Macroeconomic Effects of Financial Shocks
Online Appendix

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Data sources

Financial data is from the Flow of Funds Accounts of the Federal Reserve Board. We report the references to the FFA items using the 'Coded Tables' released on September 17, 2010. New editions of the Coded Tables may use different coding which should be taken into account by future uses. All series, financial and real, are seasonally adjusted.

Data source for Figure 1 and Table 1

Equity Payout is 'Net dividends of nonfarm, nonfinancial business' (Table F.102, line 3), plus 'Net dividends of farm business' (Table F.7, line 24), minus 'Net increase in corporate equities of nonfinancial business' (F.101, line 35), minus 'Proprietors' net investment of nonfinancial business' (F.101, line 39). *Debt Repurchase* is the negative of 'Net increase in credit markets instruments of nonfinancial business' (Table F.101, line 28). Equity payout and debt repurchase are both divided by business value added from the National Income and Product Accounts (Table 1.3.5). Total GDP used to compute the correlations reported in Table 1 is also from NIPA (Table 1.1.6).

Construction of financial shocks

The ξ_t series are constructed from the enforcement constraint $\xi_t(k_{t+1} - b_{t+1}^e) = y_t$, where $b_{t+1}^e = b_{t+1}/(1 + r_t)$ is the end of period liability. The linearized version of this constraint can be written as

$$(1) \qquad \hat{\xi}_t = \phi_k \hat{k}_{t+1} + \phi_b \hat{b}_{t+1}^e + \hat{y}_t,$$

where $\phi_k = -\bar{\xi}\bar{k}/\bar{y}$ and $\phi_b = \bar{\xi}\bar{b}^e/\bar{y}$. The hat sign denotes percent deviations from the steady state and the bar sign denotes steady state values. After parameterizing the model using steady state targets (see first set of parameters in Table 2 in the paper), we determine the coefficients $\phi_k = -1.5489$ and $\phi_b = 0.5489$. We can then use the above equation to construct the $\hat{\xi}_t$ series once we have empirical measurements for the end of period capital, $\hat{k}_{t+1}$, the end of period liabilities, $\hat{b}_{t+1}^e$, and output $\hat{y}_t$. Following is the description of how we construct these series.

The *Capital Stock* is constructed using the equation

$$k_{t+1} = k_t - Depreciation + Investment.$$

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Depreciation is measured as 'Consumption of fixed capital in nonfinancial corporate business' (Table F.8, line 14) plus 'Consumption of fixed capital in nonfinancial noncorporate business' (Table F.8, line 15). *Investment* is measured as 'Capital expenditures in nonfinancial business' (Table F.101, line 4). Both variables are deflated by the price index for business value added from NIPA (Table 1.3.4). We start the recursion in the first quarter of 1952. The initial k_t is chosen so that the capital-output ratio in the business sector does not display any trend during the sample period 1952-2010. Since the initial value of k_t is important only for the early dates, this is not relevant for our results based on the subperiod 1984-2010. The $\hat{k}_{t+1}$ series used in equation (1) is constructed by linearly detrending the log of k_{t+1} over the period 1984.I-2010.II.

The *Debt Stock* is constructed using the equation

$$b_{t+1}^e = b_t^e + Net\, New\, Borrowing.$$

Notice that we use the variable $b_{t+1}^e = b_{t+1}/(1+r_t)$ instead b_{t+1} because this is the model equivalent of the end-of-period debt reported in the data. *Net new borrowing* is measured by the 'Net increase in credit markets instruments of nonfinancial business' (Table F.101, line 28). Since the constructed stock of debt is measured in nominal terms, we deflate it by the price index for business value added from NIPA (Table 1.3.4). The initial (nominal) stock of debt is set to 94.12, which is the value reported in the balance sheet data from the Flow of Funds in 1952.I for the nonfarm nonfinancial business (Table B.102, line 22 and Table B.103, line 24). Since we start the recursion in 1952, the initial value is not important for the results of the paper based on the subperiod 1984-2010. We do not use directly the series for the stocks of debt because they are not seasonally adjusted. The $\hat{b}_{t+1}^e$ series used in equation (1) is constructed by linearly detrending the log of b_{t+1}^e over the period 1984.I-2010.II.

For *Output* we use business value added from NIPA (Table 1.3.5) deflated by the price index for business value added also from NIPA (Table 1.3.4). This gives us y_t. The $\hat{y}_t$ series used in equation (1) is constructed by linearly detrending the log of y_t over the period 1984.I-2010.II.

Construction of productivity shocks

To construct the TFP series from the production function $y_t = z_t k_t^\theta n_t^{1-\theta}$, we need three data series: capital k_t, labor n_t and output y_t.

The *Capital Stock* and *Output* are constructed through the same procedures as described above. *Labor* is measured as 'total private aggregate weekly hours' from the Current Employment Statistics, national survey.

Data series for the structural estimation

GDP is 'Real gross domestic product' from NIPA (Table 1.1.6). *Consumption* is 'Real personal consumption expenditures' from NIPA (Table 1.1.6). *Investment* is 'Real gross private domestic investment' from NIPA (Table 1.1.6). *Working Hours* is 'total private aggregate weekly hours' from the Current Employment Statistics, national survey. *Wages* are 'Real hourly compensation in the business sector' from the Bureau of Labor Statistics. *Nominal Price* is the 'implicit price deflator for GDP' from NIPA (Table 1.1.9). *Interest Rate* is the 'Federal fund rate' published by the Federal Reserve Board. We construct the quarterly series from the monthly data by averaging over three months. *Debt Repurchase* is defined in the data source for Figure 1 and Table 1.

First order conditions for the firm's problem

Consider the optimization problem (3) in the paper and let λ and μ be the Lagrange multipliers for the budget and enforcement constraints. Differentiating we get:

$$
\begin{aligned}
d &: \quad 1 - \lambda \varphi_d(d) = 0 \\
n &: \quad \lambda F_n(z, k, n) - \lambda w - \mu F_n(z, k, n) = 0 \\
k' &: \quad Em'V_k(\mathbf{s}'; k', b') - \lambda + \mu \xi = 0 \\
b' &: \quad Em'V_b(\mathbf{s}'; k', b') + \frac{\lambda}{R} - \frac{\mu \xi}{1+r} = 0.
\end{aligned}
$$

The envelope conditions are

$$
\begin{aligned}
V_k(\mathbf{s}; k, b) &= \lambda \Big[1 - \delta + F_k(z, k, n) \Big] - \mu F_k(z, k, n), \\
V_b(\mathbf{s}; k, b) &= -\lambda.
\end{aligned}
$$

Using the first condition to eliminate λ plus the envelope conditions we get equations (4)-(6) reported in the paper.

Numerical solution

We solve the model using a linear approximation under the assumption that the enforcement constraint is always binding. To check the accuracy of the linear solution and especially the assumption that the enforcement constraint is always binding, we also solve the model nonlinearly using a global approximation method. The linear and nonlinear methods are described below.

LINEAR APPROXIMATION

If the enforcement constraint is always binding, we can solve the model by log-linearizing the dynamic system around the steady state. The equilibrium is characterized by the

following system of dynamic equations:

$$wU_c(c,n) + U_n(c,n) = 0,$$

$$U_c(c,n) = \beta \left(\frac{R-\tau}{1-\tau} \right) EU_c(c',n'),$$

$$wn + b - \frac{b'}{R} + d - c = 0,$$

$$F_n(z,k,n) = w \left(\frac{1}{1 - \mu\varphi_d(d)} \right),$$

$$E\, \tilde{m}(c,n,d,c',n',d') \left[1 - \delta + (1 - \mu'\varphi_d(d'))F_k(z',k',n') \right] + \xi\mu\varphi_d(d) = 1,$$

$$RE\, \tilde{m}(c,n,d,c',n',d') + \xi\mu\varphi_d(d) \left(\frac{R(1-\tau)}{R-\tau} \right) = 1,$$

$$(1-\delta)k + F(z,k,n) - wn - b + \frac{b'}{R} - k' - \varphi(d) = 0,$$

$$\xi \left(k' - b' \frac{1-\tau}{R-\tau} \right) = F(z,k,n).$$

The first three equations are the first order conditions and budget constraint for households. In equilibrium the tax payments of households is accounted by a lower interest earned on bonds, R, and the gross (before tax) interest rate is $1 + r = (R - \tau)/(1 - \tau)$. The next three equations are the first order conditions for firms. The term $\tilde{m}(c,n,d,c',n',d') = \beta(U_c(c',n')/U_c(c,n))(\varphi_d(d)/\varphi_d(d'))$ is the effective discount factor. The remaining two equations are the firms' budget and enforcement constraints.

We have eight dynamic equations. After linearizing around the steady state we can solve the system for the eight variables c_t, d_t, n_t, w_t, R_t, μ_t, k_{t+1}, b_{t+1}, as linear functions of the states, z_t, ξ_t, k_t, b_t.

NONLINEAR APPROXIMATION

The nonlinear approach approximates the three conditional expectations in the second, fifth and sixth equations with functions that interpolate linearly between the grid points of the four-dimensional state space (z, ξ, k, b). Starting with initial guesses for the conditional expectations at the grid points, we can compute all variables of interest by solving a system of nonlinear equations.

At each grid point, we first solve the system assuming that the enforcement constraint is binding. If the solution for the multiplier is negative, we set it equal to zero, and then solve the system ignoring the enforcement constraint. In doing so we essentially check the Kuhn-Tucker conditions at each grid point. Once we have solved for all of the grid points, we update the guesses for the conditional expectations and keep iterating until convergence. The new guesses are produced through Gauss-Hermite quadrature (z_t and ξ_t are lognormal).

We found that the values of the Lagrange multiplier obtained from the simulation of the model using the nonlinear solution are almost indistinguishable to those obtained with the linear solution, after an initial period of adjustment. The same holds true for output, hours and financial flows.

The solution method described here is also used to solve the version of the model with an alternative specification of the enforcement constraint (see the sensitivity section in

the paper). As shown in Figure 9, the Lagrange multiplier for this version of the model becomes zero in some of the simulation periods.

Deriving the linearized wage equation in the extended model

Differentiating the objective (16) with respect to $w_{j,t}$ we get:

$$E_t \sum_{s=0}^{\infty} (\beta\omega)^s \gamma_{t+s} \left\{ -U_{j,3,t+s} \frac{v_{t+s}}{v_{t+s}-1} w_{j,t}^{-\frac{v_{t+s}}{v_{t+s}-1}-1} \left(\frac{1}{W_{t+s}} \right)^{-\frac{v_{t+s}}{v_{t+s}-1}} n_{t+s} - \right.$$

$$\left. \lambda_{t+s} \left(\frac{1}{v_{t+s}-1} \right) w_{j,t}^{-\frac{v_{t+s}}{v_{t+s}-1}} \left(\frac{1}{W_{t+s}} \right)^{-\frac{v_{t+s}}{v_{t+s}-1}} n_{t+s} \right\} = 0$$

where the second subscript in the utility function denotes the derivative.

Using the fact that $\lambda_{t+s} = U_{2,t+s}$, this can be re-arranged as

$$E_t \sum_{s=0}^{\infty} (\beta\omega)^s \gamma_{t+s} n_{t+s|t} U_{2,t+s} \left(\frac{1}{v_{t+s}-1} \right) \left\{ \frac{w_t}{P_{t+s}} - v_{t+s}\Lambda_{t+s|t} \right\} = 0,$$

where $\Lambda_{j,t+s|t} \equiv -\frac{U_{j,3,t+s|t}}{U_{2,t+s}}$ is the marginal rate of substitution for workers that reset their wage in period t, and $n_{j,t+s|t}$ is the labor supply of a worker that has changed the wage at time t. The j index can be dropped since all households that reset their wage at time t will make the same choice. Furthermore, because of separability in the utility function and the assumption that households can purchase contingent claims, $U_{2,t+s}$ is the same for all agents independently of whether they are able to reset their wages.

Linearizing the first order condition around the steady state we obtain

$$E_t \sum_{s=0}^{\infty} (\beta\omega)^s \left[\frac{w}{P} \frac{\partial w_{j,t}}{w} - \frac{w}{P} \frac{\partial P_{t+s}}{P} - v\Lambda \frac{\partial v_{t+s}}{v} - v\Lambda \frac{\partial \Lambda_{t+s|t}}{\Lambda} \right] = 0,$$

which takes into account that in a steady state $w/P = W/P = v\Lambda$.

Re-arranging, the log-deviation of the wage, $\partial w_{j,t}/w = \hat{w}_t$, can be written as

$$\hat{w}_t = (1-\beta\omega) E_t \sum_{s=0}^{\infty} (\beta\omega)^s \left[\hat{P}_{t+s} + \hat{v}_{t+s} + \hat{\Lambda}_{t+s|t} \right].$$

Consider now the marginal rate of substitution in log form, $\ln\Lambda_{t+s|t} = \ln(-U_{3,t+s}) - \ln U_{2,t+s}$. Because $-U_{3,t+s} = An_{t+s|t}^{1/\varepsilon}$ we have

$$\ln \Lambda_{t+s|t} = -\ln U_{2,t+s} + \frac{1}{\varepsilon} \ln n_{t+s|t} + \ln A.$$

Replacing $n_{t+s|t}$ by the labor demand (equation (15) in the paper), we obtain

$$\ln \Lambda_{t+s|t} = -\ln U_{2,t+s} + \frac{1}{\varepsilon} \ln n_{t+s} - \frac{1}{\varepsilon} \left(\frac{v_t}{v_t - 1} \right) (\ln w_t - \ln W_{t+s}) + \ln A.$$

The linearized version is then

$$\widehat{\Lambda}_{t+s|t} \;\;=\;\; -\widehat{U}_{2,t+s} + \frac{1}{\varepsilon}\,\widehat{n}_{t+s} - \left(\frac{\upsilon}{(\upsilon-1)\varepsilon}\right)\widehat{w}_t + \left(\frac{\upsilon}{(\upsilon-1)\varepsilon}\right)\widehat{W}_{t+s}.$$

We can now substitute this term in the linearized equation for the wage. After collecting terms and re-arranging we get

$$\widehat{w}_t = \Phi E_t \sum_{s=0}^{\infty}(\beta\omega)^s \left[\widehat{P}_{t+s} + \widehat{v}_{t+s} - \widehat{U}_{2,t+s} + \frac{1}{\varepsilon}\,\widehat{n}_{t+s} + \left(\frac{\upsilon}{(\upsilon-1)\varepsilon}\right)\widehat{W}_{t+s}\right],$$

where $\Phi = \frac{\varepsilon(\upsilon-1)(1-\beta\omega)}{\varepsilon(\upsilon-1)+\upsilon}$.

The linearized wage equation can be written recursively as follows:

$$\widehat{w}_t = \Phi\widehat{P}_t + \Phi\widehat{v}_t - \Phi\widehat{U}_{c,t} + \frac{\Phi}{\varepsilon}\,\widehat{n}_t + \frac{\upsilon\Phi}{(\upsilon-1)\varepsilon}\widehat{W}_t + \beta\omega E_t\widehat{w}_{t+1}$$

Using the functional form of the utility function, the marginal utility of consumption in linearized form can be written as

$$\widehat{U}_{2,t} = \left(\frac{h\sigma}{1-h}\right)\hat{c}_{t-1} - \left(\frac{\sigma}{1-h}\right)\hat{c}_t.$$

Thus, the final expression for the linearized wage equation is

$$\widehat{w}_t = -\left(\frac{h\sigma\Phi}{1-h}\right)\hat{c}_{t-1} + \left(\frac{\sigma\Phi}{1-h}\right)\hat{c}_t + \Phi\widehat{P}_t + \Phi\widehat{v}_t + \frac{\Phi}{\varepsilon}\,\widehat{n}_t + \frac{\upsilon\Phi}{(\upsilon-1)\varepsilon}\widehat{W}_t + \beta\omega E_t\widehat{w}_{t+1}.$$

Extended model

This section of the appendix describes the full set of equations that characterize the equilibrium of the extended model studied in Section IV of the paper. This extends the model estimated by Smets and Wouters (2007) by adding financial frictions and financial shocks. The log-linearized version of the model is estimated structurally using Bayesian methods. We first derive the first order conditions of the firm and then we provide the full list of equations.

First order conditions for the firm's problem

Equation (25) in the paper reports the optimization problem faced by the firm. Denoting by λ, μ, χ, Q the Lagrange multipliers associated with the four constraints, the

first order conditions are:

$$\lambda_t = \frac{1}{P_t \varphi_d(d_t)},$$

$$\left(1 - \frac{\mu_t}{\lambda_t P_t}\right) F_{l,t} = \frac{W_t}{P_t} + \frac{\chi_t D_{l,t}}{\lambda_t P_t},$$

$$\left(1 - \frac{\mu_t}{\lambda_t P_t}\right) F_{u,t} = \frac{\Psi_{u,t} k_t}{\lambda_t P_t} + \frac{\chi_t D_{u,t}}{\lambda_t P_t},$$

$$\lambda_t P_t G_{p,t} + E\, m_{t+1} \lambda_{t+1} P_{t+1} G_{p-1,t+1} = \frac{\chi_t}{P_t}$$

$$Q_t \Upsilon_{i,t} + E m_{t+1} Q_{t+1} \Upsilon_{-i,t+1} = \lambda_t P_t,$$

$$Q_t = E m_{t+1} \left\{ (1-\delta) Q_{t+1} + \lambda_{t+1} P_{t+1} \left(F_{k,t+1} - \Psi(u_t) \right) - \mu_{t+1} F_{k,t+1} - \chi_{t+1} D_{k,t+1} \right\} + \xi_t \mu_t,$$

$$1 = R_t E m_{t+1} \frac{\lambda_{t+1}}{\lambda_t} + \frac{\mu_t \xi_t}{\lambda_t P_t} \left(\frac{R_t}{1 + r_t} \right).$$

Dynamic system

We can now list the complete set of dynamic equations. We also provide the list of variables that enter each individual equation. In reporting the first order conditions for the firm we eliminate the Lagrange multiplier λ_t using the condition $\lambda_t = 1/P_t \varphi_d(d_t)$.

1) Households' euler equation for bonds:

$$(1 + r_t) E m_{t+1} \frac{P_t}{P_{t+1}} - 1 = 0$$

$$\mathtt{f}(\gamma_t, c_{t-1}, R_t, P_t, c_t, \gamma_{t+1}, P_{t+1}, c_{t+1}) = 0$$

2) Capital utilization:

$$\left(1 - \mu_t \varphi_{d,t}\right) F_{u,t} - \Psi_{u,t} k_t \varphi_{d,t} - \chi_t D_{u,t} \varphi_{d,t} = 0$$
$$\mathtt{f}(z_t, \eta_t, k_t, n_t, u_t, d_t, \mu_t, \chi_t) = 0$$

3) Euler equation for capital:

$$E m_{t+1} \left\{ (1-\delta) Q_{t+1} + \frac{F_{k,t+1} - \Psi_t}{\varphi_{d,t+1}} - \mu_{t+1} F_{k,t+1} - \chi_{t+1} D_{k,t+1} \right\} + \xi_t \mu_t - Q_t = 0$$

$$\mathtt{f}(\gamma_t, \xi_t, c_{t-1}, c_t, \mu_t, Q_t, z_{t+1}, \gamma_{t+1}, \eta_{t+1}, k_{t+1}, n_{t+1}, u_{t+1}, d_{t+1}, c_{t+1}, \mu_{t+1}, \chi_{t+1}, Q_{t+1}) = 0$$

4) Price of capital:

$$Q_t \Upsilon_{i,t} + E m_{t+1} Q_{t+1} \Upsilon_{i-1,t+1} - \frac{1}{\varphi_{d,t}} = 0$$

$$\mathtt{f}(\zeta_t, \gamma_t, i_{t-1}, c_{t-1}, d_t, c_t, i_t, Q_t, \zeta_{t+1}, \gamma_{t+1}, c_{t+1}, i_{t+1}, Q_{t+1}) = 0$$

5) Law of motion for capital:

$$(1 - \delta)k_t + \Upsilon_t - k_{t+1} = 0$$

$$\mathtt{f}(\zeta_t, i_{t-1}, k_t, i_t, k_{t+1}) = 0$$

6) New wage (linearized):

$$-\left(\frac{h\sigma\Phi}{1-h}\right)c_{t-1} + \left(\frac{\sigma\Phi}{1-h}\right)c_t + \Phi P_t + \Phi v_t + \frac{\Phi}{\varepsilon}n_t + \frac{\upsilon\Phi}{(\upsilon-1)\varepsilon}W_t + \beta\omega E_t w_{t+1} - w_t = 0$$

where $\Phi = \frac{\varepsilon(\upsilon-1)(1-\beta\omega)}{\varepsilon(\upsilon-1)+\upsilon}$.

$$\mathtt{f}(v_t, c_{t-1}, w_t, W_t, P_t, n_t, c_t, w_{t+1}) = 0$$

7) Wage index:

$$\left[\omega W_{t-1}^{\frac{1}{1-v_t}} + (1-\omega)w_t^{\frac{1}{1-v_t}}\right]^{1-v_t} - W_t = 0$$

$$\mathtt{f}(W_{t-1}, w_t, W_t) = 0$$

8) Labor demand:

$$(1 - \mu_t\varphi_{d,t})F_{n,t} - \frac{W_t}{P_t} - \chi_t\varphi_{d,t}D_{n,t} = 0$$

$$\mathtt{f}(z_t, \eta_t, k_t, W_t, P_t, n_t, u_t, d_t, \mu_t, \chi_t) = 0$$

9) Bond demand:

$$R_t E\, m_{t+1}\left(\frac{P_t\varphi_{d,t}}{P_{t+1}\varphi_{d,t+1}}\right) + \xi_t\mu_t\varphi_{d,t}\left(\frac{R_t}{1+r_t}\right) - 1 = 0$$

$$\mathtt{f}(\gamma_t, \xi_t, c_{t-1}, R_t, P_t, d_t, c_t, \mu_t, \gamma_{t+1}, P_{t+1}, d_{t+1}, c_{t+1}) = 0$$

10) Nominal price:

$$P_t\left[G_{2,t} + E\, m_{t+1}\left(\frac{\varphi_{d,t}}{\varphi_{d,t+1}}\right)G_{1,t+1}\right] - \chi_t\varphi_{d,t} = 0$$

$$\mathtt{f}(\gamma_t, P_{t-1}, c_{t-1}, P_t, d_t, c_t, Y_t, \chi_t, \gamma_{t+1}, P_{t+1}, d_{t+1}, c_{t+1}, Y_{t+1}) = 0$$

11) Firm's value:

$$d_t + Em_{t+1}V_{t+1} - V_t = 0$$

$$\mathtt{f}(\gamma_t, c_{t-1}, d_t, c_t, V_t, \gamma_{t+1}, c_{t+1}, V_{t+1}) = 0$$

12) Enforcement constraint:

$$\xi_t\left(k_{t+1} - \frac{b_{t+1}}{P_t(1+r_t)}\right) - F_t = 0$$

$$\mathtt{f}(z_t, \xi_t, k_t, R_t, P_t, n_t, u_t, k_{t+1}, b_{t+1}) = 0$$

13) Firm's budget:

$$P_t\left[F_t - \Psi_t k_t\right] + \frac{b_{t+1}}{R_t} - b_t - W_t n_t - P_t G_t - P_t \varphi_t - P_t i_t = 0$$

$$\mathtt{f}(z_t, P_{t-1}, k_t, b_t, W_t, R_t, P_t, n_t, u_t, d_t, i_t, Y_t, b_{t+1}) = 0$$

14) Household's budget:

$$W_t n_t + P_t d_t - \frac{b_{t+1}}{1 + r_t} + b_t - P_t c_t - T_t = 0$$

$$\mathtt{f}(b_t, W_t, R_t, P_t, n_t, d_t, c_t, T_t, b_{t+1}) = 0$$

15) Government budget:

$$P_t G_t + B_{t+1}\left(\frac{1}{R} - \frac{1}{1 + r_t}\right) - T_t = 0$$

$$\mathtt{f}(G_t, R_t, P_t, T_t, B_{t+1}) = 0$$

16) Monetary policy (linearized):

$$a_1 r_{t-1} + a_2(P_t - P_{t-1}) + a_3(Y_t - Y_t^*) + a_4(Y_{t-1} - Y_{t-1}^*) + \varsigma_t - r_t = 0$$

where $a_1 = \rho_R$, $a_2 = (1 - \rho_R)\nu_1$, $a_3 = (1 - \rho_R)\nu_2 + \nu_3$, $a_4 = -\nu_3$.

$$\mathtt{f}(\varsigma_t, R_{t-1}, P_{t-1}, Y_{t-1}, R_t, P_t, Y_t) = 0$$

17) Output:
$$F_t - Y_t = 0$$
$$\mathtt{f}(z_t, k_t, n_t, u_t, Y_t) = 0$$

18) Debt repurchase:
$$\frac{b_t/(1 + r_{t-1}) - b_{t+1}/(1 + r_t)}{Y_t} - x_t = 0$$

$$\mathtt{f}(R_{t-1}, b_t, R_t, Y_t, x_t, b_{t+1}) = 0$$

Taking into account that $1 + r = (R - \tau)/(1 - \tau)$, we can use the linearized version of these equations to solve for 18 variables (R_t, P_t, c_t, n_t, u_t, d_t, μ_t, χ_t, x_t, Q_t, i_t, w_t, W_t, Y_t, V_t, T_t, k_{t+1}, b_{t+1}) as a function of 16 states (z_t, ζ_t, γ_t, η_t, υ_t, G_t, ς_t, ξ_t, p_{t-1}, i_{t-1}, c_{t-1}, W_{t-1}, R_{t-1}, Y_{t-1}, k_t, b_t).