xi$. We have to show next that the sequence of prices remains constant if firms do not change $n$ and $k'$. This becomes obvious once we recognize that changes in debt issuance and equity payout associated with fluctuations in $\xi$ cancel out in the household’s budget. Therefore, prices do not change.

Thus, when $\tau = 0$ and $\kappa = 0$, business cycle fluctuations are driven only by productivity. The model becomes a standard RBC where firms are indifferent between debt and equity.

III. Quantitative Analysis

The goal of this section is to evaluate the quantitative effects of productivity and financial shocks. To do so we construct series for the two shocks using some of the model restrictions as described below. The macroeconomic effects are then captured by the responses of the model to the shocks. Through the simulation of the model we will be able to show that financial shocks are important not only for capturing the dynamics of financial flows but also for the dynamics of real business cycle quantities, especially labor.

Before proceeding we shall clarify two points about the nature of our exercise and results. First, the finding that financial shocks have played an important role in the US business cycle does not mean that other shocks are not important.
The exercise is not designed to replicate exactly the empirical series of interest. Second, the fact that we abstract from other shocks does not bias our results since the approach we use to identify the financial shocks is independent of how many shocks we add to the model.

### A. Parameterization

The parameters can be grouped into two sets. The first set includes parameters that can be calibrated using steady state targets, some of which are typical in the business cycle literature. The second group includes parameters that cannot be calibrated using steady state targets. Since the model cannot be solved analytically, we use numerical methods. In the computation we conjecture that the enforcement constraint is always binding and solve a linear approximation of the dynamic system (see the online Appendix for the list of equations). The model solution is then used to check the initial conjecture of binding constraints. We have also solved the model with a more general nonlinear approach that accommodates occasionally binding constraints and found that the linear solution is quite accurate. The nonlinear method is described in the online Appendix.

**Parameters Set with Steady State Targets.**—The period in the model is a quarter. We set $\beta = 0.9825$, implying that the annual steady state return from holding shares is 7.32 percent. The utility function takes the form $U(c,n) = \ln(c) + \alpha \cdot \ln(1 - n)$ where $\alpha = 1.8834$ is chosen to have steady state hours equal to 0.3. The Cobb-Douglas parameter in the production function is set to $\theta = 0.36$ and the depreciation to $\delta = 0.025$. The mean value of $z$ is normalized to 1. These values are standard, and the quantitative properties of the model are not very sensitive to this first group of parameters.

The tax wedge is set to $\tau = 0.35$, which corresponds to the benefit of debt over equity if the marginal tax rate is 35 percent. This parameter is important for the quantitative performance of the model because it determines whether the enforcement constraint is binding. As we will see, with this value of $\tau$ (and the remaining parameterization of the model), the enforcement constraint is always binding in our simulations.

The mean value of the financial variable, $\xi$, is chosen to have a steady state ratio of debt over quarterly GDP equal to 3.36. This is the average ratio over the period 1984:I–2010:II for the nonfinancial business sector based on data from the Flow of Funds (for debt) and National Income and Product Accounts (for business GDP). The required value is $\xi = 0.1634$.

**Parameters that Cannot Be Set with Steady State Targets.**—The parameters that cannot be set with steady state targets are those determining the stochastic properties of the shocks and the cost of equity payout—the parameter $\kappa$. Of course, in a steady state equilibrium the stochastic properties of the shocks do not matter, and the equity payout is always equal to the long-term target (steady state). Therefore, we use an alternative procedure to construct the series of productivity and financial shocks.

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5 The labor income share, i.e., the ratio of wages over output, is not constant but equal to $(1 - \theta)(1 - \mu \varphi(d))$. However, since $\mu \varphi(d)$ is on average small, the labor share is not very different from $1 - \theta$. 
For the productivity variable $z_t$ we follow the standard Solow residuals approach. Using the production function we derive

\begin{equation}
\hat{z}_t = \hat{y}_t - \theta \hat{k}_t - (1 - \theta) \hat{n}_t,
\end{equation}

where $\hat{z}_t$, $\hat{y}_t$, $\hat{k}_t$, and $\hat{n}_t$ are the percentage or log-deviations from the deterministic trend. Given the value of $\theta$ and the empirical series for $\hat{y}_t$, $\hat{k}_t$, and $\hat{n}_t$, we construct the $\hat{z}_t$ series.

To construct the series for the financial variable $\xi_t$, we follow a similar approach but using the enforcement constraint under the assumption that it is always binding, that is,

\begin{equation}
\xi_t \left( k_{t+1} - \frac{b_{t+1}}{1 + r_t} \right) = y_t.
\end{equation}

The variable $\xi_t$ is determined residually using empirical series for $k_{t+1}$, $b_{t+1}/(1 + r_t)$, and $y_t$. Of course, the validity of the procedure depends on the validity of the assumption that the enforcement constraint is always binding—a condition that we verify ex post: after constructing the series for the shocks, we feed the shocks into the model and check whether the constraint is always binding. Notice that we do not directly force any endogenous variable to perfectly match an individual data series.

For the empirical series of capital, $k_{t+1}$, and debt, $b_{t+1}/(1 + r_t)$, we use end-of-period balance sheet data from the Flow of Funds Accounts. For the variable $y_t$, we use GDP data from the National Income and Product Accounts. All series are in real terms, and the log value is linearly detrended. A more detailed description is provided in the online Appendix.

After constructing the series for the productivity and financial variables over the period 1984:I–2010:II, we estimate the autoregressive system

\begin{equation}
\begin{pmatrix}
\hat{z}_{t+1} \\
\xi_{t+1}
\end{pmatrix}
= A \begin{pmatrix}
\hat{z}_t \\
\xi_t
\end{pmatrix} + \begin{pmatrix}
\epsilon_{z,t+1} \\
\epsilon_{\xi,t+1}
\end{pmatrix},
\end{equation}

where $\epsilon_{z,t+1}$ and $\epsilon_{\xi,t+1}$ are i.i.d. with standard deviations $\sigma_z$ and $\sigma_{\xi}$, respectively.

At this point we are left with the equity cost parameter $\kappa$. This is chosen to have a standard deviation of equity payout (normalized by output) generated by the model over the period 1984:I–2010:II equal to the empirical standard deviation. The full set of parameters is reported in Table 2.

Now that we have described the procedure used to construct the series of productivity and financial shocks, it should be clear that these series do not depend on the number of shocks included in the model. No matter how many shocks we add to the model, equations (10) and (11) will not be affected. Thus, given empirical measurements for $k_{t+1}$, $b_{t+1}/(1 + r_t)$, and $y_t$, we would generate the same series for the financial shocks. Similarly, given the observable variables $k_t$, $n_t$, and $y_t$, we would generate the same series for productivity.$^6$

$^6$The only way additional shocks could alter the $\xi_t$ series is in the eventuality that they affect the tightness of the enforcement constraint. Even if the model with only two shocks predicts that the enforcement constraint is always
B. Findings

The first two panels of Figure 2 plot the variables $z_t$ and $\xi_t$ constructed using our procedure. The bottom panels plot the innovations $\epsilon_{z,t}$ and $\epsilon_{\xi,t}$.

It is important to point out that the macroeconomic effects of financial shocks are mostly driven by the unexpected “changes” in $\xi_t$, not the “level” of this variable. A low value of $\xi_t$ may have moderate effects on hours and investment if the decline has not taken place recently, that is, if the economy had time to adjust to the lower $\xi_t$. This helps us understand the effects of financial shocks in the recent crisis where the decline in $\xi_t$ has been the largest since the early 1980s (see middle panel in the bottom section of Figure 2). Because the negative financial shock emerged when $\xi_t$ was high, the level of this variable is still high even after the shock. However, it is the change that matters, not the level. It is in this sense that the current crisis is characterized by the most severe financial conditions experienced by the US economy during the last two and a half decades.

Next we show that the constructed series of financial shocks tracks reasonably well qualitative indicators of credit tightness. The Federal Reserve Board conducts a survey among senior loan officers of banks (Senior Loan Officer Opinion Survey on Bank Lending Practices) asking whether they have recently tightened the credit standards for commercial and industrial loans. It then constructs an index of credit tightness as the percentage of officers with tightening standards. Notice that this is a measure of the changes in credit standard, not of the level. The index has a similar interpretation as the changes in the variable $\xi_t$ constructed from the model. A proxy for the changes in $\xi_t$ is given by the innovations $\epsilon_{\xi,t}$. Therefore, in the model we can define the index of credit tightening as the negative of $\epsilon_{\xi,t}$.

The last panel of Figure 2 plots the tightness indices constructed from the model and from the survey. For the sample period taken into consideration, the survey of senior loan officers is available starting in the second quarter of 1990. To facilitate the comparison we have rescaled the survey index by a factor of 0.04. As can be seen, our measure of credit tightness tracks quite well the
survey index. In particular, we see a sharp increase in both indices during the last recession. The same pattern can be observed in the 1990–91 recession and, to some extent, in the 2001 recession.7

To study the dynamics of the model induced by the constructed series of shocks, we conduct the following simulation. Starting with initial values of \( \zeta_{1984,I} \) and \( \xi_{1984,I} \), we feed the innovations into the model and compute the responses for key macroeconomic and financial variables. Although we use the actual sequence of shocks, they are not perfectly anticipated by the agents. They forecast future values of \( z_t \) and \( \xi_t \) using the autoregressive system (12). The right panel in the top section of Figure 2 reports the Lagrange multiplier for the enforcement constraint, \( \mu_t \). The negative deviations of this variable from the steady state never exceed −100 percent, implying that the multiplier is always positive during the simulation period. This is further checked by solving the model nonlinearly. See the online Appendix.

**Productivity Shocks.**—We show first the dynamics induced by the series of productivity shocks \( \epsilon_{z,t} \). The financial variable \( \xi_t \) is kept constant at its unconditional mean \( \bar{\xi} \).

---

7Levin, Natalucci, and Zakrajsek (2004) estimate the external finance premium within a costly verification model assuming time-varying recovery values for the period 1997–2003. They find that the external premium increased significantly during the 2001 recession, which is consistent with our finding of a higher financial tightness during this recession. The financial shocks constructed in our article are also consistent with those identified by Gilchrist, Yankov, and Zakrajsek (2009) using corporate bond spreads.
Figure 3 plots the series of output, hours worked, and financial flows. To highlight the importance of financial frictions, the figure also reports the responses generated by the model without financial frictions obtained by setting $\tau = 0$ and $\kappa = 0$. In this version of the model the financial flows become indeterminate because firms are indifferent between debt and equity financing. Thus the bottom graphs report the financial flows only for the baseline model with financial frictions. The empirical series of GDP and working hours are in logs and linearly detrended over the period 1984:I–2010:II. The debt repurchase and the equity payout are also linearly detrended over the same period but not logged.

As can be seen from the figure, there is a substantial divergence between the series generated by the model and the empirical counterparts. In particular, while the data show an output boom during the 1990s, the simulated series displays a decline for most of the 1990s. The model also misses the expansion in working hours, a fact also emphasized in McGrattan and Prescott (2010). Jermann and Quadrini (2007) propose an explanation of the 1990s expansion driven by the stock market boom. It is also worth noting that the drop in output generated by the model during the previous two recessions, 1990–91 and 2001, are significantly smaller than in the data. In the most recent recession productivity shocks capture some of the drop in output but not in hours. More importantly, the model with only productivity shocks does not generate enough volatility of hours. This finding is robust to an alternative specification of preferences based on indivisible labor. The movements
in debt flows generated in response to productivity shocks are also quite different from the data (see lower panels of Figure 3). The properties of the model with financial frictions are further illustrated by the impulse responses to a one-time productivity shock reported in Figure 6.

**Financial Shocks.**—Figure 4 plots the responses of output, hours, and financial flows to the sequence of financial shocks. With financial shocks only, the dynamics of output and labor are quite close to the data. In particular, we see a boom in output and hours during the 1990s. Furthermore, financial shocks generate sharp drops in output and labor in all three recessions: 1990–1991, 2001, and 2008–2009. The drop in hours generated by financial shocks in the recent recession is more than half the decline in the data.

The performance of the model in response to financial shocks relies on the impact that these shocks have on the demand for labor. As shown in the upper right panel, financial shocks generate large fluctuations in working hours. Also, they generate large drops in labor during the three recessions and an upward trend during the 1990s.

The importance of the financial shocks for the demand of labor can be seen from the first-order condition (4), which for convenience we rewrite here,

\[ F_n(z, k, n) = w \cdot \left( \frac{1}{1 - \mu \varphi_d(d)} \right). \]
The variable $\mu$ is the multiplier for the enforcement constraint and the term $\mu \varphi_d(d)$ determines the labor wedge. A negative financial shock makes the enforcement constraint tighter, increasing the term $\mu \varphi_d(d)$ and, therefore, the labor wedge. Intuitively, if the firm wants to keep the same scale and hire the same number of workers, it has to reduce the equity payout. Because this is costly, the firm chooses in part to reduce the equity payout and in part the input of labor. Impulse responses to a one-time financial shock are reported in Figure 6.

The model with financial shocks also captures the dynamics of the financial flows as shown in the lower panels of Figure 4. The series generated by the model broadly mimics the main features of the empirical series for debt and equity flows. Of course, we would not expect this parsimonious model to fit the data perfectly. In particular, the volatility of debt repurchases is somewhat higher than in the data.

Both Shocks. Figure 5 plots the series generated by the model in response to both shocks: productivity and financial. Overall, the model does a reasonable job in replicating the dynamics of output and hours worked as well as the dynamics of the financial flows. For financial flows and labor the performance of the model is very similar to the case with only financial shocks. For output, the performance during the 1990s is somewhat worse than the case with only financial shocks. However, the model continues to predict sharp drops in output during each of the three major recessions. In particular it captures most of the output decline observed in the recent crisis.
We close this section observing that the cost of deviating from the equity payout $d$ is small. Over the whole simulation period the average cost is only 0.01 percent of output. The highest cost was incurred in the first two quarters of 2009 (about 0.08 percent of output).

C. Sensitivity

In this section we explore the sensitivity of our results to (i) adjustment costs in investment, (ii) different forms for working capital, (iii) alternative specification of the enforcement constraint.

Adjustment Costs in Investment.——As we can see from the impulse responses presented in Figure 6, a positive financial shock induces a fall in the equity value of firms. This derives from the impact that the shock has on the stochastic discount factor. Since asset prices are typically procyclical and they tend to comove with credit, this is an unattractive property of the model. However, this feature can be easily changed by adding adjustment costs in investment.

Suppose that the law of motion for the stock of capital takes the form

$$k_{t+1} = (1 - \delta) k_t + \left[ \frac{\varphi_1 (i_t/k_t)^{1-\nu}}{1 - \nu} + \varphi_2 \right] k_t,$$
where $\nu$ determines the sensitivity of the cost to investment and the parameters $\varrho_1$ and $\varrho_2$ are set by imposing steady state targets. In particular, we impose that in the steady state the depreciation rate is equal to $\delta$ and $\partial k_{t+1}/\partial i_t = 1$. The second condition implies that the Tobin’s $q$ is equal to 1 in the steady state. Besides this, the model retains the baseline structure. In particular, the enforcement constraint remains the one specified in equation (2). Therefore, even though we now have a Tobin’s $q$ that is different from 1, this is not the market price of capital in the event of liquidation.

Figure 7 plots the responses to the constructed shocks when $\nu = 0.5$. As can be seen from the left and middle panels, adjustment costs do not change the main findings of this paper. However, as shown in the right panel, the value of the firm increases in response to a one-time financial shock. Therefore, the adjustment costs in investment improve the asset price performance of the model without changing the basic results of the article.

**Alternative Specification of Working Capital.**—Working capital financing plays an important role in our model since this determines the intraperiod loan $l_t = z_t k^\theta_t n^{1-\theta}_t$. Because the enforcement constraint takes the form $\xi_t (k_{t+1} - B_{t+1}/(1 + r_t)) \geq l_t$, the fact that $l_t = z_t k^\theta_t n^{1-\theta}_t$ implies that a positive productivity shock makes the constraint tighter. Therefore, financial frictions could dampen rather than amplify the shock.

To eliminate the direct impact of a productivity shock on the enforcement constraint, we now consider alternative specifications of working capital. What is crucial for our results is that, directly or indirectly, working capital depends on the input of...
labor. This would be the case, for example, if the loan is required to finance only the payment of wages. Alternatively, we could assume the production process requires an intermediate input that is complementary to labor.

Figure 8 plots the simulation of the model when the intraperiod loan is equal to the wage bill, that is, \( l_t = w_t n_t \). The figure also plots the simulation when the working capital is required for the financing of an intermediate input that is complementary to labor. To simplify the analysis we consider an extreme case of complementarity where the gross production function takes the form

\[
z_t k_t^\theta \left( \min \{n_t, x_t\} \right)^{1-\theta} + x_t.
\]

Here \( x_t \) is an intermediate input. We assume that \( x_t \) also increases output additively so that the net production (value added) remains \( z_t k_t^\theta \left( \min \{n_t, x_t\} \right)^{1-\theta} \). However, this is not essential for the results. Given this production function, the firm always chooses \( x_t = n_t \). Therefore, the intraperiod loan is \( l_t = n_t \).

As can be seen from the first four panels of Figure 6, the simulation results are not very different from the baseline simulation. One feature that changes, however, is the response of labor to a productivity shock as shown by the two panels on the right-hand side of Figure 8. While in the baseline model the response to a negative productivity shock was positive (see Figure 6), with the new specifications of working capital the response is negative. However, since productivity shocks do not
generate much movement in labor, whether the response is positive or negative has minor implications for the overall macroeconomic dynamics.

**Alternative Enforcement Constraint.**—The enforcement constraint considered earlier is derived from particular assumptions about the liquidation value of the firm’s capital. We now consider an alternative specification that is more in line with enforcement constraints used in the literature.

As in the baseline model, we continue to assume that in case of default the lender has the right to liquidate the firm’s capital. What is different is that the liquidation value is known at the moment of contracting the loan, and it is equal to \( \xi_t k_{t+1} \). Under the same assumptions about renegotiation, the enforcement constraint takes the form

\[
\xi_t k_{t+1} \geq \frac{b_{t+1}}{1 + r_t} + l_t.
\]

Essentially, the total debt (inter- and intratemporal) cannot be larger than the collateral, that is, the liquidation value of capital.\(^8\)

This specification brings some computational complications. Using the same calibration targets as those used in the baseline model, we found that the constraint is not always binding. Thus, we can not solve the model with a linear approximation, but we have to use global approximation methods that are computationally involved. This is not a problem for the parsimonious model considered here. However, for the large scale model we will consider in the next section, a nonlinear approximation becomes impractical. Before going into the large scale model, however, we want to show that the key message of the article remains valid if we use this alternative specification of the enforcement constraint. The nonlinear approximation solution is described in the online Appendix.

In addition to the higher complexity of solving the model nonlinearly, the identification of the financial shocks is also more involved. Since the enforcement constraint is not always binding, we cannot use the binding constraint to construct the \( \xi_t \) series. Therefore, we adopt a different approach. First we guess the parameter \( \kappa \) and the parameters for the stochastic process of \( \xi_t \), that is, \( \rho_\xi \) and \( \sigma_\xi \). Given these parameters, we solve the model and obtain the decision rules. At this point we simulate the model forward starting in the first quarter of 1984. In each quarter we find the value of \( \xi_t \) for which the decision rule exactly replicates the empirical debt repurchase. We repeat this step forward until the second quarter of 2010. By doing so we generate a sequence of \( \xi_t \). We then check whether the properties of the \( \xi_t \) series are close to the properties of the guessed process (that is the autocorrelation and standard deviation of the innovations are equal to the guesses for \( \rho_\xi \) and \( \sigma_\xi \)) and whether the standard deviation of equity payout is equal to the data. We iterate until convergence.

Figure 9 plots the resulting simulation. The first panel reports the \( \xi_t \) and \( z_t \) series, and the first panel at the bottom plots the Lagrange multiplier \( \lambda_t \). The multiplier takes the

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\(^8\)There are many studies that have used similar enforcement constraints. In some models the stock of capital is multiplied by the market price, which in our case is always one. More recently a similar constraint is used by Gertler and Karadi (2011) to model the agency problems faced by banks.
value of zero in several quarters during the period that preceded the recent recession. This was a period of easy credit that in the model is captured by nonbinding constraints. The remaining panels plot GDP, hours, and financial flows from the simulation of the model with only financial shocks and with both productivity and financial shocks. The quantitative impact of financial shocks on output and labor is slightly smaller compared to the baseline model, but the general pattern is consistent with what we have seen earlier.

IV. Extended Model and Structural Estimation

It has become common to estimate macroeconomic models using likelihood based techniques. The relative contributions of the various shocks are then evaluated through variance decompositions. In this section we investigate the effects of financial shocks using this approach.

Before describing the details of the estimation we would like to clarify one important difference between the approach we adopted in the previous section and the approach based on the structural estimation of the model. The approach used earlier to investigate the macroeconomic effects of financial shocks is independent of how many shocks we add to the model. This is true for two reasons. First, the sequence of financial shocks we constructed from the data is invariant to the consideration of other shocks (see earlier discussion). Second, the effects of financial shocks are evaluated in “absolute” terms, not relatively to other shocks. Thus, the use of a
parsimonious model with only two shocks is not problematic. In fact, we could have also abstracted from productivity shocks.

With the structural estimation, however, the effects of a particular shock depend, in general, on the shocks we include in the model. Since the estimation is designed to replicate in a likelihood sense the empirical series of interest (for example GDP, labor, investment, etc.), if we ignore shocks that are quantitatively important, the contribution of the included shocks may be over-estimated. It becomes apparent then that the reliability of the results requires the inclusion of many shocks, at least those that have received more attention in the literature. For that reason we now consider a more general model with more structural shocks. In doing so we also enrich the model with other frictions widely used in the literature.

We start with the model estimated by Smets and Wouters (2007). This is a model with seven shocks—productivity, investment-specific, intertemporal preferences, labor supply, price mark-up, government spending, and monetary policy. The estimation has used seven empirical variables—GDP, investment, working hours, wage rate, federal fund rate, government spending, and nominal prices. Our contribution is to extend the model by adding financial frictions and financial shocks along the lines described earlier. By doing so we end up with eight structural shocks. We will then estimate the model using eight empirical series: the seven series used by Smets and Wouters plus a variable representative of the financial flows.

A. The Model

In this section we describe the various sectors of the model starting with the household sector. We will then describe the household and government sectors.

Household Sector.—There is a continuum of households indexed by \( j \in [0, 1] \), supplying specialized labor services \( n_{j,t} \). They maximize the expected lifetime utility

\[
E_t \sum_{s=0}^{\infty} \beta^s \gamma_{t+s} \left[ \left( \frac{c_{t+s} - h c_{t+s-1}}{1 - \sigma} \right)^{1-\sigma} - \alpha \frac{n_{j,t+s}^{1+1/\varepsilon}}{1 + \frac{1}{\varepsilon}} \right],
\]

where \( c_t \) is consumption, \( n_{j,t} \) is labor of type \( j \), and \( \beta \) is the discount factor. The variable \( \gamma_{t+s} \) evolves stochastically and captures shocks to the intertemporal margin. The parameter \( \varepsilon \) is the elasticity of labor supply, and \( h \) determines the degree of “external” habit in consumption. We denote the period utility by \( U(c_{t-1}, c_t, n_{j,t}) \).

The household’s budget constraint is

\[
w_{j,t} n_{j,t} + d_t + B_t + a_{j,t} = P_t q_t s_{t+1} + \frac{B_{t+1}}{1 + r_t} + P_t c_t + T_t
\]

\[+ \int q_{j,t+1}^\omega a_{j,t+1} d \omega_{j,t+1,1},\]

where \( r_t \) is the nominal interest rate on bonds and \( w_{j,t} \) is the nominal wage rate set by household \( j \). The variable \( B_t \) is the one-period nominal bond, \( d_t \) is the equity
payout received from the ownership of firms, and $T_t$ denotes nominal lump-sum taxes. Households can buy state-contingent claims $a_{j,t+1}$ at the price $q_{j,t+1}$ to insure against wage shocks.

Individual households are monopolistic suppliers of specialized labor and set the wage taking the demand function as given. The demand for labor of type $j$ derives from the aggregation of the inputs demanded by all firms. As we will see in the description of the firm’s problem, the demand for type $j$ labor is given by

\begin{equation}
\begin{aligned}
n_{j,t} &= \left( \frac{w_{j,t}}{W_t} \right)^{-\frac{\nu_t}{\nu_t - 1}} N_t,
\end{aligned}
\end{equation}

where $N_t$ is the aggregate demand of labor and $W_t = \left( \int_0^1 W_t^{1/(1-\nu_t)} \, dj \right)^{1-\nu_t}$ is the aggregate nominal wage index. The variable $\nu_t$ is stochastic and captures shocks to the wage mark-up.

Households post nominal wages and supply the specialized services as determined by the demand function (15). Wage rigidities derive from the assumption that households can change their posted wage only with probability $1 - \omega$ (Calvo’s price rigidity).

Consider a household which is allowed to post a new wage in period $t$. Using the labor demand function (15), the new posted wage solves

\begin{equation}
\begin{aligned}
\max_{w_{j,t}} E_t \sum_{s=0}^{\infty} (\beta \omega)^s \gamma_{t+s} U \left( c_{t+s-1}, c_{t+s}, \left( \frac{w_{j,t}}{W_{t+s}} \right)^{-\frac{\nu_t}{\nu_t - 1}}, L_{t+s} \right),
\end{aligned}
\end{equation}

subject to the sequence of budget constraints (14).

After choosing $w_{j,t}$ at time $t$, the wage remains constant until the household is allowed to post a new wage. Only the periods preceding the resetting of a new wage are relevant for the choice of the wage today, explaining the probability $\omega$ in the discount factor.

Following the literature, we derive a wage equation by differentiating (16) with respect to $w_{j,t}$ and taking a log-linear approximation around the steady state. As shown in the online Appendix, the log-linearized wage equation is

\begin{equation}
\begin{aligned}
\hat{w}_t &= -\left( \frac{h \sigma \Phi}{1 - h} \right) \hat{c}_{t-1} + \left( \frac{\sigma \Phi}{1 - h} \right) \hat{c}_t + \Phi \hat{P}_t + \Phi \hat{v}_t + \frac{\Phi}{\varepsilon} \hat{n}_t \\
&\quad + \frac{v \Phi}{(v - 1)\varepsilon} \hat{W}_t + \beta \omega E_t \hat{w}_{t+1},
\end{aligned}
\end{equation}

where $\Phi = \frac{\varepsilon (v - 1)(1 - \beta \omega)}{\varepsilon (v - 1) + v}$, and the hat sign denotes log-deviations from steady state.

Since all households that reoptimize choose the same $w_{j,t}$, the aggregate wage index evolves according to

\begin{equation}
\begin{aligned}
W_t &= \left[ \omega W_{t-1}^{1/(1-\nu_t)} + (1 - \omega)w_{t}^{1/(1-\nu_t)} \right]^{1-\nu_t}.
\end{aligned}
\end{equation}
In addition to the nominal wage and, implicitly, the supply of labor, households choose nominal bonds. The first-order condition for $B_{t+1}$ is

$$1 = \beta(1 + r_t)E_t \left( \frac{\gamma_{t+1} U_{t+1}}{\gamma_t U_t} \right) \left( \frac{P_t}{P_{t+1}} \right).$$

Firms’ optimization is consistent with households’ optimization, and the stochastic discount factor is $m_{t+1} = \beta \left( \frac{\gamma_{t+1} U_{t+1}}{\gamma_t U_t} \right)$.

### B. Business Sector

There is a continuum of firms in the $[0, 1]$ interval, each producing an intermediate good $x_i$. The intermediate good is used as an input in the final goods production,

$$y_t = \left( \int_0^1 x_i^{1/\eta} \, di \right)^{\eta_t}.$$

The variable $\eta_t$ is stochastic, capturing shocks to the nominal price mark-up.

The first-order condition for the maximization of profits, $P_t y_t - \int_0^1 p_i x_i \, di$, returns the inverse demand function for the intermediate good $i$,

$$p_{i,t} = P_t y_t \left( \frac{\eta_t - 1}{\eta_t} \right) x_i^{1-\eta_t/\eta_t},$$

where $p_{i,t}$ is the nominal price set by the producer of good $i$ and $P_t = \left( \int_0^1 p_i^{1/(1-\eta_t)} \, di \right)^{1-\eta_t}$ is the aggregate nominal price index.

The intermediate good is produced with capital and labor according to

$$x_i,t = z_t(u_{i,t}, k_{i,t})^\theta n_i^{1-\theta},$$

where $z_t$ is the aggregate productivity, $k_{i,t}$ the input of capital, $u_{i,t}$ the capital utilization rate, and $n_i,t = \left( \int_0^1 n_{j,t}^{1/\nu_j} \, di \right)^{\eta_j}$ is the aggregation of all labor inputs used by firm $i$. The variable $\nu_j$ is stochastic and affects the demand elasticity for the different types of labor.

From the cost minimization problem we can derive the demand for labor of type $j$ for each firm. Aggregating over all firms gives the aggregate demand for type $j$ labor as reported in equation (15). Substituting the production into the inverse demand for the intermediate input, the price charged by firm $i$ can be expressed as

$$p_{i,t} x_i,t = P_t y_t \left( \frac{\eta_t - 1}{\eta_t} \right) \left[ z_t(u_{i,t}, k_{i,t})^\theta n_i^{1-\theta} \right]^{1-\eta_t/\eta_t} \equiv P_t D(k_{i,t}, u_{i,t}, n_{i,t}; s_t).$$

To take into account the dependence on the aggregate production $Y_t$, we have included the term $s_t$, which is the vector of aggregate states.

Using (23) the real revenues of the firm can also be expressed as a function of the production inputs and aggregate states, that is,

$$p_{i,t} x_i,t = P_t y_t \left( \frac{\eta_t - 1}{\eta_t} \right) \left[ z_t(u_{i,t}, k_{i,t})^\theta n_i^{1-\theta} \right]^{1/\eta_t} \equiv P_t F(k_{i,t}, u_{i,t}, n_{i,t}; s_t).$$
Physical capital is accumulated by firms and evolves according to 
\[ k_{t+1} = (1 - \delta)k_t + \Upsilon(i_{t-1}, i_t; \zeta_t), \]
where \( \zeta_t \) is a stochastic variable affecting the transformation of final goods in new capital goods (investment specific technology shock). The function \( \Upsilon(i_{t-1}, i_t; \zeta_t) \) takes the form

\[
\Upsilon(i_{t-1}, i_t; \zeta_t) = \zeta_t \left[ 1 - g \left( \frac{i_t}{i_{t-1}} \right) \right] i_t,
\]

with \( g(1) = 0, g'(1) = 0, g''(\cdot) > 0 \). This cost function is not standard in the investment literature but has become popular in New Keynesian models. The function \( g(i_t/i_{t-1}) \) is specified as \( g(i_t/i_{t-1}) = (i_t - i_{t-1})^2 \).

Capital utilization is also costly. Denoting by \( u_t \) the fraction of used capital over the owned capital, the utilization cost is \( \Psi(u_t)k_t \) where we impose that \( \Psi(1) = 0, \Psi'(1) > 0 \) and \( \Psi''(1) > 0 \). The functional form for \( \Psi(u_t) \) is specified as \( \vartheta (u_t^{1+\psi} - 1)/(1 + \psi) \) where \( \vartheta = 1/\beta - 1 + \delta \) so that the steady state utilization is 1.

There are different ways of generating nominal price rigidity. A popular approach is based on Calvo’s staggered prices, which generates heterogeneity in firms’ prices. The price heterogeneity can be easily handled in the case of complete markets. With incomplete markets, however, the characterization of the equilibrium is much more complex because the price heterogeneity generates heterogeneity in the financial structure of firms. Thus, we would not be able to aggregate and work with a “representative firm.”

This problem does not arise with Rotemberg’s approach, which is based on a convex cost of adjusting the nominal price. This is the only change we make to the model estimated by Smets and Wouters, besides adding financial frictions and financial shocks.

Given the nominal price \( p_{i,t-1} \) set in the previous period, the adjustment cost is

\[
G(p_{i,t-1}, p_i; s_t) \equiv \frac{\phi}{2} \left( \frac{p_{i,t}}{p_{i,t-1}} - 1 \right)^2 Y_t.
\]

We should think of the model as already detrended by long-term inflation.

The financial structure and frictions are the same as those described in the simpler model studied earlier. In particular, they are characterized by two parameters: \( \tau \) and \( \kappa \). The first parameter determines the tax advantage of using debt. Given \( r_t \) the nominal interest rate, the effective gross rate paid by firms is \( R_t = 1 + r_t(1 - \tau) \). The second parameter determines the cost of changing the equity payout. Given the equity payout \( d_t \) received by shareholders, the cost for the firm is \( \varphi(d_t) = d_t + \kappa \cdot (d_t - \bar{d})^2 \). As in the simpler model, if we set these two parameters to zero, the model collapses to a New Keynesian model with complete markets.

The individual state variables for the firm are the nominal price chosen in the previous period, \( p_{-1} \), the previous period investment, \( i_{-1} \), the stock of capital, \( k_t \), and the debt, \( b_t \). Since in equilibrium all firms make the same choices (assuming that they
start with the same states), from now on we omit the subscript \(i\). The optimization problem is

\[
V(s; p_{-1}, i_{-1}, k, b) = \max_{d, n, u, p, i, k', b'} \{d + Em'V(s'; p, i, k', b')\}
\]

subject to

\[
P[F(k, u, n; s) - \Psi(u)k + \frac{b'}{R} - b = Wn + PG(p_{-1}, p; s) + P\varphi(d) + Pi
\]

\[
\xi\left(k' - \frac{b'}{P(1 + r)}\right) \geq F(k, u, n; s)
\]

\[
\frac{p}{P} = D(k, u, n; s)
\]

\[
(1 - \delta)k + \Upsilon(i_{-1}, i; \zeta) = k'.
\]

The problem is subject to the budget constraint, the enforcement constraint, the demand for the firm’s product and the law of motion for capital. The first-order conditions are derived in the online Appendix.

**Public Sector.**—The government faces the budget constraint

\[
P_tG_t + B_{t+1}\left(\frac{1}{R_t} - \frac{1}{1 + r_t}\right) = T_t,
\]

where \(G_t\) is real (unproductive) government purchases, \(r_t\) the nominal interest rate, and \(R_t = 1 + r_t(1 - \tau)\) is the effective gross interest rate paid by firms. The cost of the interest deduction is \(B_{t+1}/[1/R_t - 1/(1 + r_t)]\). Total expenditures are financed with lump-sum taxes \(T_t\) paid by households. Government purchases follow the stochastic process

\[
\hat{G}_t = \rho_g \hat{G}_{t-1} + \rho_gc(\hat{z}_t - \hat{z}_{t-1}) + \epsilon_{g,t},
\]

where \(\epsilon_{g,t} \sim N(0, \sigma_G)\).

Monetary policy takes the form of an interest rate rule,

\[
\frac{1 + r_t}{1 + \bar{r}} = \left(\frac{1 + r_{t-1}}{1 + \bar{r}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\nu_1} \left(\frac{Y_t}{Y_{t-1}}\right)^{\nu_2}\right]^{1-\rho_R} \zeta_t,
\]

where \(\rho_R, \nu_1,\) and \(\nu_2\) are parameters and \(\zeta_t \sim N(0, \sigma_R)\). The monetary authority targets inflation and output growth deviations from the steady state.\(^9\)

\(^9\)To simplify the numerical solution of the model we assume that the monetary authority targets output deviations from the steady state rather than deviations from full capacity. The latter is usually defined as the equilibrium output that would prevail in absence of frictions. It is well known, however, that the two ways of defining the output gap do not affect significantly the quantitative results.
Stochastic Processes for the Shocks and Relation to Simpler Model.—The stochastic processes for government purchases (fiscal policy) and the nominal interest rate (monetary policy) have already been specified in (27) and (28). The remaining stochastic variables follow the process $\hat{x}_{t+1} = \rho_x \hat{x}_t + \epsilon_{x,t+1}$, with $\epsilon_x \sim N(0, \sigma_x)$ and $x \in \{z, \zeta, \gamma, \eta, \nu, \xi\}$. The hat sign denotes log deviations from steady state.

Now that we have completed the description of the theoretical framework, we would like to emphasize that the simpler model studied in the first part of the paper is just a special case of the more general model studied here. The simpler model is obtained by replacing the interest rate rule used by the monetary authority with a policy that stabilizes the price level and by imposing the following parameter values: $\lambda = 0$, $\omega = 0$, $\phi = 0$, $g(\cdot) = 0$, $\eta = 1$, $\nu = 1$, $\bar{G} = 0$, $\sigma_\zeta = 0$, $\sigma_\gamma = 0$, $\sigma_\eta = 0$, $\sigma_\nu = 0$, $\sigma_\xi = 0$.

B. Estimation

A small number of the model parameters are pinned down using the standard calibration technique based on steady state targets. The remaining parameters are estimated using Bayesian methods as described in An and Schorfheide (2007).

Calibrated Parameters.—The period in the model is a quarter and the calibration targets for the few calibrated parameters are the same as those in the simpler model studied earlier. More specifically, we set $\beta = 0.9825$, $\tau = 0.35$, $\theta = 0.36$, $\delta = 0.025$, and $\alpha$ is chosen to have an average working time of 0.3. The average value of the enforcement variable $\xi$ is chosen to have a steady state ratio of debt over quarterly output of 3.36. The final parameter we calibrate is the average value of government purchases $\bar{G}$. This is chosen to have a steady state ratio of government purchases over output of 0.18.

Estimated Parameters.—The model is estimated using eight empirical series: growth rate of GDP, growth rate of personal consumption expenditures, growth rate of private domestic investment, growth rate of implicit price deflator for GDP, growth rate of working hours in the private sector, growth rate of hourly wages in the business sector, federal fund rate, and debt repurchases in the nonfinancial business sector. The first seven variables are similar to the variables used in Smets and Wouters (2007). Debt repurchases is added because we have an additional shock, $\xi$. The sample period is 1984:I–2010:II. We start in 1984 to avoid the issue of possible structural breaks associated with the so called “Great Moderation.” See the online Appendix for a more detailed description of the data.

To generate artificial series, we solve the model numerically after log-linearizing around the steady state. This is possible because the enforcement constraint is always binding in the neighborhood of the steady state equilibrium. The whole set of equations is listed in the online Appendix.

The choice of the prior distributions are the same as those used in Smets and Wouters (2007) with the exception, of course, of the parameters that were not present in that model—in particular, the parameters that govern the stochastic process for the financial shock, $\rho_\xi$ and $\sigma_\xi$, and the flexibility in equity payout, $\kappa$. For the persistence and standard deviation of the financial shocks we use the same priors.
as those used for the other shocks. For the parameter $\kappa$ we use an inverse gamma distribution with a mean of 0.146 (the calibration value used in the simpler model) and a standard deviation of 0.05. Table 3 reports the parameters, calibrated and estimated. For the subset of the estimated parameters we also report the prior densities, the mode, and the cutoff values for the fifth and ninety-fifth percentiles of the posterior distribution. The posterior density is constructed by simulation using the Random-Walk Metropolis algorithm as described in An and Schorfheide (2007).

C. Findings

Table 4 reports the variance decomposition for the eight variables used in the estimation. The most important result is that financial shocks contribute significantly to the volatility of the growth rate of output (46 percent), investment (25 percent), and labor (33 percent). Especially important is the contribution to the volatility of labor. Financial shocks, however, contribute only marginally to the volatility of consumption. Movements in consumption are mostly driven by shocks to the intertemporal margin. Somewhat surprising is that financial shocks are not the major driving force for the volatility of debt repurchases. Although the contribution of financial shocks is sizable (13 percent), the movements in debt repurchases are mostly driven by shocks to the nominal price mark-up.

The first panel of Figure 10 plots the series of output growth generated by the model in response to the financial shocks when the parameters of the model are set to the mode values, that is, the values that maximize the posterior density. The panel also reports the empirical growth rates of GDP. As can be seen, the growth rates induced by financial shocks are quite consistent with the data. This is especially true during the three major recessions experienced by the US economy since the mid-1980s.

To illustrate this point more clearly, the three panels at the bottom of Figure 10 plot the growth rates of GDP for each of the NBER recessionary dates. During the 1990 and 2001 recessions, the output growth series generated by financial shocks tracks quite closely the empirical series. In the most recent recession financial shocks play a crucial role during the deepest face of the recession, that is, 2008:III–2009:I. In particular, financial shocks capture almost all of the decline in GDP in the third quarter of 2008 and about half of the decline in the fourth quarter of 2008 and first quarter of 2009.

Figure 10 also plots the “level” of GDP (see top panel on the right-hand side) constructed by compounding the growth rates shown in the first panel. This allows us to compare the simulated series of the estimated model with the series generated with the simpler model (see Figure 4). Also from this panel it is evident that financial shocks replicate the output boom of the 1990s and the downturns experienced by the US economy in the three major recessions. The output decline generated during the most recent recession is almost half the decline observed in the data. The most

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10 This can be explained by looking at the enforcement constraint. Let’s consider a positive mark-up shock. The increase in the mark-up is associated with lower production. With higher market power firms have an incentive to reduce production to maximize profits. The lower production reduces the right-hand side of the enforcement constraint and, therefore, the tightness of this constraint. This allows firms to raise debt and pay more dividends. If we reestimate the model without the price mark-up shock, a larger share of the movement in debt repurchases will be captured by the financial shock.
notable deviation of the simulated data from the actual series for output is the period before the recent crisis, 2006–2007. The model generates an output boom that is not seen in the data. However, this is fully consistent with the credit expansion experienced by the US economy before the recession.

Overall, the dynamics of the GDP level are very similar to the first panel of Figure 4, which we constructed using the alternative approach. Despite the simpler model and a different procedure to construct the $\xi_t$ series, the results are similar. To understand why we reach similar results, it would be helpful to look again at the equation used to identify the financial shocks, that is, the enforcement constraint $\xi_t(k_{t+1} - b_{t+1}/(1 + r_t)) = y_t$. The key point we would like to emphasize is

<table>
<thead>
<tr>
<th>Table 3—Parameterization</th>
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<tbody>
<tr>
<td><strong>Calibrated parameters</strong></td>
</tr>
<tr>
<td>Discount factor, $\beta$</td>
</tr>
<tr>
<td>Tax advantage, $\tau$</td>
</tr>
<tr>
<td>Utility parameter, $\alpha$</td>
</tr>
<tr>
<td>Production technology, $\theta$</td>
</tr>
<tr>
<td>Depreciation rate, $\delta$</td>
</tr>
<tr>
<td>Enforcement parameter, $\xi$</td>
</tr>
<tr>
<td>Average gov. purchases, $G$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Estimated parameters</strong></th>
<th><strong>Prior[mean,std]</strong></th>
<th><strong>Mode</strong></th>
<th><strong>Below 5%</strong></th>
<th><strong>Below 95%</strong></th>
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<tbody>
<tr>
<td>Utility parameter, $\sigma$</td>
<td>Normal $[1.5, 0.37]$</td>
<td>1.090</td>
<td>1.082</td>
<td>1.091</td>
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<tr>
<td>Elasticity of labor, $\varepsilon$</td>
<td>Normal $[2.0, 0.75]$</td>
<td>1.761</td>
<td>1.759</td>
<td>1.765</td>
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<td>Habit in consumption, $\lambda$</td>
<td>Beta $[0.5, 0.30]$</td>
<td>0.608</td>
<td>0.609</td>
<td>0.616</td>
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<td>Wage adjustment, $\omega$</td>
<td>Beta $[0.5, 0.30]$</td>
<td>0.278</td>
<td>0.276</td>
<td>0.285</td>
</tr>
<tr>
<td>Price adjustment cost, $\phi$</td>
<td>IGamma $[0.1, 0.30]$</td>
<td>0.031</td>
<td>0.032</td>
<td>0.043</td>
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<tr>
<td>Investment adjustment cost, $\theta$</td>
<td>IGamma $[0.1, 0.30]$</td>
<td>0.021</td>
<td>0.016</td>
<td>0.020</td>
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<tr>
<td>Capital utilization cost, $\psi$</td>
<td>Beta $[0.5, 0.15]$</td>
<td>0.815</td>
<td>0.811</td>
<td>0.820</td>
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<td>Equity payout cost, $\kappa$</td>
<td>IGamma $[0.2, 0.10]$</td>
<td>0.426</td>
<td>0.420</td>
<td>0.431</td>
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<tr>
<td>Average price mark-up, $\eta$</td>
<td>Beta $[1.2, 0.10]$</td>
<td>1.137</td>
<td>1.125</td>
<td>1.138</td>
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<td>Average wage mark-up, $\upsilon$</td>
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<td>1.021</td>
<td>1.027</td>
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<td>Productivity shock persistence, $\rho_z$</td>
<td>Beta $[0.5, 0.20]$</td>
<td>0.902</td>
<td>0.899</td>
<td>0.907</td>
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<td>Investment shock persistence, $\rho_\zeta$</td>
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<td>0.921</td>
<td>0.935</td>
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<td>Intertemporal shock persistence, $\rho_\gamma$</td>
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<td>0.794</td>
<td>0.796</td>
<td>0.804</td>
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<td>Price mark-up shock persistence, $\rho_\eta$</td>
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<td>0.902</td>
<td>0.907</td>
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<td>Wage mark-up shock persistence, $\rho_\upsilon$</td>
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<td>0.625</td>
<td>0.636</td>
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<td>Government shock persistence, $\rho_G$</td>
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<td>0.945</td>
<td>0.952</td>
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<td>Interest policy shock persistence, $\rho_\varsigma$</td>
<td>Beta $[0.5, 0.20]$</td>
<td>0.203</td>
<td>0.195</td>
<td>0.204</td>
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<td>Financial shock persistence, $\rho_\xi$</td>
<td>Beta $[0.5, 0.20]$</td>
<td>0.969</td>
<td>0.967</td>
<td>0.973</td>
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<td>Interaction prod-government, $\rho_{Gz}$</td>
<td>Beta $[0.5, 0.20]$</td>
<td>0.509</td>
<td>0.510</td>
<td>0.531</td>
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<td>Productivity shock volatility, $\sigma_z$</td>
<td>IGamma $[0.001, 0.05]$</td>
<td>0.005</td>
<td>0.004</td>
<td>0.005</td>
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<tr>
<td>Investment shock volatility, $\sigma_\zeta$</td>
<td>IGamma $[0.001, 0.05]$</td>
<td>0.006</td>
<td>0.005</td>
<td>0.007</td>
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<td>Intertemporal shock volatility, $\sigma_\gamma$</td>
<td>IGamma $[0.001, 0.05]$</td>
<td>0.016</td>
<td>0.014</td>
<td>0.018</td>
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<td>Price mark-up shock volatility, $\sigma_\eta$</td>
<td>IGamma $[0.001, 0.05]$</td>
<td>0.019</td>
<td>0.018</td>
<td>0.021</td>
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<td>Wage mark-up shock volatility, $\sigma_\upsilon$</td>
<td>IGamma $[0.001, 0.05]$</td>
<td>0.085</td>
<td>0.082</td>
<td>0.097</td>
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<td>0.026</td>
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<tr>
<td>Financial shock volatility, $\sigma_\xi$</td>
<td>IGamma $[0.001, 0.05]$</td>
<td>0.008</td>
<td>0.007</td>
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<td>Monetary policy, $\rho_R$</td>
<td>Beta $[0.75, 0.10]$</td>
<td>0.745</td>
<td>0.733</td>
<td>0.744</td>
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<td>Monetary policy, $\rho_1$</td>
<td>Normal $[1.50, 0.25]$</td>
<td>2.410</td>
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<td>Monetary policy, $\rho_2$</td>
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<td>Monetary policy, $\rho_3$</td>
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<td>0.121</td>
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that this constraint must also be satisfied in the structural estimation. If the variables used in the estimation included \( y_t, k_{t+1}, \) and \( b_{t+1}/(1 + r_t) \), we would get exactly the same time series for \( \xi_t \) as in the first part of the article. We did use time series data for \( y_t \) and \( b_{t+1}/(1 + r_t) \), but not for \( k_{t+1} \). Thus, we do not get exactly the same \( \xi_t \). However, since we used investment, which is obviously related to the dynamics of the capital stock, the \( \xi_t \) series obtained through the structural estimation can not be very different. The response of output and other variables to the \( \xi_t \) series may differ, though, because the estimated model incorporates more frictions such as sticky prices and sticky wages, and the estimated value of the parameter \( \kappa \) is different.

### Appendix: Derivation of the Enforcement Constraint

The decision to default arises after the realization of revenues but before repaying the intraperiod loan. The total liabilities are \( l_t + b_{t+1}/(1 + r_t) \), that is, the

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<tbody>
<tr>
<td>( z )</td>
<td>( \zeta )</td>
<td>( \gamma )</td>
<td>( \eta )</td>
<td>( \nu )</td>
<td>( G )</td>
<td>( \varsigma )</td>
<td>( \xi )</td>
</tr>
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<td>GDP</td>
<td>4.1</td>
<td>4.1</td>
<td>1.1</td>
<td>24.9</td>
<td>12.9</td>
<td>0.8</td>
<td>5.9</td>
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<td>Consum</td>
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<td>27.8</td>
<td>56.6</td>
<td>2.9</td>
<td>2.7</td>
<td>7.1</td>
<td>0.2</td>
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<td>Invest</td>
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<td>16.5</td>
<td>13.3</td>
<td>13.8</td>
<td>9.6</td>
<td>15.2</td>
<td>4.4</td>
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<td>GDPdefl</td>
<td>2.2</td>
<td>24.0</td>
<td>2.0</td>
<td>3.7</td>
<td>5.2</td>
<td>2.8</td>
<td>50.6</td>
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<tr>
<td>FF rate</td>
<td>3.6</td>
<td>61.9</td>
<td>4.1</td>
<td>3.4</td>
<td>8.1</td>
<td>9.7</td>
<td>4.5</td>
</tr>
<tr>
<td>Hours</td>
<td>19.4</td>
<td>5.1</td>
<td>0.8</td>
<td>16.0</td>
<td>17.7</td>
<td>1.1</td>
<td>6.5</td>
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<tr>
<td>Wages</td>
<td>0.5</td>
<td>2.9</td>
<td>3.1</td>
<td>5.4</td>
<td>83.3</td>
<td>0.7</td>
<td>3.1</td>
</tr>
<tr>
<td>DebtPay</td>
<td>6.9</td>
<td>5.8</td>
<td>0.5</td>
<td>51.3</td>
<td>15.3</td>
<td>5.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>
intraperiod loan plus the new intertemporal debt. At this stage the firm also holds liquidity \( l_t = F(z_t, k_t, n_t) \) from selling its products.

If the firm defaults, the lender acquires the right to liquidate the firm’s capital. Suppose that at the moment of contracting the loan the liquidation value of physical capital is uncertain. With probability \( \xi_t \) the lender will be able to recover the whole value \( k_{t+1} \), but with probability \( 1 - \xi_t \) the recovery value is zero. Neither the lender nor the firm is able to observe the liquidation value before the actual default. Therefore, to derive the renegotiation outcome, we have to consider these two cases separately. In doing so, we assume that the firm has all the bargaining power in the renegotiation and the lender gets only the threat value. Let us consider the two cases separately.

**Liquidation value is** \( k_{t+1} \): Since the lender can expropriate the whole capital, the firm has to make a payment that leaves the lender indifferent between liquidation and keeping the firm in operation. This requires the firm to make the payment \( k_{t+1} - b_{t+1}/(1 + r_t) \) and promise to pay \( b_{t+1} \) at the beginning of the next period, when the intertemporal debt is due.\(^{11}\) Therefore, the ex post value of defaulting is

\[
l_t + Em_{t+1} V_{t+1} - k_{t+1} + \frac{b_{t+1}}{1 + r_t},
\]

\(^{11}\)The required payment \( k_{t+1} - b_{t+1}/(1 + r_t) \) could be bigger than the liquidity \( l_t \). In this case we assume that the extra cash is raised from shareholders without additional costs.
**Liquidation value is zero:** If the liquidation value is zero, liquidation is clearly not the best option for the lender. Instead, the best option is to wait to the next period when $b_{t+1}$ is due. In the current period the lender gets no payments, and the firm retains the liquidity $l_t = F(z_t, k_t, n_t)$. Thus, the ex post default value is

$$l_t + E m_{t+1} V_{t+1}.$$ 

When the debt is contracted, the expected liquidation value is

$$l_t + E m_{t+1} V_{t+1} - \xi_t \left(k_{t+1} - \frac{b_{t+1}}{1 + r_t}\right).$$

Enforcement requires that the value of not defaulting is not smaller than the expected value of defaulting, that is,

$$E m_{t+1} V_{t+1} \geq l_t + E m_{t+1} V_{t+1} - \xi_t \left(k_{t+1} - \frac{b_{t+1}}{1 + r_t}\right),$$

which can be rearranged as in equation (2).

We would like to point out that the particular timing about the payments and decision to default is made only for analytical convenience. For example, the assumption that the firm contracts an intraperiod loan is a shortcut to the fact that firms carry “cash” or “liquidity” to the next period. The cash is then used to pay the equity holders (including dividends) and to finance working capital (wages and investment). When interpreted this way, the payment of dividends comes from previous period earnings, which is a more natural interpretation. All of this can be formalized by explicitly adding cash but at the cost of having an additional state variable.

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