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**Technical Appendix:**  
**Unmeasured Investment and the Puzzling U.S. Boom in the 1990s\***

ELLEN R. MCGRATTAN  
Federal Reserve Bank of Minneapolis  
and University of Minnesota

EDWARD C. PRESCOTT  
Arizona State University  
and Federal Reserve Bank of Minneapolis

\* The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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# Chapter 1.

## Introduction

In this appendix, we provide supporting evidence for claims made in our paper “Unmeasured Investment and the Puzzling U.S. Boom in the 1990s” and address issues raised in seminars. Here, we summarize our findings as responses to four common myths (which arise in most discussions of the paper).

The first common myth is that intangible investment is simply a free parameter that makes up for whatever is missing to make standard theory work. To dispel this myth, we apply our methodology to three theories of the U.S. boom in the 1990s. *All three theories generate paths of GDP, consumption, investment, and hours that match U.S. data perfectly.* Despite the perfect fit for all theories, only one of the three theories satisfies our criteria for a successful theory.<sup>1</sup> One of the unsuccessful theories does include intangible capital and does generate a boom in the 1990s, but does not satisfy our criteria.

We also demonstrate that we would get a very different result if the data-generating mechanism were in fact inconsistent with our theory of intangible investment and non-neutral technological change. This is a slightly different way of making the point that intangible investment is not a free parameter that makes up for whatever is missing. We set up the following experiment. First, we generate artificial data from a model with *no* intangible investment that has hours fluctuating only because there have been changes in labor market distortions (which are not due to changes in government labor tax rates).

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<sup>1</sup> Our *input justification criterion* requires the exogenous inputs to be consistent with micro and macro evidence. Our *prediction criterion* requires conformity of theory and observed time series that were not used to set parameters or exogenous inputs.

Second, with these data, we assess the theory *with* intangible investment and non-neutral technology. In this case, we find that our theory would satisfy neither our input justification criterion nor our prediction criterion.

The second common myth is that the neoclassical growth model does poorly over the *entire postwar period*—especially with regard to movements in hours of work—and not just in the 1990s as we claimed. For example, Schmitt-Grohe and Uribe (2005) motivated their work in an interview with the *NBER Reporter* by noting that “by the late 1990s empirical research using macroeconomic data from industrialized countries had cast compelling doubts on the ability of the neoclassical growth model to provide a satisfactory account of aggregate fluctuations” (p. 19). In dispelling the myth that neoclassical theory is doomed, we draw heavily on the work of Uhlig (2003) and Chen, İmrohoroğlu, and İmrohoroğlu (2007).<sup>2</sup>

A third common myth is that the class of new Keynesian models that Schmitt-Grohe, Uribe, and others have adopted provides a better understanding of business cycles. With a new Keynesian model developed and used by Smets and Wouters (2007) to study U.S. business cycles, we show that this current-generation model, *which is designed to perfectly fit seven U.S. time series—GDP, consumption, investment, business hours, business wages, the federal funds rate, and inflation—fails to satisfy our criteria for a successful theory.*

The fourth and final myth is that conclusions based on perfect-foresight analyses are not robust. We use models with a reasonable amount of stochastic variation in the key exogenous variables to show that the specific realizations of these exogenous stochastic

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<sup>2</sup> We thank Harold Uhlig, Kaiji Chen, and Rafael Wouters for providing us with data and codes used in their papers that we discuss in this appendix. This made it relatively easy to reproduce their research and to make direct comparisons between our work and theirs.

variables, not the choice of household expectations, are what is important. We also do sensitivity analysis with respect to choices of parameters of preferences and technologies to show that our findings and conclusions are robust.

## Chapter 2.

### Assessing Three Theories

In this chapter we consider three theories. All three generate equilibrium paths for GDP, consumption, investment, and hours *that exactly match the U.S. time series during the 1990s*. The fact that all three theories can generate the boom of the 1990s—the phenomenon that is central to our paper—does not mean, however, that we view all of them as “successful.” In fact, we will demonstrate later on that two of these theories are actually unsuccessful *in our sense of the word* because they do not satisfy the input justification criterion or the prediction criterion described in the paper. We deem them unsuccessful *even though they can generate the U.S. boom of the 1990s*. We finish the chapter by addressing the question, Could our preferred theory with intangible investment and non-neutral technological change ever fail to satisfy the criteria for a successful theory that we propose, or are we simply setting the bar too low for ourselves?

To satisfy the input justification criterion, the exogenous inputs of the theoretical model must be consistent with micro and macro empirical evidence. This criterion requires a theory for the exogenous inputs. To satisfy the prediction criterion, the model must not make counterfactual predictions. This is a minimum requirement. A stronger requirement is that the theory must predict time series that were not used to set parameters or exogenous inputs. For example, we can use the theories to make predictions for incomes and capital gains—data that were not used in setting any of the exogenous variables.

For each theory that we investigate, we generate an exact match between predicted and actual paths for GDP, consumption, investment, and hours by introducing either a

*labor wedge* or an *investment wedge* or both. The labor wedge is an exogenous input that results in an exact fit for the household's intratemporal first-order condition relating the marginal rate of substitution between leisure and consumption and the marginal product of labor.<sup>3</sup> The investment wedge is an exogenous input that results in an exact fit for the household's intertemporal first-order condition relating the intertemporal marginal rate of substitution and the intertemporal marginal rate of transformation.

For a theory to satisfy the input justification criterion, we require either (i) the variation in U.S. time series attributed to the wedges is small or (ii) some empirical justification for these wedges. If no empirical support is available, then theory is, for all practical purposes, vacuous. For a theory to successfully resolve the puzzling U.S. boom of the 1990s, it must then satisfy the more demanding prediction criterion.

The first theory we analyze in this section is the standard *theory without intangible capital*, commonly referred to as neoclassical growth theory. We consider a specific model with fluctuations driven by TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. We consider a version of the model with one sector that combines business and non-business activity and another version that distinguishes between them. We do both because the behavior of economy-wide TFP and of business-sector TFP was quite different during the 1990s. The economy-wide TFP was a little below trend throughout the 1990s, and the business-sector TFP started below trend and rose rapidly in the late 1990s. To give the simple theory the best chance of success, we want to allow for a rapid increase in business TFP.

Neither model for the standard theory satisfies our criteria for a successful theory.

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<sup>3</sup> Unlike Chari, Kehoe, and McGrattan (2007), we separately include tax rates on labor. The labor wedge would have to be a proxy for labor distortions other than taxes.



The labor wedge has to be huge and is inconsistent with all measures of effective tax rates and all measures of worker benefits available (e.g., tax credits and welfare). Furthermore, the model's predictions of factor incomes and capital gains are not consistent with U.S. observations.

The second and third theories include intangible capital and also distinguish between business and non-business activity. The second theory assumes that technological change is neutral with respect to two activities: production of final goods and services and production of new intangible investment goods. We refer to this as the *theory with intangible capital and neutral technology*. The specific model we analyze has fluctuations driven by the same exogenous variables used for the standard model: TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. Like the standard model without intangible capital, this extension with intangible capital *does not* satisfy our criteria for a successful theory. The labor wedge and the implied intangible investment are wildly oscillatory and inconsistent with all micro evidence on labor distortions and all direct measures of intangible investments. Furthermore, the model's predictions are also grossly inconsistent with the U.S. data. This theory shows that intangible capital is not “making up” for whatever is missing in standard theory. In fact, the theory of intangible capital with neutral technology does considerably worse than the standard theory.

The third theory assumes that technological change is non-neutral. The non-neutrality is at the heart of the theoretical contribution. We consider two different activities within the business sector and refer to the theory as the *theory with intangible capital and non-neutral technology*. The specific model that we analyze has fluctuations driven by TFP in the final goods and services sector, TFP in the intangible-investment sector, tax rates on

hours and consumption, and an investment wedge. The effect of the investment wedge—which is simply an addition to get a perfect fit—is small and well within the range of estimates of capital tax rate changes. The other inputs are consistent with micro evidence of this period in which a technology boom occurred. Specifically, they are consistent with micro evidence on R&D, which is an important component of intangible investment, and they are consistent with the shift in employment to occupations in which sweat equity is important. We compare the model’s predictions for factor incomes and capital gains, which were not used in the determination of sectoral TFPs. We show that the model does well here too.

These findings lead us to conclude that there is now one theory of the 1990s boom, whereas before there was none.

## 2.1. Theory without Intangible Capital

In this section we describe a particular growth model without intangible capital. Fluctuations in the model are driven by changes in TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. We use the model to demonstrate that this theory fails to satisfy the criteria we propose for a successful theory.

### 2.1.1. A Specific Model

Given an initial capital stock  $k_0$ , the stand-in household chooses sequences of consumption  $\{c_t\}$ , investment  $\{x_t\}$ , and hours  $\{h_t\}$  to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$c_t + x_t = r_t k_t + w_t h_t - \tau_{ct} c_t - \tau_{ht} w_t h_t - \tau_{kt} k_t - \tau_{pt} (r_t - \delta - \tau_{kt}) k_t - \tau_{xt} x_t - \tau_{dt} \{r_t k_t - x_t - \tau_{kt} k_t - \tau_{pt} (r_t - \delta - \tau_{kt}) k_t - \tau_{xt} x_t\} + Tr_t \quad (2.1.1)$$

$$k_{t+1} = [(1 - \delta) k_t + x_t] / (1 + \eta), \quad (2.1.2)$$

where variables are written in per capita terms,  $N_t$  is population at  $t$  which grows at rate  $\eta$ , and  $r_t$  and  $w_t$  are rental and wage rates. Taxes are assessed on consumption ( $\tau_c$ ), investment ( $\tau_x$ ), property ( $\tau_k$ ), profits ( $\tau_p$ ), dividends ( $\tau_d$ ), and labor income ( $\tau_h$ ). Transfers are given by  $Tr_t$ .

In equilibrium factors are paid their marginal products. Per capita output is given by

$$y_t = k_t^\theta (Z_t h_t)^{1-\theta},$$

and, therefore,  $r_t = \theta y_t / k_t$  and  $w_t = (1 - \theta) y_t / h_t$  are the rental rate and wage rate, respectively, in equilibrium. The parameter  $Z_t$  is labor-augmenting technology change which grows at rate  $\gamma$ , that is,  $Z_t = z_t (1 + \gamma)^t$  with  $z_t$  stationary.

Suppose  $U(c, h) = \log c + \psi \log(1 - h)$ . In this case, the household's first-order conditions—after substituting for  $r_t$  and  $w_t$ —are given by

$$\frac{\psi (1 + \tau_{ct}) \hat{c}_t}{1 - h_t} = (1 - \tau_{ht}) \frac{(1 - \theta) \hat{y}_t}{h_t} \quad (2.1.3)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} [R_{t,t+1} + (1 - \delta) \xi_{t,t+1}], \quad (2.1.4)$$

where  $\hat{\beta} = \beta / (1 + \gamma)$ ,  $\mu_t = 1 / [(1 + \tau_{ct}) \hat{c}_t]$ , and

$$R_{t,t+1} = \frac{1 - \tau_{d,t+1}}{(1 - \tau_{dt}) (1 + \tau_{xt})} \left[ (1 - \tau_{p,t+1}) \left( \theta \frac{\hat{y}_{t+1}}{\hat{k}_{t+1}} - \tau_{k,t+1} \right) + \delta \tau_{p,t+1} \right] \quad (2.1.5)$$

$$\xi_{t,t+1} = \frac{1 - \tau_{d,t+1}}{1 - \tau_{dt}} \cdot \frac{1 + \tau_{x,t+1}}{1 + \tau_{xt}}. \quad (2.1.6)$$

The hat on a variable indicates that it has been detrended by  $(1+\gamma)^t$ , e.g.,  $\hat{c}_t = c_t/(1+\gamma)^t$ .

To close the model, we add the resource constraint,

$$\hat{c}_t + \hat{x}_t + \hat{g}_t = \hat{y}_t.$$

When setting parameters for our numerical experiments, we use 1990 estimates from U.S. data for  $\hat{y}$ ,  $\hat{c}$ ,  $\hat{g}$ ,  $h$ , and  $\hat{k}$  along with estimates for the growth rates  $\gamma$ ,  $\eta$ , the tax rate on labor  $\tau_h$ , the tax rate on consumption  $\tau_c$ , tax rates on capital  $\tau_p$ ,  $\tau_d$ ,  $\tau_x$ ,  $\tau_k$ , and an interest rate  $i$ . We can use these estimates to evaluate the following expressions for  $\beta$ ,  $\delta$ ,  $\theta$ ,  $\psi$ , and  $z$ :

$$\beta = \frac{1 + \gamma}{1 + i} \tag{2.1.7}$$

$$\delta = \hat{x}/\hat{k} + 1 - (1 + \eta)(1 + \gamma) \tag{2.1.8}$$

$$\theta = \frac{\left(1 - \hat{\beta}(1 - \delta)\right)(1 + \tau_x) - \hat{\beta}\delta\tau_p + \hat{\beta}(1 - \tau_p)\tau_k \frac{\hat{k}}{\hat{y}}}{\hat{\beta}(1 - \tau_p)} \tag{2.1.9}$$

$$\psi = \frac{(1 - \tau_h)(1 - \theta)(1 - h)\hat{y}}{(1 + \tau_c)\hat{c}h} \tag{2.1.10}$$

$$z = \left(\hat{k}/\hat{y}\right)^{\theta/(\theta-1)} \frac{\hat{y}}{h} \tag{2.1.11}$$

with  $\hat{x} = \hat{y} - \hat{c} - \hat{g}$ .

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are as follows:  $\hat{y} = 1$  (which is a normalization),  $\hat{c} = .7626$ ,  $\hat{x} = .2377$ ,  $\hat{g} = 0$ ,  $h = .2751$ , and  $\hat{k} = 3.91$ .<sup>4</sup> The growth rates are set equal to  $\eta = 1\%$  and  $\gamma = 2\%$  and the interest rate to  $i = 4.1\%$ . Tax rates on labor and consumption are  $\tau_h = .3109$  and

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<sup>4</sup> All estimates are based on the data described in the paper. Note that we included public consumption in  $\hat{c}$  and public investment in  $\hat{x}$ . See the data appendix for further details.

$\tau_c = .0657$ , respectively. When we compute equilibrium paths for the 1990s, we assume that these tax rates affecting the intratemporal margin (2.1.3) are varying. We assumed that capital tax rates were roughly constant throughout the 1990s, since there was little change in corporate tax policy and capital tax rates have little effect on hours. The constant rates we use are as follows:  $\tau_k = .0073$ ,  $\tau_x = 0$ ,  $\tau_p = 0.1487$ , and  $\tau_d = 0.0637$ . The tax on profits  $\tau_p$  and distributions  $\tau_d$  are computed by multiplying effective corporate tax rates times the ratio of business capital to total capital. Substituting these values in the expressions (2.1.7)–(2.1.11) implies  $\beta = .98$ ,  $\delta = 0.0306$ ,  $\theta = .3358$ ,  $\psi = 1.4841$ , and  $z = 1.8243$ .

### 2.1.2. Business Cycle Accounting in the 1990s

We observe sequences for  $\hat{y}_t, \hat{c}_t, \hat{x}_t, \hat{g}_t, h_t, \tau_{ct}$ , and  $\tau_{ht}$ , and an initial capital stock for capital  $\hat{k}_0$ . The initial capital stock plus sequence of investments imply a sequence of capital stocks if we apply the capital accumulation equation in (2.1.2). Given inputs for capital and labor, we have a measure of (detrended) total factor productivity,  $A_t = \hat{y}_t / (\hat{k}_t^\theta h_t^{1-\theta})$ , which is also equal to  $z_t^{1-\theta}$ .

If we compute a perfect-foresight equilibrium path for this model, assuming households take as given time paths for TFP and tax rates on hours and consumption, we cannot get a perfect match between the model predictions and the data.<sup>5</sup> For example, if we substitute U.S. data for  $\hat{c}_t, h_t, \hat{y}_t, \tau_{ht}$ , and  $\tau_{ct}$  into (2.1.3), the relation does not hold exactly.

We could get a perfect match if we introduce a labor wedge that forces (2.1.3) to hold and an investment wedge that forces (2.1.4) to hold. Specifically, we define the labor wedge

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<sup>5</sup> Later, we relax that assumption to determine if our results are sensitive to this specification of expectations.

$L_{wt}$  and investment wedge  $X_{wt}$  as follows:

$$L_{wt} = \frac{\psi(1 + \tau_{ct})\hat{c}_t}{1 - h_t} \cdot \frac{h_t}{(1 - \theta)\hat{y}_t} \cdot \frac{1}{1 - \tau_{ht}} \quad (2.1.12)$$

$$X_{w,t+1} = \frac{\hat{\beta}(1 - \delta)\mu_{t+1}X_{wt}}{\mu_t - \hat{\beta}R_{t,t+1}\mu_{t+1}X_{wt}}\xi_{t,t+1} \quad (2.1.13)$$

with equation (2.1.13) solved recursively starting with  $X_{w0} = 1$ . Then we replace  $1 - \tau_{ht}$  in (2.1.3) with  $(1 - \tau_{ht})L_{wt}$  and  $1/(1 + \tau_{xt})$  in (2.1.4) with  $X_{wt}/(1 + \tau_{xt})$ . If there is some mismeasurement in the effective rates on labor and capital, these wedges will pick it up. Ideally, they should be quantitatively insignificant.

Increases in the wedges have a positive effect on output and hours. The labor wedge has the same effect as a tax on labor (in the form  $1 - \tau_{ht}$ ), and the investment wedge has the same effect as a tax on investment (in the form  $1/(1 + \tau_{xt})$ ). We distinguish movements in the wedges from movements in these tax rates because we want to set  $\tau_{ht}$  and  $\tau_{xt}$  equal to effective rates set by the government. Without further interpretation, these time-varying inputs are just wedges that force first-order conditions to hold. Thus, it is desirable, unless we have some theory of these wedges, that their effect be quantitatively insignificant. They should be interpreted as small measurement errors in constructing national accounts and tax data.

In Table 1, we report the values of the implied exogenous variables; when all are fed into the model, the model exactly reproduces the U.S. sequences for detrended output  $\hat{y}_t$ , detrended consumption  $\hat{c}_t$ , detrended investment  $\hat{x}_t$ , and hours of work  $h_t$ . This is true by construction.

Figure 1 is a comparison of U.S. per capita hours and the model's prediction of per capita hours in the case that only TFP and tax rates on labor and consumption are varying

Year ( $t$ )	$A_t$	$\tau_{ht}$	$\tau_{ct}$	$L_{wt}$	$X_{wt}$
1990	1.4909	0.3109	0.0657	1.0000	1.0000
1991	1.4651	0.3070	0.0675	0.9838	1.0132
1992	1.4760	0.3028	0.0678	0.9716	1.0101
1993	1.4609	0.3034	0.0678	0.9859	1.0164
1994	1.4544	0.3068	0.0702	1.0076	1.0196
1995	1.4435	0.3116	0.0686	1.0228	1.0255
1996	1.4441	0.3190	0.0674	1.0403	1.0274
1997	1.4448	0.3254	0.0674	1.0561	1.0316
1998	1.4530	0.3327	0.0670	1.0730	1.0252
1999	1.4612	0.3335	0.0662	1.0870	1.0119
2000	1.4568	0.3424	0.0649	1.0960	1.0112
2001	1.4469	0.3472	0.0625	1.0874	1.0156
2002	1.4459	0.3076	0.0617	1.0119	1.0159
2003	1.4328	0.2885	0.0621	0.9990	1.0153

TABLE 1. EXOGENOUS VARIABLES FOR MODEL WITHOUT INTANGIBLE CAPITAL

(i.e.,  $L_{wt} = X_{wt} = 1$  for all  $t$ ). By construction, if the wedges were varying, then the model would fit exactly and the predicted and actual series would lie on top of each other. The difference in the actual and predicted series is therefore attributed to the wedges. Clearly, this difference is large.

Figure 2 compares U.S. per capita real GDP and the model's prediction for per capita real GDP. We divide both series by  $1.02^t$ , since our technological growth rate is chosen to be  $\gamma = .02$ . The model predicts a depressed economy (relative to a 2 percent trend), but the U.S. economy boomed. Figure 3 shows GDP per hour relative to the 2 percent trend for both the data and the model. This figure shows that the deviations in Figures 1 and 2 are not offsetting and, therefore, the prediction for labor productivity is also inconsistent with observations.

Figures 4 and 5 show the model predictions for per capita real investment and consumption along with U.S. data. Neither match up well.

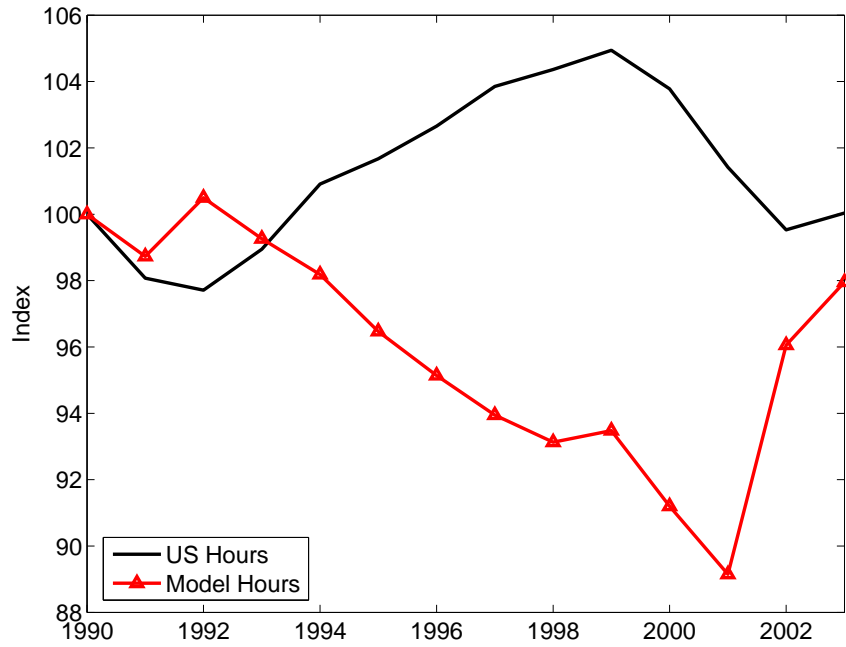


FIGURE 1. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL  
(Labor and investment wedges constant)

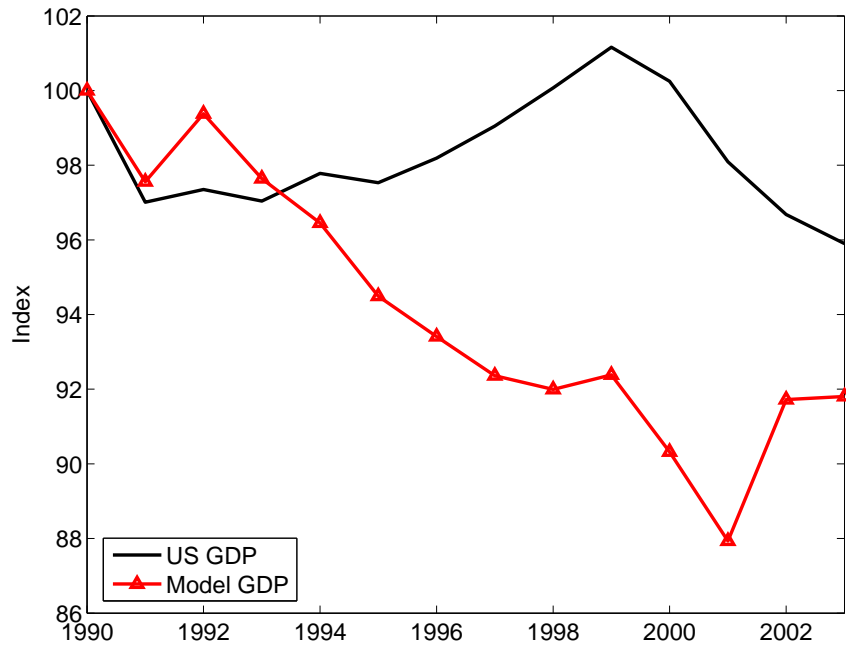


FIGURE 2. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL,  
SERIES DIVIDED BY  $1.02^t$   
(Labor and investment wedges constant)



In Figures 6 and 7, we examine the model's predictions for per capita hours without TFP or tax rates varying. In Figure 6, we plot U.S. per capita hours along with the model's prediction for hours in the case that only  $L_{wt}$  is varying. Figure 7 is the prediction when only  $X_{wt}$  is varying.<sup>6</sup> These figures show that the labor wedge is key to getting the hours boom. To generate an hours boom of 7 percent between 1992 and 1999, the labor wedge has to rise nearly 10 percent.

### *Summary*

The main problem with the standard theory driven by the labor wedge is the interpretation of the wedge. It certainly cannot be interpreted as mismeasurement of effective labor tax rates, which were rising—not falling—for all estimates we have seen. Could it be that other policies such as the Earned Income Tax Credit (EITC) or welfare benefits were affecting how much people work?<sup>7</sup> The answer, given the aggregate spending and coverage of these programs, is most surely no. For example, in 1990, the EITC total amount of credit was 7.5 billion, or roughly 0.13 times GDP. (See the U.S. House of Representatives (2004), Table 13-14.) That figure rose over the 1990s to 0.34 times GDP and then flattened. It is not clear whether it had a positive or negative effect on hours, but the upper bound of the effect on tax rates is tiny. Furthermore, the EITC credits did not decline after 1999, but hours did. There are other tax credits and income-tested benefit programs that affect hours but are much smaller than the EITC.<sup>8</sup>

Without some other empirical motivation for this wedge, the theory does not satisfy

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<sup>6</sup> When summed, the predicted series in Figures 1, 6, and 7 are approximately but not exactly equal to the U.S. series. It is not exact because there are endogenous movements in the capital stock.

<sup>7</sup> These policies appear in transfers to persons and do not come into the calculation of  $\tau_{ht}$ .

<sup>8</sup> Examples include the Work Opportunity Tax Credit, the Welfare-to-Work Tax Credit, the Welfare-to-Work Grant Program, and work-related Temporary Assistance for Needy Families. For further details, see Appendix K of the U.S. House of Representatives (2004).

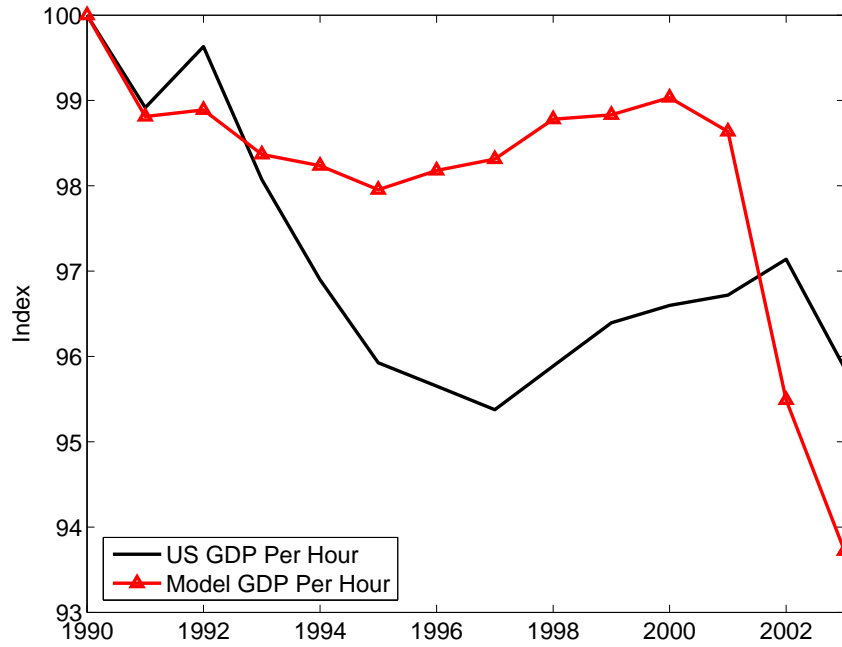


FIGURE 3. U.S. REAL GDP PER HOUR AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)

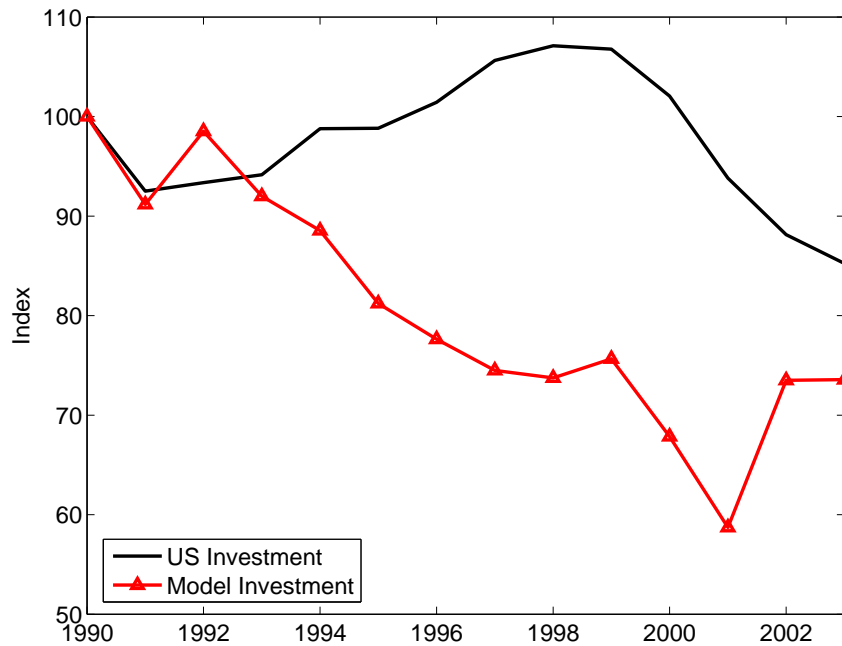


FIGURE 4. U.S. PER CAPITA REAL INVESTMENT AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)

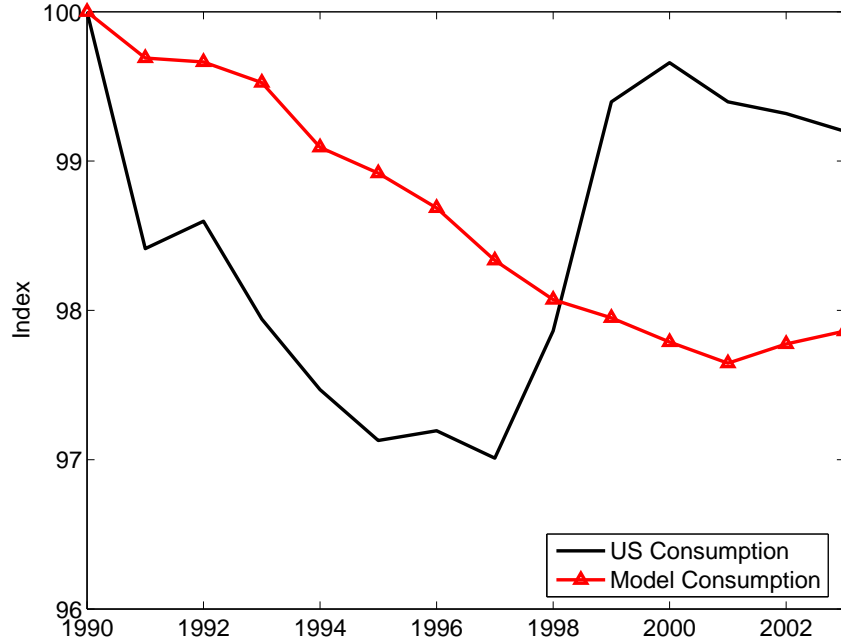


FIGURE 5. U.S. PER CAPITA REAL CONSUMPTION AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)

the input justification criterion and does not provide a plausible answer to the question, Why did hours boom in the 1990s? It also does not satisfy the prediction criterion. The model's predictions for factor incomes and capital gains are inconsistent with U.S. observations. The model's estimate of compensation is  $(1 - \theta)y_t$ . U.S. wages rose by more than U.S. output. The model's estimate for the market value of capital is  $(1 - \tau_d)k_t$ . Changes in the reproducible stock of tangible capital are much too small to rationalize the large U.S. capital gains in the late 1990s.

### 2.1.3. Investment-Specific Technical Change Consistent with NIPA

Above we chose the investment wedge to ensure an exact fit for the household's intertemporal first-order condition which relates the intertemporal marginal rate of substitution

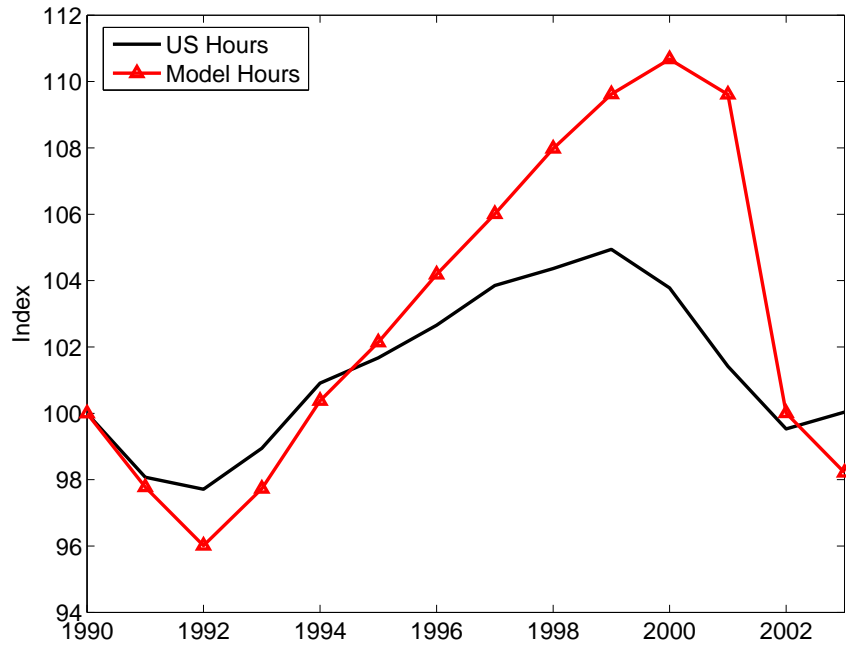


FIGURE 6. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL  
(Labor wedge only)

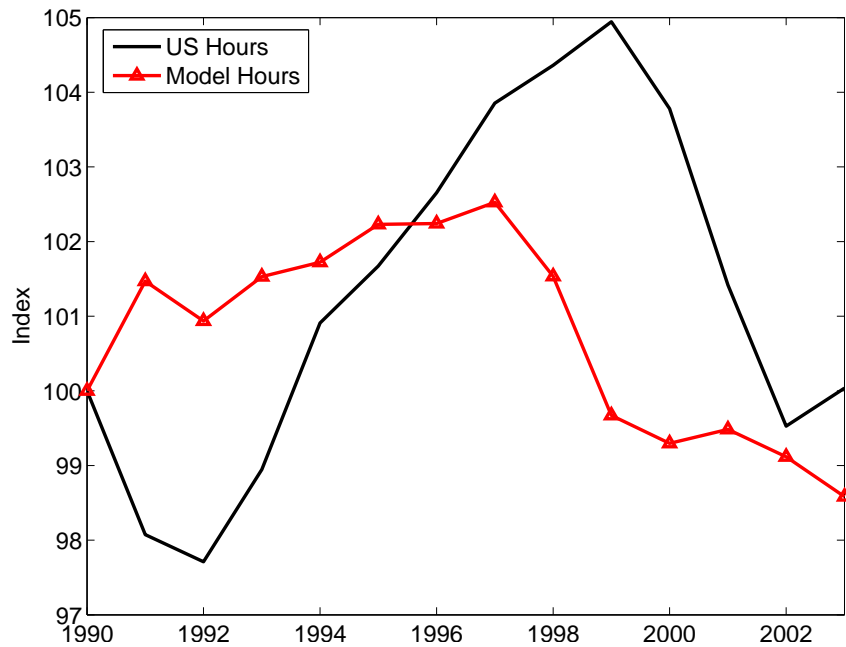


FIGURE 7. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITHOUT INTANGIBLE CAPITAL  
(Investment wedge only)

and the intertemporal marginal rate of transformation. In this section, we run a slightly different experiment and use the relative price of consumption to investment from the U.S. national accounts, which rose 22 percent between 1990 and 2003, as our input for  $X_{wt}$ . The dramatic rise in this relative price has led some to conjecture that the puzzle of the 1990s boom can be resolved by extending the standard model to permit non-neutral technology change with respect to producing *tangible* investment goods—like computer and electronic products. In this section, we use U.S. data on the relative price of consumption to tangible investment to show that this non-neutrality is not the central factor behind the puzzling boom of the 1990s.

In Figure 8, we plot an index of the ratio of the U.S. implicit price deflator for personal consumption expenditures to the implicit price deflator for gross private domestic investment with 1990 equal to 1. We use this relative price for the input  $X_{wt}$  along with measures for  $A_t$ ,  $\tau_{ht}$ , and  $\tau_{ct}$  in Table 1. We set the labor wedge to 1 in all periods, which effectively turns it off. We also set detrended government consumption equal to  $(1/X_{wt} - 1)\hat{x}_t$  so that the resource constraint is consistent with a two-sector model and non-neutral technical change.

In Figure 9, we plot the results for per capita hours along with the actual series and the results of the business cycle accounting exercise shown in Figure 1. With the investment wedge added, the model predicts a counterfactually large rise in hours between 1991 and 1992 and a decline between 1992 and 2001. The overall pattern is similar to the model with no investment wedge, although the depression in hours is less severe with the investment wedge included.

In Figure 10, we plot the results for detrended real GDP. Because real GDP in the

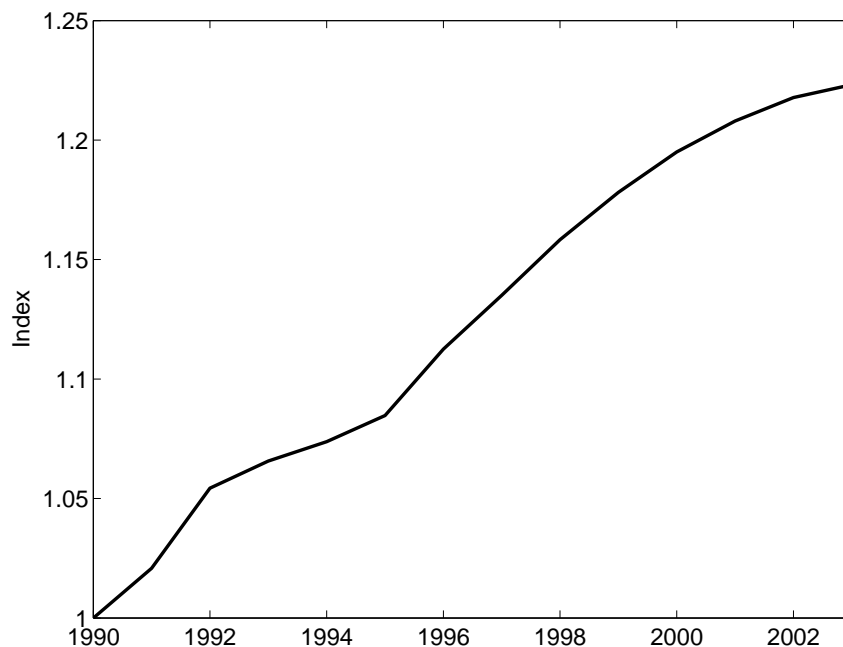


FIGURE 8. RATIO OF U.S. IMPLICIT DEFLATORS FOR PERSONAL CONSUMPTION EXPENDITURES AND GROSS PRIVATE DOMESTIC INVESTMENT

U.S. national accounts is chain-weighted, we chain-weight the model's GDP as well. This is relevant for the version of the model with non-neutral technical change in investment. Including the non-neutrality does improve the predictions for real GDP but large deviations between theory and data exist. The predicted rise in GDP between 1991 and 1992 is too large and the subsequent fall is counterfactual. There is some increase in GDP relative to trend at the end of the 1990s, but it is too small.

#### 2.1.4. A Version with a Business Sector

Next, we extend the standard model slightly to include both a business and non-business sector, where the latter includes households, government, and nonprofits. As we saw from Table 1, the TFP of the aggregate economy is falling relative to trend. Business TFP, on the other hand, rose rapidly at the end of the 1990s. Here, we investigate whether focusing on the business sector helps the standard theory satisfy our criteria for a successful theory.

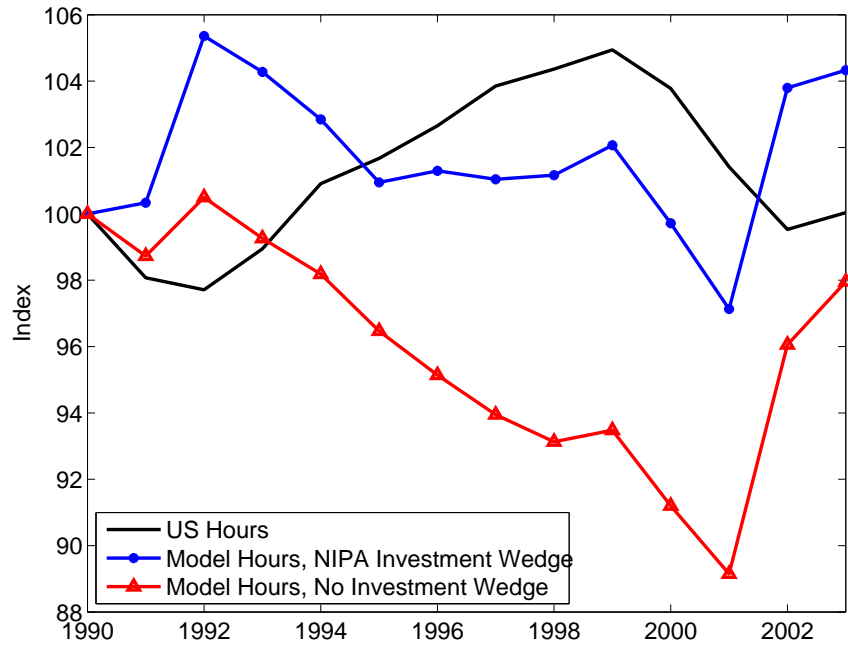


FIGURE 9. U.S. PER CAPITA HOURS AND PREDICTIONS OF MODELS WITHOUT INTANGIBLE CAPITAL  
(Labor wedge constant)

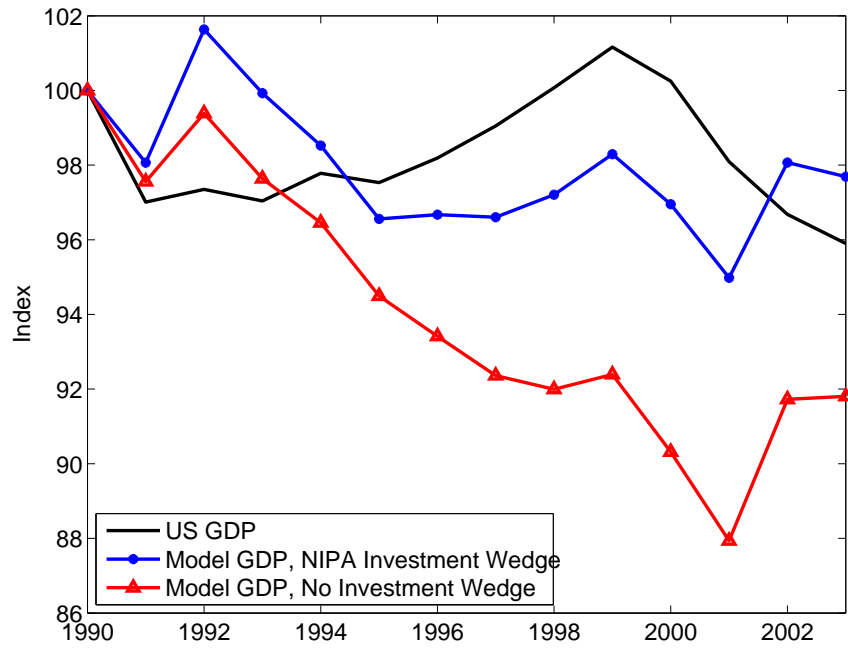


FIGURE 10. U.S. PER CAPITA REAL GDP AND PREDICTIONS OF MODELS WITHOUT INTANGIBLE CAPITAL,  
SERIES DIVIDED BY  $1.02^t$   
(Labor wedge constant)

Assume now that measured investment includes business-sector investment  $x_{bt}$  plus non-business investment  $\bar{x}_{nt}$ . Throughout, we will use  $b$  as a subscript for business and  $n$  for non-business. The problem for the household given the initial capital stock  $k_{b0}$  is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$\begin{aligned} c_t + x_{bt} &= r_t k_{bt} + w_t h_{bt} - \tau_{ct} c_t - \tau_{ht} w_t h_{bt} - \tau_{kt} k_{bt} \\ &\quad - \tau_{pt} (r_t - \delta - \tau_{kt}) k_{bt} - \tau_{xt} x_{bt} \\ &\quad - \tau_{dt} \{ r_t k_{bt} - x_{bt} - \tau_{kt} k_{bt} - \tau_{pt} (r_t - \delta - \tau_{kt}) k_{bt} - \tau_{xt} x_{bt} \} \\ &\quad + [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t \end{aligned} \tag{2.1.14}$$

$$k_{b,t+1} = [(1 - \delta) k_{bt} + x_{bt}] / (1 + \eta) \tag{2.1.15}$$

$$h_t = h_{bt} + \bar{h}_{nt}, \tag{2.1.16}$$

where  $\bar{y}_{nt}$ ,  $\bar{x}_{nt}$ ,  $\bar{h}_{nt}$ , and  $\bar{\tau}_{nt}$  are value added, investment, hours, and taxes paid, respectively, in the non-business sector.

Here and later, we assume that sequences for non-business investment, hours, and output are taken as given by the households. Essentially we are assuming that prices in the non-business sector are such that households optimally chose the U.S. levels. Treating the non-business sector this way simplifies the modeling and allows us to directly compare the model national accounts and U.S. national accounts. Furthermore, our interest is in the U.S. boom in the 1990s, which occurred in the business sector.

The resource constraint is now

$$c_t + x_{bt} + \bar{x}_{nt} + g_t = y_{bt} + \bar{y}_{nt} = y_{mt},$$



where model GDP is the measured output  $y_{mt}$ . Value added in the business sector is

$$y_{bt} = k_{bt}^\theta (Z_{bt}h_{bt})^{1-\theta},$$

where  $Z_{bt} = z_{bt}(1 + \gamma)^t$ . A NIPA accountant in this economy would measure the following product and income:

$$\text{NIPA product} = c + x_b + \bar{x}_n + g$$

$$\text{Private consumption} = c$$

$$\text{Public consumption} = g$$

$$\text{Investment} = x_b + \bar{x}_n$$

$$\text{NIPA income} = y_b + \bar{y}_n$$

$$\text{Business profits} = (r - \tau_k - \delta)k_b$$

$$\text{Business wages} = wh_b$$

$$\text{Business depreciation} = \delta k_b$$

$$\text{Business production tax} = \tau_k k_b$$

$$\text{Non-business income} = \bar{y}_n.$$

The first-order conditions for the household's problem (assuming factors are paid their marginal product) in the case of log utility are as follows:

$$\frac{\psi(1 + \tau_{ct})\hat{c}_t}{1 - h_t} = (1 - \tau_{ht}) \frac{(1 - \theta)\hat{y}_{bt}}{h_{bt}} \quad (2.1.17)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} [R_{t,t+1}^b + (1 - \delta)\xi_{t,t+1}], \quad (2.1.18)$$

where  $\hat{\beta} = \beta/(1 + \gamma)$  and  $\mu_t = 1/[(1 + \tau_{ct})\hat{c}_t]$  as before,

$$R_{t,t+1}^b = \frac{1 - \tau_{d,t+1}}{(1 - \tau_{dt})(1 + \tau_{xt})} \left[ (1 - \tau_{p,t+1}) \left( \theta \frac{\hat{y}_{b,t+1}}{\hat{k}_{b,t+1}} - \tau_{k,t+1} \right) + \delta \tau_{p,t+1} \right] \quad (2.1.19)$$

and  $\xi_{t,t+1}$  is defined in (2.1.6). Notice that the only difference between the first-order conditions (2.1.3)–(2.1.4) and (2.1.17)–(2.1.18) are the marginal products of labor and

capital, which in the first case is economy-wide and in the second case is for the business sector.

We'll again assume that we have values for some of the endogenous variables in 1990 and use them to set some of the parameters. Suppose we observe  $\hat{y}_b$ ,  $\hat{y}_n$ ,  $\hat{c}$ ,  $\hat{g}$ ,  $h$ ,  $h_b$ , and  $\hat{k}_b$  along with estimates for the growth rates  $\gamma$ ,  $\eta$ , the tax on labor  $\tau_h$ , the tax on consumption  $\tau_c$ , tax rates on capital,  $\tau_p$ ,  $\tau_d$ ,  $\tau_x$ ,  $\tau_k$ , and an interest rate  $i$ . We can use these estimates to evaluate the following expressions for  $\delta$ ,  $\theta$ ,  $\psi$ , and  $z_b$ :

$$\delta = \hat{x}_b/\hat{k}_b + 1 - (1 + \eta)(1 + \gamma) \quad (2.1.20)$$

$$\theta = \frac{\left(1 - \hat{\beta}(1 - \delta)\right)(1 + \tau_x) - \hat{\beta}\delta\tau_p + \hat{\beta}(1 - \tau_p)\tau_k \frac{\hat{k}_b}{\hat{y}_b}}{\hat{\beta}(1 - \tau_p)} \quad (2.1.21)$$

$$\psi = \frac{(1 - \tau_h)(1 - \theta)(1 - h)\hat{y}_b}{(1 + \tau_c)\hat{c}h_b} \quad (2.1.22)$$

$$z_b = \left(\hat{k}_b/y_b\right)^{\theta/(\theta-1)} \frac{\hat{y}_b}{h_b}, \quad (2.1.23)$$

where  $\hat{x}_b = \hat{y}_b + \bar{y}_n - \bar{x}_n - \hat{g} - \hat{c}$  and  $\beta = (1 + \gamma)/(1 + i)$  as before.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are as follows:  $\hat{y}_m = 1$  (which is a normalization),  $\hat{y}_b = .6621$ ,  $\bar{y}_n = .3379$ ,  $\hat{c} = .7626$ ,  $\hat{x} = .2377$ ,  $\hat{g} = 0$ ,  $h = .2751$ , and  $\hat{k}_b = 1.66$ . Growth rates, the interest rate, the discount factor, and taxes on labor and consumption are as before. The capital tax rates are now  $\tau_k = .0144$ ,  $\tau_x = 0$ ,  $\tau_p = 0.35$ , and  $\tau_d = 0.15$ , which are the effective rates for the business sector. Substituting these values in the expressions (2.1.20)–(2.1.23) implies  $\delta = 0.0331$ ,  $\theta = .277$ ,  $\psi = 1.375$ , and  $z_b = 2.176$ .

Year ( $t$ )	$A_t$	$\tau_{ht}$	$\tau_{ct}$	$L_{wt}$	$X_{wt}$
1990	1.7544	0.3109	0.0657	1.0000	1.0000
1991	1.7136	0.3070	0.0675	0.9882	1.0122
1992	1.7267	0.3028	0.0678	0.9761	1.0081
1993	1.7127	0.3034	0.0678	0.9866	1.0135
1994	1.7066	0.3068	0.0702	1.0088	1.0162
1995	1.7069	0.3116	0.0686	1.0163	1.0224
1996	1.7118	0.3190	0.0674	1.0327	1.0253
1997	1.7269	0.3254	0.0674	1.0403	1.0316
1998	1.7639	0.3327	0.0670	1.0394	1.0284
1999	1.7837	0.3335	0.0662	1.0449	1.0185
2000	1.7985	0.3424	0.0649	1.0402	1.0218
2001	1.7559	0.3472	0.0625	1.0488	1.0285
2002	1.7330	0.3076	0.0617	0.9864	1.0298
2003	1.7118	0.2885	0.0621	0.9746	1.0297

TABLE 2. EXOGENOUS VARIABLES FOR MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL

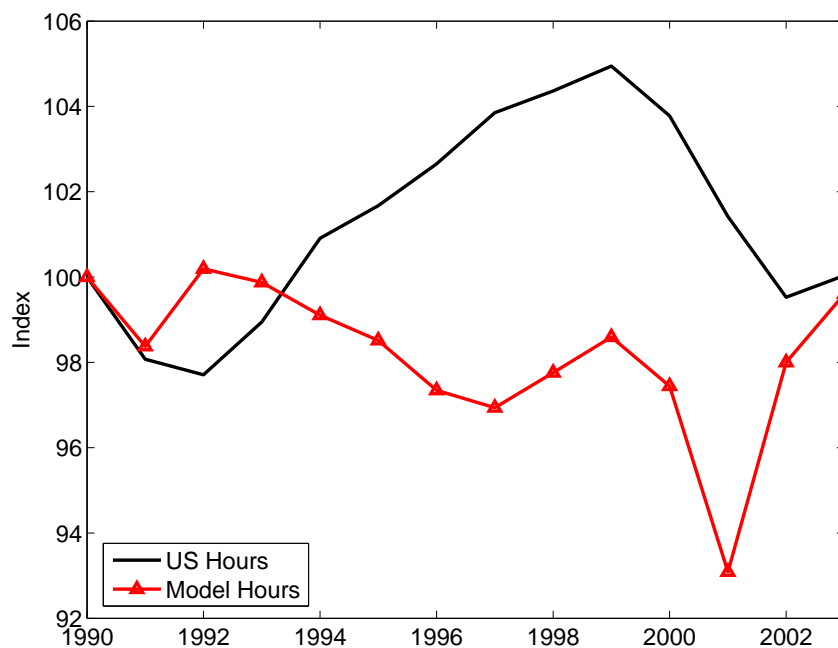


FIGURE 11. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL (Labor and investment wedges constant)

### 2.1.5. Business Cycle Accounting in the 1990s

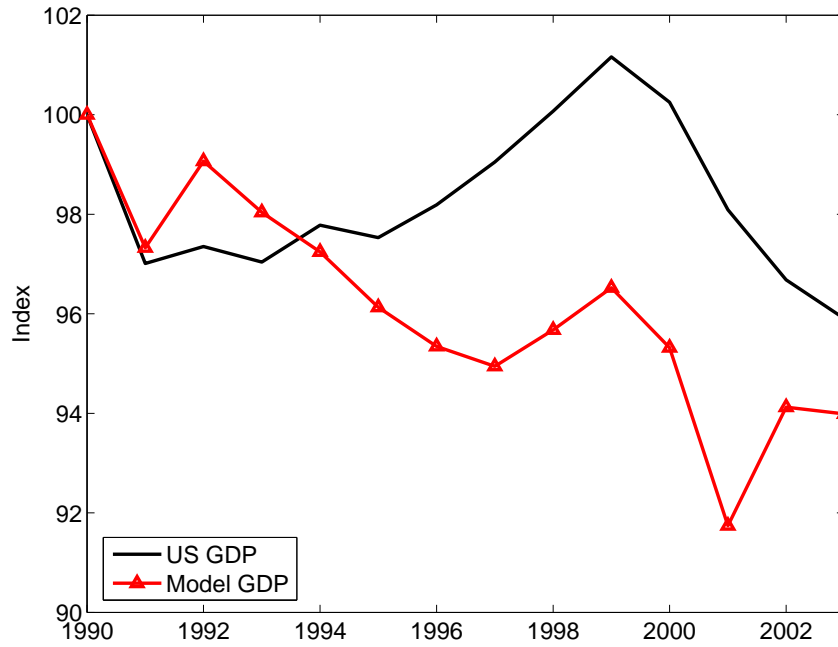


FIGURE 12. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)

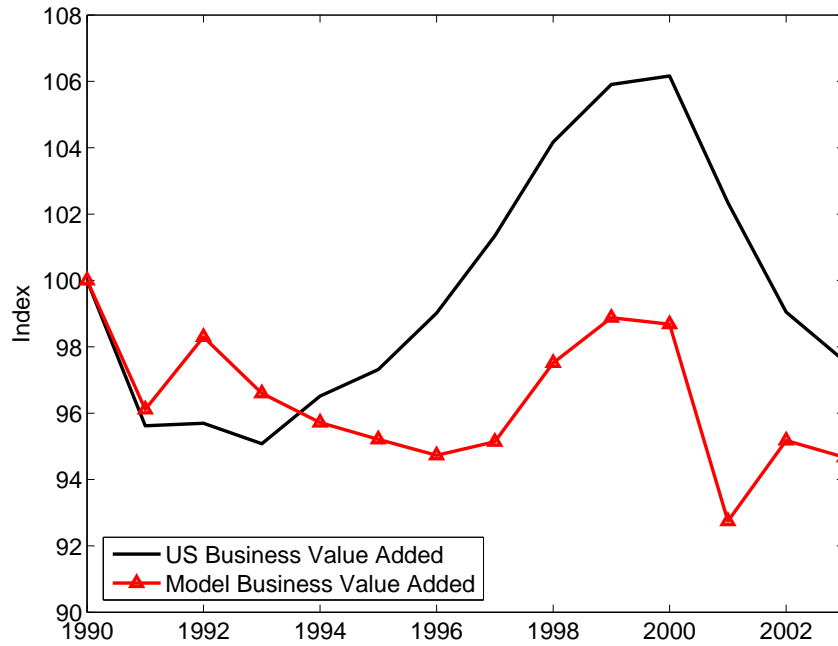


FIGURE 13. U.S. PER CAPITA REAL BUSINESS VALUE ADDED AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)

We observe sequences for  $\hat{y}_{bt}$ ,  $\bar{y}_{nt}$ ,  $\hat{c}_t$ ,  $\bar{x}_{bt}$ ,  $\bar{x}_{nt}$ ,  $\hat{g}_t$ ,  $h_t$ ,  $h_{bt}$ ,  $\bar{h}_{nt}$ ,  $\tau_{ht}$ , and  $\tau_{ct}$ , and an initial business capital stock  $\hat{k}_{b0}$ . Given  $\hat{k}_{b0}$  and the sequence for  $\hat{x}_{bt}$ , we can use the law of motion for business capital (2.1.15) to derive the sequence of stocks  $\{k_{bt}\}$ . Then, we have business TFP as follows:  $A_t = y_{bt}/[k_{bt}^\theta h_{bt}^{1-\theta}]$ .

As in the one-sector version of the model, we can define the labor wedge as follows:

$$L_{wt} = \frac{\psi(1 + \tau_{ct})\hat{c}_t}{1 - h_t} \cdot \frac{\hat{h}_{bt}}{(1 - \theta)\hat{y}_{bt}} \cdot \frac{1}{1 - \tau_{ht}} \quad (2.1.24)$$

and the investment wedge is as in (2.1.13), except that the capital return appearing in this expression is now the return on business capital  $R_{t,t+1}^b$  instead of  $R_{t,t+1}$ .

We conduct the same experiment as before of computing equilibrium paths for per capita hours. In Table 2, we report the values of the implied exogenous variables.

Figures 11–17 show the results in the case that only TFP ( $A_t$ ) and tax rates ( $\tau_{ht}, \tau_{ct}$ ) vary. Figure 11 is a comparison of per capita hours for the United States and for the model. As in the one-sector version of the model, there is a large deviation. Figures 12 and 13 show per capita real GDP and value added in the business sector. As before, the model predicts that the U.S. economy should have been depressed (relative to trend). Because TFP in the business sector rises at the end of the 1990s, there is a rise in business value added. However, the growth is too modest relative to what we observed in the actual economy.

Figures 14–15 show labor productivity for the aggregate economy and for the business sector. The prediction of business labor productivity is reasonable, but the prediction of overall labor productivity is no better than in the one-sector version of the growth model.

For completeness, we also include figures for the components of GDP. These are shown

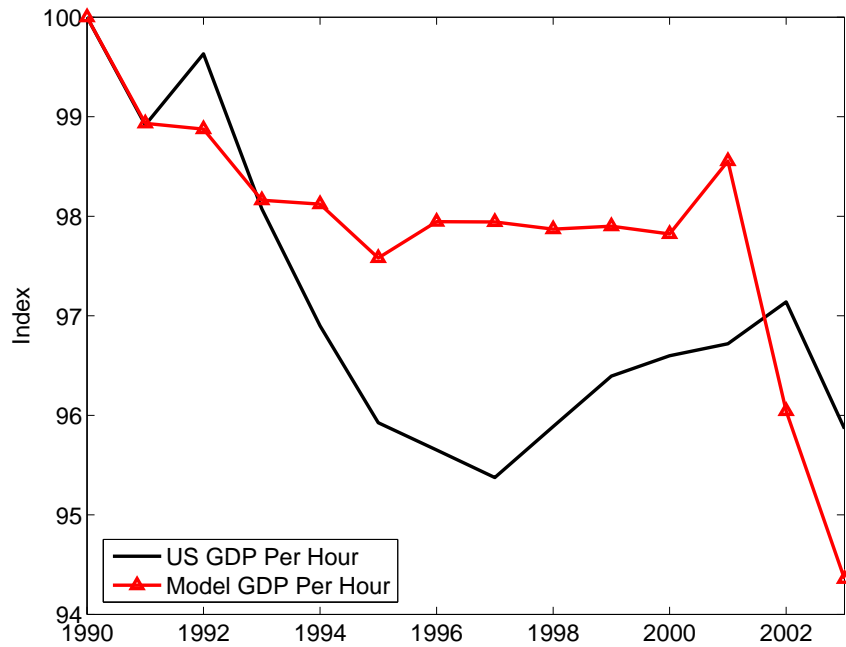


FIGURE 14. U.S. REAL GDP PER HOUR AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)

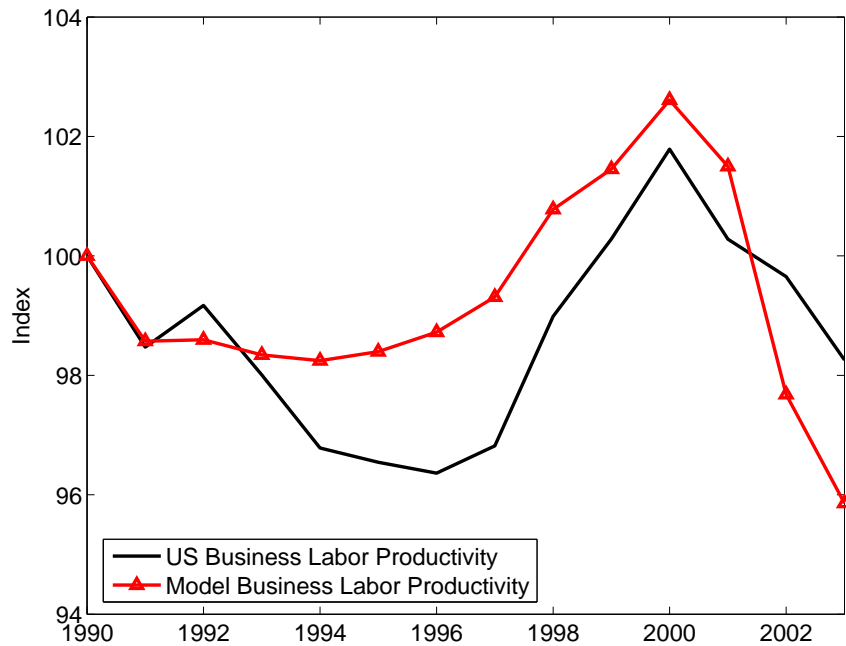


FIGURE 15. U.S. REAL BUSINESS VALUE ADDED PER HOUR AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)

in Figures 16 and 17. In this case, there is little improvement in the model's predictions relative to the one-sector model.

In Figures 18 and 19, we show the predictions for per capita hours when we allow only the labor wedge or only the investment wedge to vary. These results are comparable to those in Figures 6 and 7 for the one-sector version of the model. They can also be compared to Figure 11, which assumes no variation in either the labor wedge or the investment wedge. These results make it clear that to generate the hours boom, we again need an implausible boom in the labor wedge. However, as before, we have no empirical evidence supporting any theory of this labor wedge boom.

### *Summary*

To account for the boom in the U.S. economy using the standard growth model, whether we use the one-sector model or the two-sector model, we must rely on implausible movements in the labor and investment wedges.

## **2.2. Theory with Intangible Capital and Neutral Technology**

We now extend the basic theory described above by incorporating intangible capital. We have used this theory before to study the U.S. and U.K. stock markets. We show, however, that to generate a boom like that observed in the United States during the 1990s, we require wildly implausible exogenous wedges as before, thus demonstrating that intangible capital per se cannot make up for whatever is missing in standard theory.

### **2.2.1. A Specific Model**

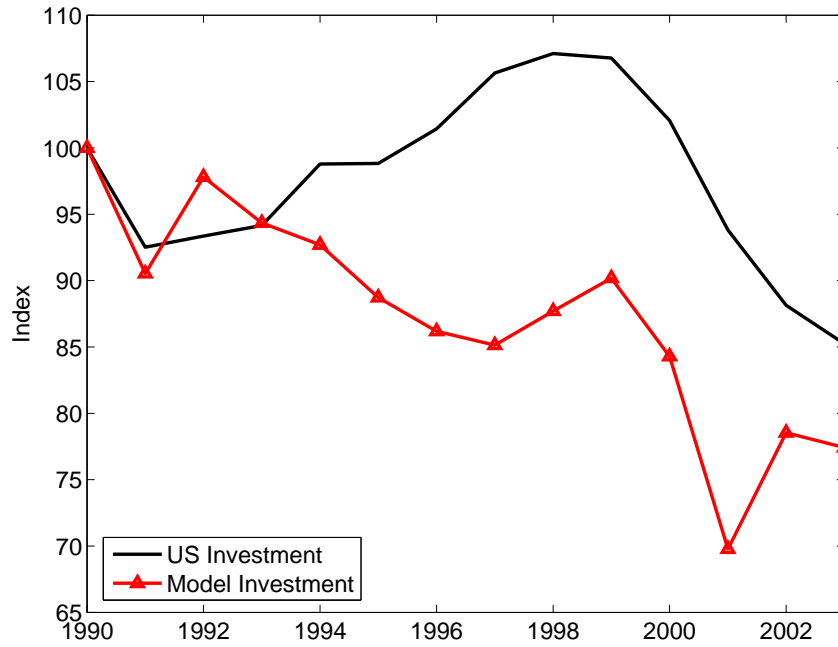


FIGURE 16. U.S. PER CAPITA REAL INVESTMENT AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)

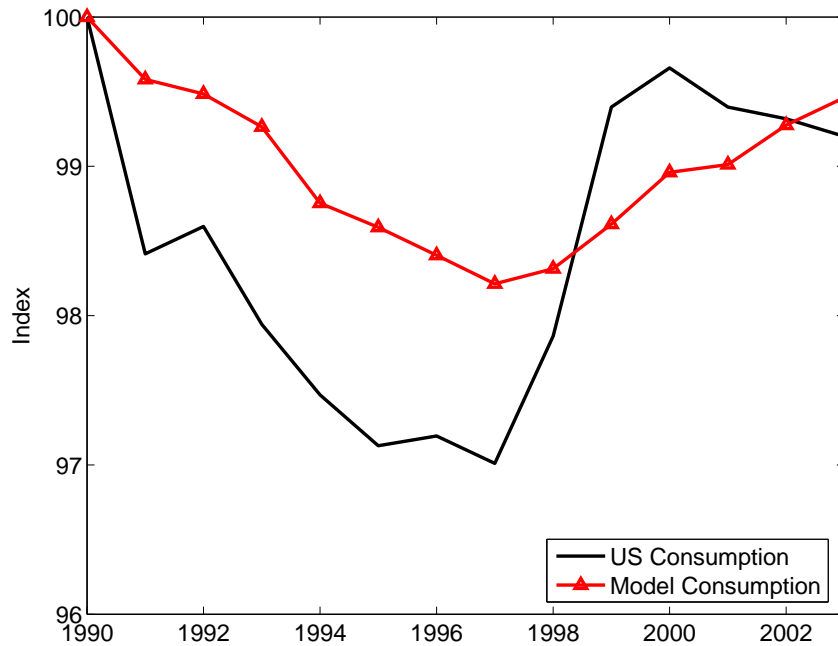


FIGURE 17. U.S. PER CAPITA REAL CONSUMPTION AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL, SERIES DIVIDED BY  $1.02^t$  (Labor and investment wedges constant)



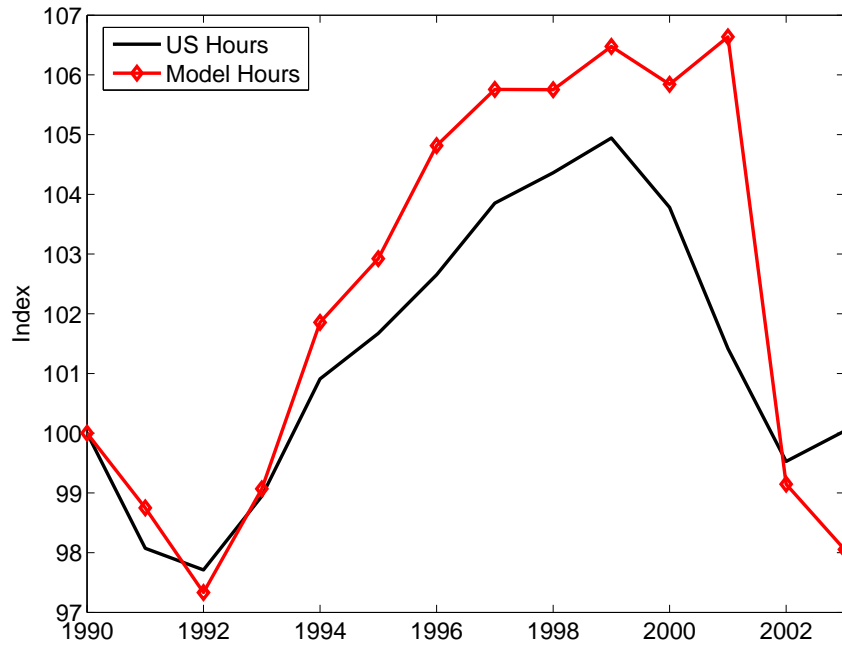


FIGURE 18. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL (Labor wedge only )

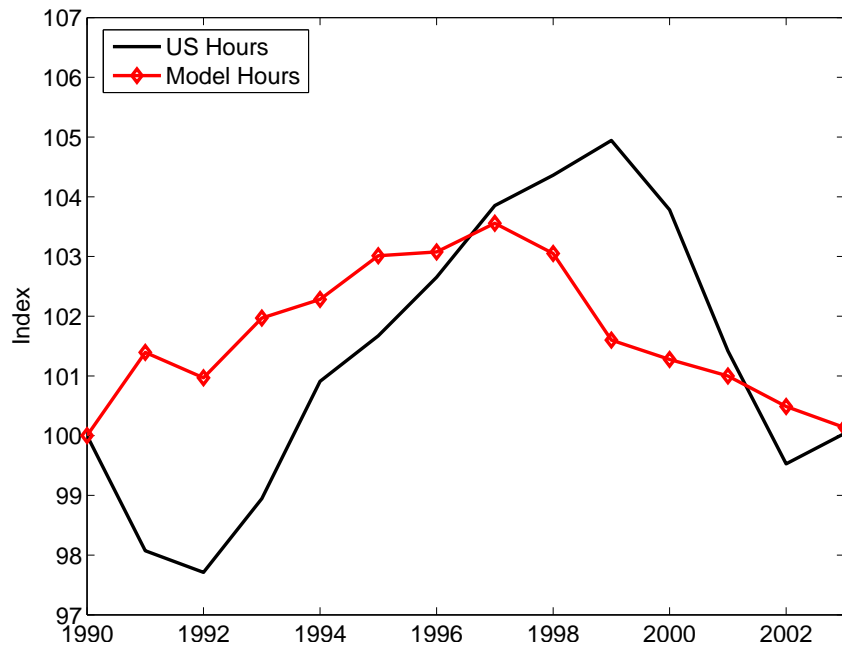


FIGURE 19. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH BUSINESS BUT NO INTANGIBLE CAPITAL (Investment wedge only)

The problem for the household given initial stocks of tangible capital  $k_T$  and intangible capital  $k_I$  (which is *unmeasured* in NIPA) is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$\begin{aligned} c_t + x_{Tt} + x_{It} &= r_{Tt}k_{Tt} + r_{It}k_{It} + w_t h_{bt} \\ &- \tau_{ct}c_t - \tau_{ht}w_t h_{bt} - \tau_{kt}k_{Tt} \\ &- \tau_{pt}\{r_{Tt}k_{Tt} + r_{It}k_{It} - \delta_T k_{Tt} - \tau_{kt}k_{Tt} - x_{It}\} - \tau_{xt}x_{Tt} \\ &- \tau_{dt}\{r_{Tt}k_{Tt} + r_{It}k_{It} - x_{Tt} - \tau_{kt}k_{Tt} - x_{It} \\ &\quad - \tau_{pt}(r_{Tt}k_{Tt} + r_{It}k_{It} - \delta_T k_{Tt} - \tau_{kt}k_{Tt} - x_{It}) - \tau_{xt}x_{Tt}\} \\ &+ [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t \end{aligned} \tag{2.2.1}$$

$$k_{T,t+1} = [(1 - \delta_T) k_{Tt} + x_{Tt}] / (1 + \eta) \tag{2.2.2}$$

$$k_{I,t+1} = [(1 - \delta_I) k_{It} + x_{It}] / (1 + \eta) \tag{2.2.3}$$

$$h_t = h_{bt} + \bar{h}_{nt}. \tag{2.2.4}$$

All variables are as defined in Section 2.1.3 except that we have added the stock  $k_{It}$  and investment  $x_{It}$  of intangibles. There are no additional exogenous variables. Notice that  $x_{It}$  is expensed and thus subtracted from taxable profits in (2.2.1).

Total output in the business sector is equal to

$$y_t = k_{Tt}^\theta k_{It}^\phi (Z_{bt} h_{bt})^{1-\theta-\phi},$$

where  $Z_{bt} = z_{bt}(1 + \gamma)^t$ , and the economy's resource constraint is

$$c_t + x_{Tt} + x_{It} + \bar{x}_{nt} + g_t = y_t + \bar{y}_{nt}.$$

*Measured* value added in the business sector is  $y_{bt} = y_t - x_{It}$  and aggregate GDP is  $y_{bt} + \bar{y}_{nt}$ .

A NIPA accountant in this economy would measure the following product and income:

$$\text{NIPA product} = c + x_T + \bar{x}_n + g$$

$$\text{Private consumption} = c$$

$$\text{Public consumption} = g$$

$$\text{Investment} = x_T + \bar{x}_n$$

$$\text{NIPA income} = y_b + \bar{y}_n$$

$$\text{Business profits} = (r_T - \tau_k - \delta_T)k_T + r_I k_I - x_I$$

$$\text{Business wages} = wh_b$$

$$\text{Business depreciation} = \delta_T k_T$$

$$\text{Business production tax} = \tau_k k_T$$

$$\text{Non-business income} = \bar{y}_n,$$

which differs from the earlier model only in the category of business profits. Business profits now include a dividend to intangible,  $r_I k_I - x_I$ .

Assuming log utility, we can derive the first-order conditions, which are given by

$$\frac{\psi(1 + \tau_{ct})\hat{c}_t}{1 - h_t} = (1 - \tau_{ht}) \frac{(1 - \theta - \phi)\hat{y}_t}{h_{bt}} \quad (2.2.5)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} [R_{t,t+1}^T + (1 - \delta_T)\xi_{t,t+1}] \quad (2.2.6)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} [R_{t,t+1}^I + (1 - \delta_I)] \zeta_{t,t+1}, \quad (2.2.7)$$

where

$$R_{t,t+1}^T = \frac{1 - \tau_{d,t+1}}{(1 - \tau_{dt})(1 + \tau_{xt})} \left[ (1 - \tau_{p,t+1}) \left( \theta \frac{\hat{y}_{t+1}}{\hat{k}_{T,t+1}} - \tau_{k,t+1} \right) - \delta_T \tau_{p,t+1} \right] \quad (2.2.8)$$

$$R_{t,t+1}^I = \phi \frac{\hat{y}_{t+1}}{\hat{k}_{I,t+1}} \quad (2.2.9)$$

$$\zeta_{t,t+1} = \frac{1 - \tau_{d,t+1}}{1 - \tau_{dt}} \cdot \frac{1 - \tau_{p,t+1}}{1 - \tau_{pt}} \quad (2.2.10)$$

and  $\xi_{t,t+1}$  is given by (2.1.6). Relative to the standard theory without intangible capital, we are adding one dynamic equation (2.2.7).

Assigning parameters for the model is done as above except we need to also assign  $\delta_I$  and  $\theta_I$ . We assume the intangible capital is long-lived (e.g., organizations) and set  $\delta_I = 0$ . This choice will not matter for our results. To set  $\theta_I$  we use one piece of additional information from the national accounts for 1990, namely business compensation. Then, given estimates for the rental rate  $r_T$ , the intangible capital stock  $\hat{k}_I$ , and the intangible investment  $\hat{x}_I$ , we have  $\theta$  and  $\phi$ :

$$r_T = \frac{\left(1 - \hat{\beta}(1 - \delta_T)\right)(1 + \tau_x) - \delta_T \hat{\beta} \tau_p + \hat{\beta}(1 - \tau_p) \tau_k}{\hat{\beta}(1 - \tau_p)} \quad (2.2.11)$$

$$\hat{k}_I = \frac{\hat{y}_b - r_T \hat{k}_T - 1990 \text{ NIPA business compensation}}{1 + i - (1 + \gamma)(1 + \eta)} \quad (2.2.12)$$

$$\hat{x}_I = ((1 + \gamma)(1 + \eta) - 1 + \delta_I) \hat{k}_I \quad (2.2.13)$$

$$\phi = \frac{(i + \delta_I) \hat{k}_I}{\hat{y}_b + \hat{x}_I} \quad (2.2.14)$$

$$\theta = \frac{r_T \hat{k}_T}{\hat{y}_T + \hat{x}_I}, \quad (2.2.15)$$

where the values for  $\hat{y}_b$ ,  $\hat{k}_T$ ,  $i$ ,  $\gamma$ , and  $\eta$  are as before.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are exactly the same as those used above. The only additional information is NIPA business compensation, which is equal to 0.443 times GDP in 1990. Growth rates, the interest rate, the discount factor, tax rates, and the depreciation rate on tangible capital are also exactly the same as in Section 2.1.3. Because total output includes intangible

investment, some parameters are changed slightly. The new parameters are  $\theta = 0.240$ ,  $\phi = 0.180$ ,  $\psi = 1.273$ , and  $z = 1.635$ .

### 2.2.2. Business Cycle Accounting for the 1990s

Suppose that we have observations on  $\hat{y}_{bt}$ ,  $\bar{y}_{nt}$ ,  $\hat{c}_t$ ,  $\hat{x}_{Tt}$ ,  $\bar{x}_{nt}$ ,  $\hat{g}_t$ ,  $h_t$ ,  $\bar{h}_{nt}$ ,  $\tau_{ht}$ , and  $\tau_{ct}$ . Then, given  $\hat{x}_{Tt}$ , we can use the law of motion for capital to get

$$\hat{k}_{T,t+1} = \left[ (1 - \delta_T) \hat{k}_{Tt} + \hat{x}_{Tt} \right] / [(1 + \gamma)(1 + \eta)]$$

given an initial condition  $\hat{k}_{T0}$ .

We can infer the magnitude of intangible capital and investment using (2.2.3) and (2.2.7) along with data on consumption, business output, and tax rates.<sup>9</sup> Given sequences for intangible investments and stocks, we can compute TFP,

$$A_t = \frac{\hat{y}_{bt} + \hat{x}_{It}}{\hat{k}_{Tt}^\theta \hat{k}_{It}^\phi h_{bt}^{1-\theta-\phi}}.$$

To get a perfect match to U.S. data, we again rely on a labor wedge and an investment wedge,

$$L_{wt} = \frac{\psi(1 + \tau_{ct}) \hat{c}_t}{1 - h_t} \cdot \frac{h_{bt}}{(1 - \theta) \hat{y}_t} \cdot \frac{1}{1 - \tau_{ht}} \quad (2.2.16)$$

$$X_{w,t+1} = \frac{\hat{\beta}(1 - \delta) \mu_{t+1} X_{wt}}{\mu_t - \hat{\beta} R_{t,t+1}^T \mu_{t+1} X_{wt}} \xi_{t,t+1}, \quad (2.2.17)$$

with equation (2.1.13) solved recursively starting with  $X_{w0} = 1$ . As before, we would replace  $1 - \tau_{ht}$  with  $(1 - \tau_{ht})L_{wt}$  and  $1/(1 + \tau_{xt})$  with  $X_{wt}/(1 + \tau_{xt})$ .

---

<sup>9</sup> We use the steady-state level of  $\hat{k}_I$  (given 1990 values of other variables) to initialize the stock. We assume the growth in the per capita stock in the last period is  $\gamma$ .

Year ( $t$ )	$A_t$	$\tau_{ht}$	$\tau_{ct}$	$L_{wt}$	$X_{wt}$
1990	1.6586	0.3109	0.0657	0.8020	1.0000
1991	0.9014	0.3070	0.0675	1.4051	0.9911
1992	1.4223	0.3028	0.0678	0.8955	0.9919
1993	1.1352	0.3034	0.0678	1.1216	0.9875
1994	1.2306	0.3068	0.0702	1.0634	0.9846
1995	1.1315	0.3116	0.0686	1.1695	0.9788
1996	1.2365	0.3190	0.0674	1.0995	0.9740
1997	1.1647	0.3254	0.0674	1.1924	0.9668
1998	1.4442	0.3327	0.0670	0.9874	0.9627
1999	1.6581	0.3335	0.0662	0.8722	0.9625
2000	1.3061	0.3424	0.0649	1.0993	0.9575
2001	1.1055	0.3472	0.0625	1.2776	0.9471
2002	1.1913	0.3076	0.0617	1.1043	0.9358
2003	1.1942	0.2885	0.0621	1.0793	0.9235

TABLE 3. EXOGENOUS VARIABLES FOR MODEL WITH INTANGIBLE CAPITAL AND NEUTRAL TECHNOLOGY

In Figures 20–22, we repeat the exercise of plotting actual hours and predicted hours for versions of the model with wedges off and wedges on. Table 3 has the values of the implied exogenous variables. Figure 20 shows the predicted hours for the case that  $L_{wt} = X_{wt} = 1$  in all periods. Clearly, this is not an improvement on standard theory because the hours prediction is wildly oscillatory.<sup>10</sup> The same strange behavior is evident in the labor-wedge-only case shown in Figure 21. Essentially, the labor wedge is canceling out the other exogenous variables in such a way that the prediction of Figure 22, with the investment-wedge-only case, is relatively smooth. Interestingly, if we input all of these exogenous variables, *we get a perfect fit*.

Why do the results look so strange? The logic that intangible capital makes up for whatever is missing is faulty. When we added a labor wedge to the theory without

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<sup>10</sup> An alternative strategy is to use (2.2.5) to infer the sequence of intangible investments. In this case, a “tangible investment wedge” and an “intangible investment wedge” are needed for (2.2.6) and (2.2.7) to hold, given U.S. observations. We applied this strategy and found again that the model’s predictions were grossly at odds with the data when the wedges were off.

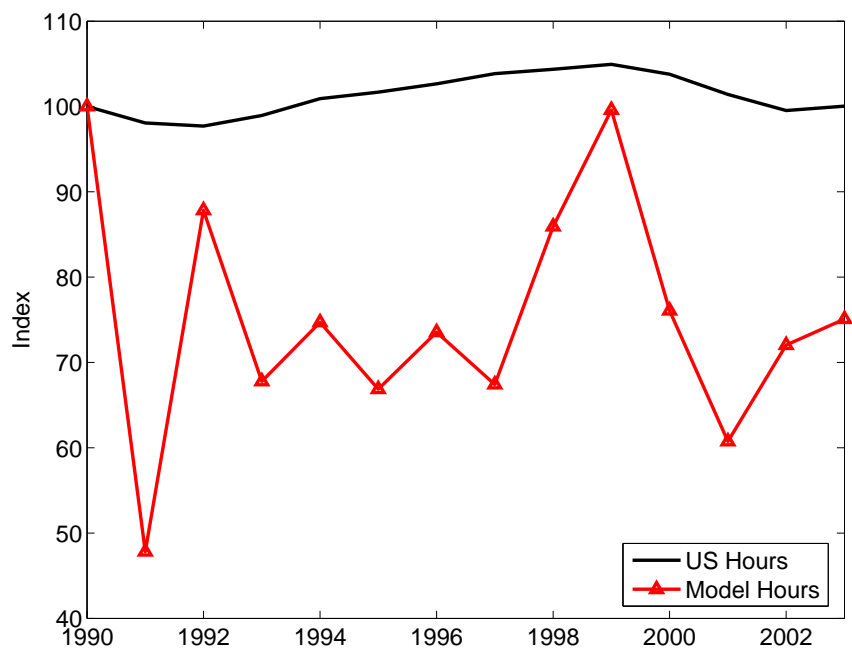


FIGURE 20. U.S. PER CAPITA HOURS AND PREDICTION FOR MODEL WITH INTANGIBLE CAPITAL AND NEUTRAL TECHNOLOGY (Labor and investment wedges constant)

intangible capital, we simply added an exogenous term to one of the existing first-order conditions and let the wedge be whatever it had to be. *In the model with intangible capital, we are adding two more endogenous variables, and, therefore, we are adding two more equations and more restrictions in the dynamical system.* The first-order conditions in the new larger system are not in block form. Thus, the intangible variables enter the original first-order conditions, and the observed variables enter the additional conditions. There is no mathematical basis for thinking that adding the intangible capital will improve the predictive power of the model with the wedges turned off. Indeed, it is worse in this case.

### Summary

We showed that intangible capital is not a free parameter that makes up for whatever

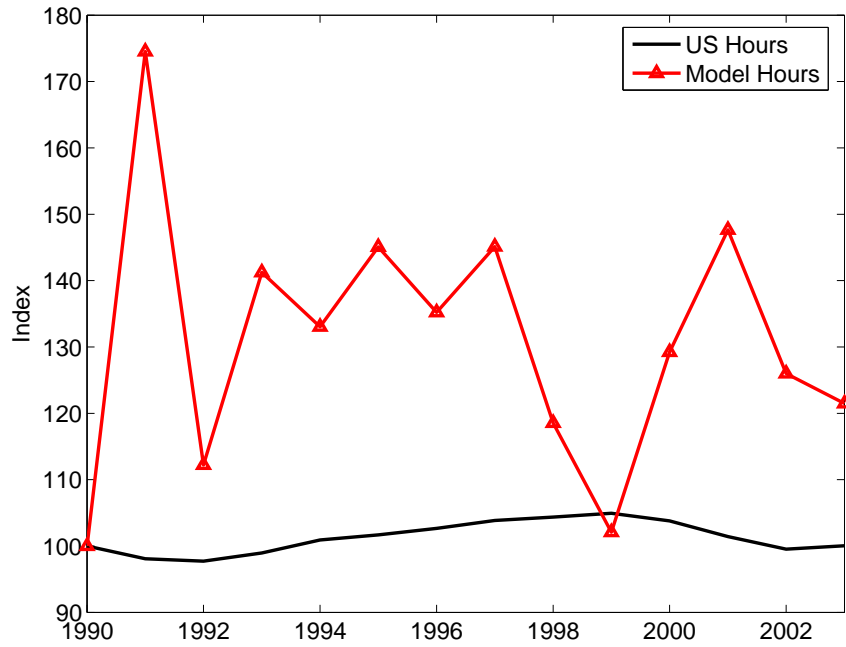


FIGURE 21. U.S. PER CAPITA HOURS AND PREDICTION FOR MODEL WITH INTANGIBLE CAPITAL AND NEUTRAL TECHNOLOGY (Labor wedge only )

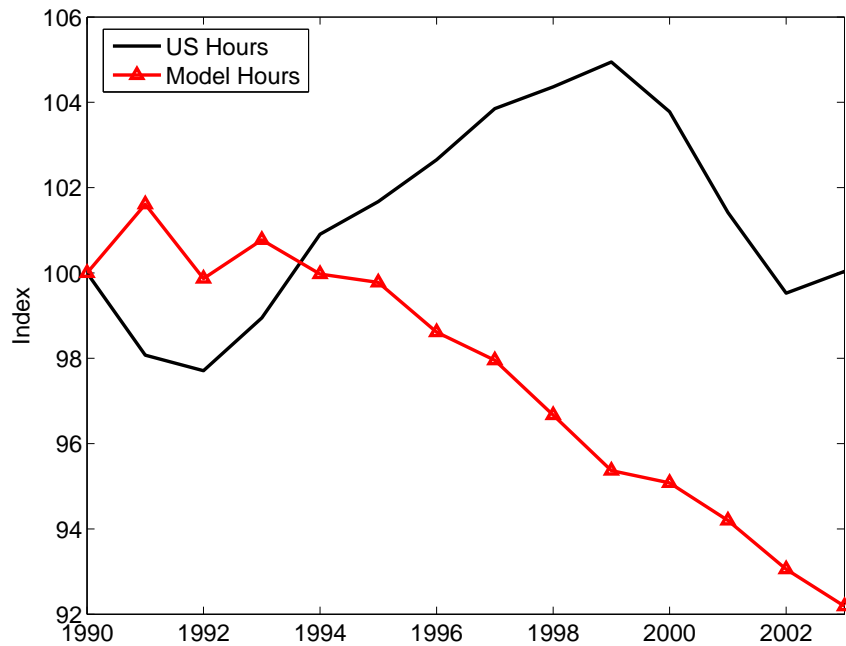


FIGURE 22. U.S. PER CAPITA HOURS AND PREDICTION FOR MODEL WITH INTANGIBLE CAPITAL AND NEUTRAL TECHNOLOGY (Investment wedge only )



is missing to make standard theory work. Next, we show that central to understanding the boom of the 1990s is non-neutral technological change.

### 2.3. Theory with Intangible Capital and Non-neutral Technology

We turn now to the “extended model” developed in our paper that has *both* intangible capital and non-neutral technological change. This theory satisfies our input justification criterion, and we show here that it also satisfies our prediction criterion. We also demonstrate that we would get a very different result if the data-generating mechanism were inconsistent with the theory proposed, implying that the bar we set is not too low.

#### 2.3.1. A Specific Model

There are two additions relative to the model of Section 2.2. First, we allow for *non-neutral technology*. The idea is that we are modeling a technology boom that is concentrated in intangible activity. Second, we allow for *sweat equity*. Some intangible investment is financed by shareholders and some is financed by worker-owners of businesses. We were motivated to include this, since the pattern of incomes suggests that both types of financing are done.

The problem for the household given initial stocks of business tangible capital  $k_{T0}$  and business intangible capital  $k_{I0}$  is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$c_t + x_{Tt} + q_t x_{It} = r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_{bt}$$

$$\begin{aligned}
& -\tau_{ct}c_t - \tau_{ht}(w_t h_{bt} - (1 - \chi)q_t x_{it}) - \tau_{kt}k_{Tt} \\
& - \tau_{pt}\{r_{Tt}k_{Tt} + r_{It}k_{It} - \delta_T k_{Tt} - \tau_{kt}k_{Tt} - \chi q_t x_{it}\} - \tau_{xt}x_{Tt} \\
& - \tau_{dt}\{r_{Tt}k_{Tt} + r_{It}k_{It} - x_{Tt} - \tau_{kt}k_{Tt} - \chi q_t x_{it} \\
& \quad - \tau_{pt}(r_{Tt}k_{Tt} + r_{It}k_{It} - \delta_T k_{Tt} - \tau_{kt}k_{Tt} - \chi q_t x_{it}) \\
& \quad - \tau_{xt}x_{Tt}\} - \tau_{gt}q_t x_{it} \\
& + [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t
\end{aligned} \tag{2.3.1}$$

$$k_{T,t+1} = [(1 - \delta_T)k_{Tt} + x_{Tt}] / (1 + \eta) \tag{2.3.2}$$

$$k_{I,t+1} = [(1 - \delta_I)k_{It} + x_{It}] / (1 + \eta) \tag{2.3.3}$$

$$h_t = h_t^1 + h_t^2 + \bar{h}_{nt} \tag{2.3.4}$$

$$h_{bt} = h_t^1 + h_t^2 \tag{2.3.5}$$

$$k_{Tt} = k_{Tt}^1 + k_{Tt}^2, \tag{2.3.6}$$

where  $q_t$  is the relative price of intangible investment goods and final output, and  $\chi$  is the fraction of intangible investment financed by shareholders. The remaining  $1 - \chi$  of intangible investment is financed by workers who own their businesses and put in sweat equity, which is uncompensated labor. The effective compensation is through capital gains when the business is sold.

The total produced in the business sector is  $y_t = y_{bt} + q_t x_{it}$ , where

$$\begin{aligned}
y_{bt} &= (k_{Tt}^1)^{\theta_1} k_{It}^{\phi_1} (Z_t^1 h_t^1)^{1-\theta_1-\phi_1} \\
x_{it} &= (k_{Tt}^2)^{\theta_2} k_{It}^{\phi_2} (Z_t^2 h_t^2)^{1-\theta_2-\phi_2}
\end{aligned}$$

and  $Z_t^1 = z_t^1(1 + \gamma)^t$ ,  $Z_t^2 = z_t^2(1 + \gamma)^t$ . If  $\chi = 1$  and technologies are neutral (so that  $q_t = 1$  in equilibrium), then we are back to the model of Section 2.2.1.

The economy's resource constraint is

$$c_t + x_{Tt} + q_t x_{It} + \bar{x}_{nt} + g_t = y_t + \bar{y}_{nt}$$

and aggregate GDP is  $y_{bt} + \bar{y}_{nt}$ . A NIPA accountant in this economy would measure the following product and income:

$$\text{NIPA product} = c + x_T + \bar{x}_n + g$$

$$\text{Private consumption} = c$$

$$\text{Public consumption} = g$$

$$\text{Investment} = x_T + \bar{x}_n$$

$$\text{NIPA income} = y_b + \bar{y}_n$$

$$\text{Business profits} = (r_T - \tau_k - \delta_T)k_T + r_I k_I - \chi q x_I$$

$$\text{Business wages} = w h_b - (1 - \chi) q x_I$$

$$\text{Business depreciation} = \delta_T k_T$$

$$\text{Business production tax} = \tau_k k_T$$

$$\text{Non-business income} = \bar{y}_n$$

Simplifying the first-order conditions for the log utility case, we get

$$\frac{\psi(1 + \tau_{ct}) \hat{c}_t}{1 - h_t} = (1 - \tau_{ht}) \frac{(1 - \theta_1 - \phi_1) \hat{y}_{bt}}{h_t^1} \quad (2.3.7)$$

$$(1 - \theta_1 - \phi_1) \frac{\hat{y}_{bt}}{h_t^1} = (1 - \theta_2 - \phi_2) \frac{q_t \hat{x}_{It}}{h_t^2} \quad (2.3.8)$$

$$\theta_1 \frac{\hat{y}_{bt}}{\hat{k}_{Tt}^1} = \theta_2 \frac{q_t \hat{x}_{It}}{\hat{k}_{Tt}^2} \quad (2.3.9)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} [R_{t,t+1}^T + (1 - \delta_T) \xi_{t,t+1}] \quad (2.3.10)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} [R_{t,t+1}^I + (1 - \delta_I) q_{t+1} \zeta_{t,t+1} / q_t], \quad (2.3.11)$$

where

$$\begin{aligned}
R_{t,t+1}^T &= \frac{1 - \tau_{d,t+1}}{(1 - \tau_{dt})(1 + \tau_{xt})} \left[ (1 - \tau_{p,t+1}) \left( \theta_1 \frac{\hat{y}_{b,t+1}}{\hat{k}_{T,t+1}^1} - \tau_{k,t+1} \right) - \delta_b \tau_{p,t+1} \right] \\
R_{t,t+1}^I &= \frac{\phi_1 \hat{y}_{b,t+1} + \phi_2 q_{t+1} \hat{x}_{I,t+1}}{q_t \hat{k}_{I,t+1}} \left( \frac{(1 - \tau_{d,t+1})(1 - \tau_{p,t+1})}{\chi(1 - \tau_{dt})(1 - \tau_{pt}) + (1 - \chi)(1 - \tau_{ht})} \right) \\
\zeta_{t,t+1} &= \frac{\chi(1 - \tau_{d,t+1})(1 - \tau_{p,t+1}) + (1 - \chi)(1 - \tau_{h,t+1})}{\chi(1 - \tau_{dt})(1 - \tau_{pt}) + (1 - \chi)(1 - \tau_{ht})} \tag{2.3.12}
\end{aligned}$$

and  $\xi_{t,t+1}$  is given by (2.1.6). Assigning parameters for the model is done as above except that we have three additional parameters,  $\chi$ ,  $\theta_2$ , and  $\phi_2$  (and we normalize  $q$  to 1). In our benchmark experiments, we set  $\chi = 1/2$  and equated capital shares,  $\theta_2 = \theta_1$  and  $\phi_2 = \phi_1$ . In this case, the shares  $\theta_1$  and  $\phi_1$  are constructed as follows:

$$r_I = \frac{q \left( 1 - \hat{\beta} (1 - \delta_I) \right) [(1 - \chi)(1 - \tau_h) + \chi(1 - \tau_d)(1 - \tau_p)]}{\hat{\beta} (1 - \tau_d) (1 - \tau_p)} \tag{2.3.13}$$

$$\hat{k}_I = \frac{\hat{y}_b - r_T \hat{k}_T - 1990 \text{ NIPA business compensation}}{r_I - \chi q [(1 + \gamma)(1 + \eta) - 1 + \delta_I]} \tag{2.3.14}$$

$$\hat{x}_I = ((1 + \gamma)(1 + \eta) - 1 + \delta_I) \hat{k}_I \tag{2.3.15}$$

$$\theta_1 = \frac{r_T \hat{k}_T}{\hat{y}_b + q \hat{x}_I} \tag{2.3.16}$$

$$\phi_1 = \frac{r_I \hat{k}_I}{\hat{y}_b + q \hat{x}_I}, \tag{2.3.17}$$

where  $r_T$  is given by (2.2.11). In Chapter 5, we will check to see if our results are sensitive to our choices of  $\chi$ ,  $\theta_1$ , and  $\phi_1$ .

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are exactly the same as those used in Section 2.2.1. Because  $\chi < 1$  and technology is

non-neutral, some parameters are changed. These are changed to  $\theta_1 = 0.263$ ,  $\phi_1 = 0.076$ ,  $\psi = 1.323$ ,  $z^1 = 2.161$ , and  $z^2 = 1.539$ .

### 2.3.2. Business Cycle Accounting for the 1990s

We have observations on  $\hat{y}_{bt}$ ,  $\bar{y}_{nt}$ ,  $\hat{c}_t$ ,  $\hat{x}_{Tt}$ ,  $\bar{x}_{nt}$ ,  $\hat{g}_t$ ,  $h_t$ ,  $h_{bt}$ ,  $\bar{h}_{nt}$ ,  $\tau_{ht}$ , and  $\tau_{ct}$ . Given  $\hat{x}_{Tt}$ , we can use the law of motion for capital as before to get the sequence of capital stocks  $\{\hat{k}_{Tt}\}$  given an initial condition  $\hat{k}_{T0}$ .

We can infer how many hours are used in final production in the business sector by rearranging the intratemporal condition as follows:

$$\hat{h}_t^1 = \frac{(1 - \theta_1 - \phi_1)(1 - \tau_{ht})\hat{y}_{bt}(1 - h_t)}{\psi(1 + \tau_{ct})\hat{c}_t}$$

in terms of observables. Since we observe total business hours, we know the hours spent accumulating intangibles,  $h_t^2 = h_{bt} - h_t^1$ . The fact that we had to put in a large  $L_{wt}$  in the standard theory works in favor of the technology boom hypothesis. If a rise in  $h_t^2$  is the source of the hours boom, then we will get a boom in  $\hat{y}_{bt}/\hat{h}_t^1$ . This is our story for the earlier labor wedge.

Total output is  $\hat{y}_{bt} + \bar{y}_{nt} + q_t\hat{x}_{It}$ , where

$$q_t\hat{x}_{It} = \left( \frac{1 - \theta_1 - \phi_1}{1 - \theta_2 - \phi_2} \right) \frac{\hat{y}_t}{h_t^1} h_t^2. \quad (2.3.18)$$

The relation follows from the fact that households equate wages in the two business-sector activities. Similarly, households equate returns to tangible business capital in the two business-sector activities and, therefore, it must be that case that

$$\hat{k}_{Tt}^1 = \frac{\theta_1\hat{y}_{bt}}{\theta_1\hat{y}_{bt} + \theta_2q_t\hat{x}_{It}}\hat{k}_{Tt}$$

and  $\hat{k}_{Tt}^2 = \hat{k}_{Tt} - \hat{k}_{Tt}^1$ .

To infer sequences for intangible capital  $\hat{k}_{It}$ , we guess a path for the price  $q_t$  and use (2.3.11) and the capital accumulation equation (2.3.3) to derive sequences for intangible flows and stocks. It must be the case that  $x_{It}$  multiplied by the guess  $q_t$  is equal to the left-hand side of (2.3.18).

Technology parameters are given as follows:

$$\begin{aligned} A_t^1 &= \hat{y}_{bt} / \left[ \left( \hat{k}_{Tt}^1 \right)^{\theta_1} \left( \hat{k}_{It} \right)^{\phi_1} \left( h_t^1 \right)^{1-\theta_1-\phi_1} \right] \\ A_t^2 &= \hat{x}_{It} / \left[ \left( k_{Tt}^2 \right)^{\theta_2} \left( k_{It} \right)^{\phi_2} \left( h_t^2 \right)^{1-\theta_2-\phi_2} \right] \end{aligned}$$

We also include an investment wedge in (2.3.10) to account for any mismeasurement in capital tax rates (which we assumed to be constant over this period). The expression is exactly as in the three earlier examples with  $R_{t,t+1}^T$  given by (2.2.8). We need the effect of the investment wedge to be quantitatively small if our theory is to satisfy the input justification criterion. Otherwise, we need to rethink our assumption of constant tax policy.<sup>11</sup>

We turn now to the numerical experiments. Table 4 has the values of the implied exogenous variables. In Figure 23, we plot U.S. per capita hours along with the model's prediction for the case with only the TFPs and tax rates on labor and consumption varying (i.e.,  $X_{wt} = 1$  for all  $t$ ). By construction, if the investment wedge were varying, then the model would fit exactly and the predicted and actual series would lie on top of each other. The difference in the actual and predicted series is therefore attributed to the wedges. Clearly, this difference is small.

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<sup>11</sup> In our sensitivity analysis, we consider feeding in some estimates of capital tax rates for the 1990s.

Year ( $t$ )	$A_t^1$	$A_t^2$	$\tau_{ht}$	$\tau_{ct}$	$X_{wt}$
1990	1.6534	1.3111	0.3109	0.0657	1.0000
1991	1.5956	1.2357	0.3070	0.0675	1.0116
1992	1.5910	1.1608	0.3028	0.0678	1.0064
1993	1.5940	1.2445	0.3034	0.0678	1.0110
1994	1.6213	1.3818	0.3068	0.0702	1.0140
1995	1.6312	1.4321	0.3116	0.0686	1.0208
1996	1.6579	1.5176	0.3190	0.0674	1.0252
1997	1.6814	1.5740	0.3254	0.0674	1.0336
1998	1.7125	1.5948	0.3327	0.0670	1.0327
1999	1.7358	1.6245	0.3335	0.0662	1.0254
2000	1.7380	1.6132	0.3424	0.0649	1.0314
2001	1.7027	1.6180	0.3472	0.0625	1.0416
2002	1.5901	1.3214	0.3076	0.0617	1.0431
2003	1.5556	1.2302	0.2885	0.0621	1.0428

TABLE 4. EXOGENOUS VARIABLES FOR MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY

In Figure 24, we plot the components of total hours for the model. In the intangible capital sector, hours start to rise starting in 1992 and fall back starting in 2001. There is a slight rise in hours allocated to final goods and services in 1999, but the increase is temporary. Non-business hours, which are set exogenously (and match the U.S. series), did not change much over the period.

Figures 25 and 26 show output for the aggregate economy and the business sector. In this case, the match is extremely close. As a contrast, compare these figures to Figures 12 and 13.

Figures 27 and 28 show labor productivity for the aggregate economy and the business sector. Given the good agreement between theory and data for both outputs and hours, it is not a surprise that predicted and actual series are close. The significant improvement in fit can be seen by comparing these figures to Figures 14 and 15.

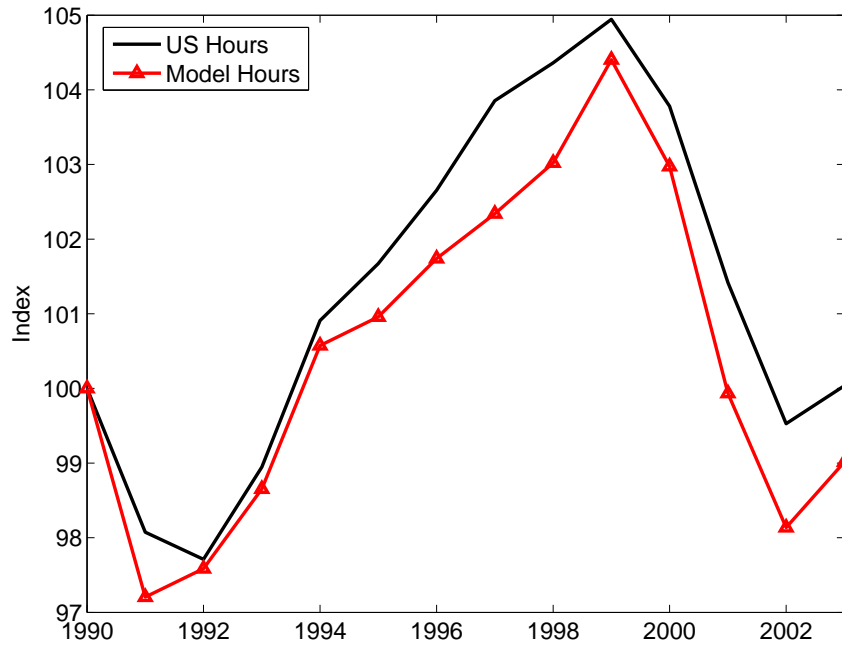


FIGURE 23. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY  
(Investment wedge constant)

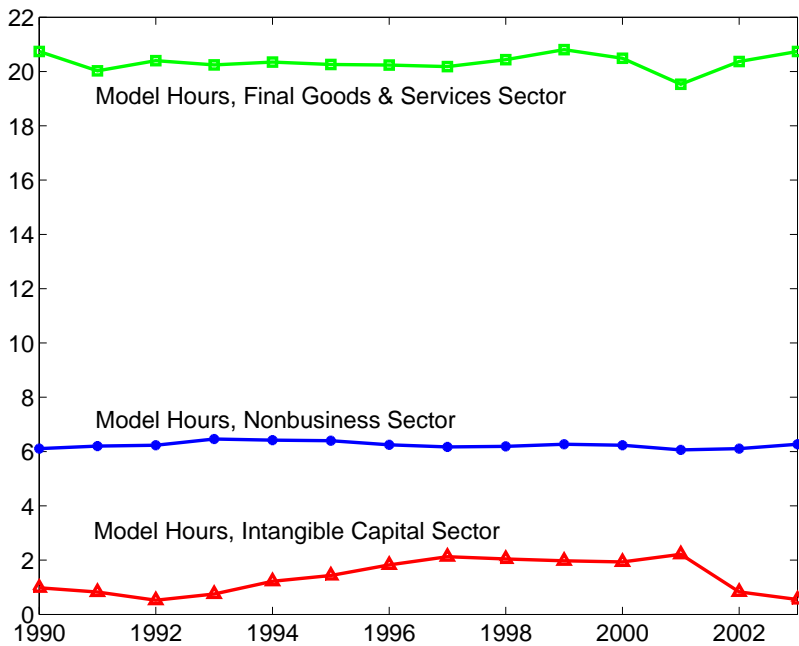


FIGURE 24. PREDICTED COMPONENTS OF HOURS IN MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY  
(Investment wedge constant)



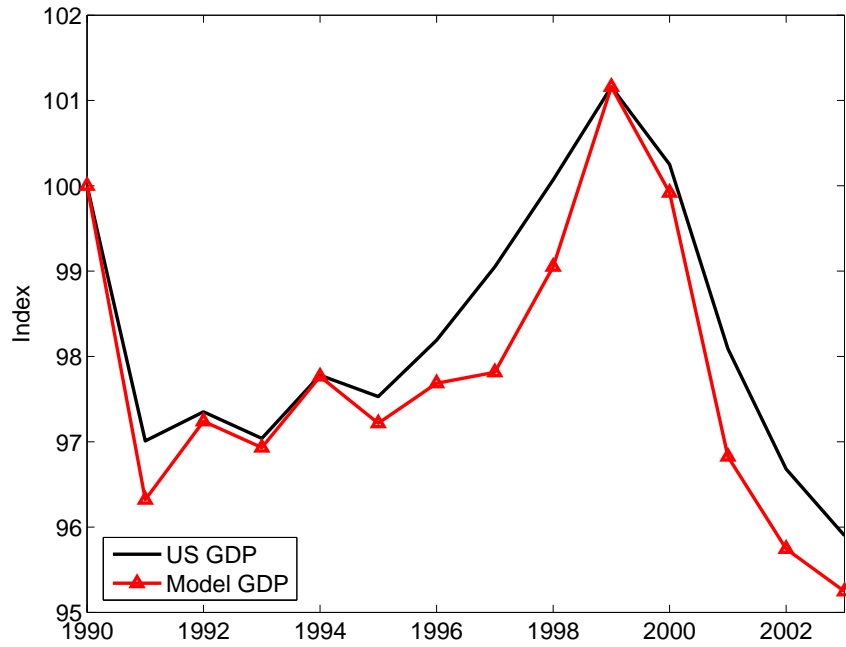


FIGURE 25. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Investment wedge constant)

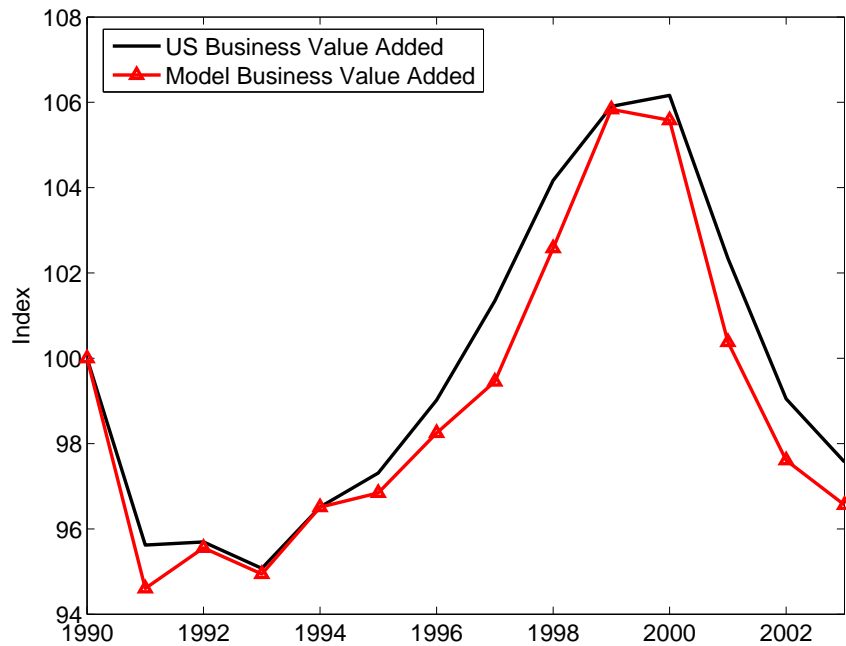


FIGURE 26. U.S. PER CAPITA REAL BUSINESS VA AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Investment wedge constant)

Figure 29 displays tangible investment for both the model and the data. The deviation from theory seems larger than it really is. For example, if we set 1991=100, the series line up nicely. The same is true for consumption displayed in Figure 30.

As a check on the model's predictions, we compared the model's prediction for factor incomes and capital gains to U.S. measures in, respectively, the NIPA and the U.S. Flow of Funds accounts. This is shown in Figures 31 and 32. In Figure 32, the data have been averaged. The line at the beginning of the 1990s is the average for the period 1953–1994, and the line at the end of the 1990s is the average for the period 1995–2003. Neither the income data nor the capital gains data are used to infer our measures of sectoral TFPs. We find that the model's predictions of both incomes and capital gains are in conformity with U.S. observations.

### 2.3.3. Excluding Software from GDP

In 1999, the BEA recognized expenditures for computer software as investment. Prior to that year, such expenditures were treated as an expense and would have been included with our measure of intangible investment  $x_I$ . In this section, we show that excluding business expenditures for software from GDP and including it with intangible investment has little impact on our quantitative results.<sup>12</sup>

To redo the business cycle accounting, we first adjust our data to remove software expenditures from private fixed investment and from gross domestic product. These expenditures are reported in NIPA Table 5.3.5. On the income side, we remove depreciation of software from consumption of fixed capital. The estimates of depreciation are reported

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<sup>12</sup> We did not exclude government expenditures because the BEA does not report government expenditures on equipment and software separately.

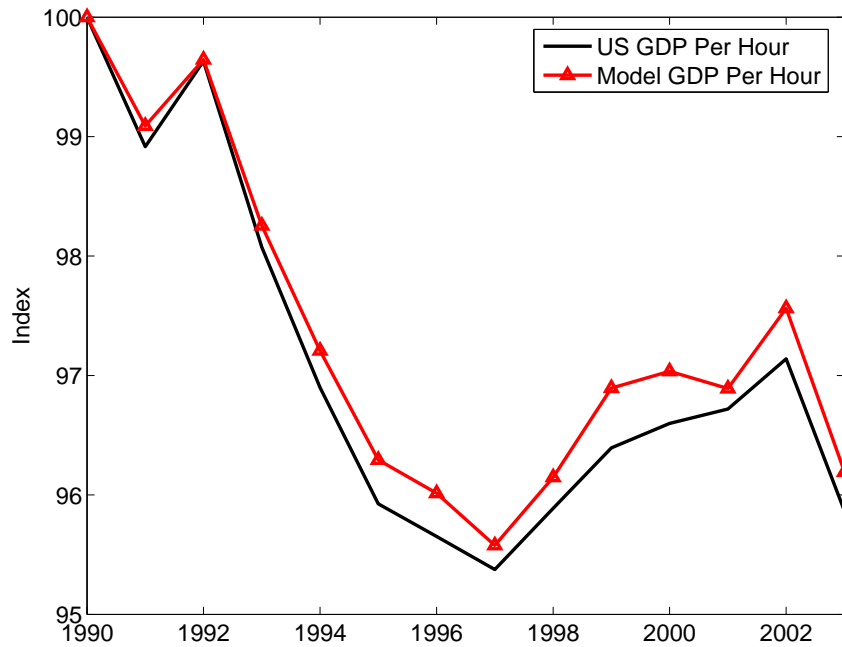


FIGURE 27. U.S. REAL GDP PER HOUR AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Investment wedge constant)

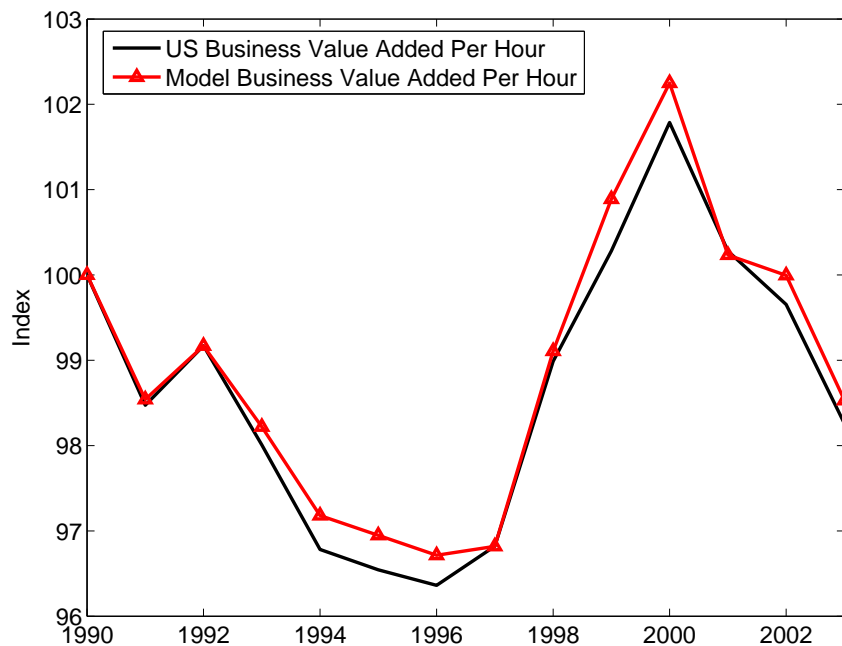


FIGURE 28. U.S. REAL BUSINESS VALUE ADDED PER HOUR AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Investment wedge constant)

in the Fixed Asset Table 2.4. The final change was made in business profits before tax. We subtracted the software investment and added back the consumption of fixed capital.

In Figures 33 through 35, we show the results. These can be directly compared to the earlier Figures 23, 25, and 26. Notice that the detrended U.S. GDP series in Figure 34 is slightly lower than that displayed in Figure 25. The difference is the expenditures on business software. We also see that the predictions of the model based on these new data change little.

#### **2.3.4. Can Our Theory Be Unsuccessful?**

An issue is whether our theory can ever be unsuccessful given that the path of intangible capital is inferred from first-order conditions of the theory. To address this issue, we carry out the following experiment. We simulate data for the model with no intangible capital, assuming that TFP is on trend, no change in capital tax rates, and a large decline in labor market distortions other than  $\tau_{ht}$  or  $\tau_{ct}$ . In other words, we simulate a large rise in the labor wedge that proxies for labor market distortions other than government taxes. (See Section 2.1.1.) We treat these simulated data as the true economic data.

With these data, we ask, If we analyzed these data using our preferred theory with intangible capital and non-neutral technology, would we say that the theory satisfies our two criteria for a successful theory?

The short answer is no: the model with intangible capital and non-neutral technology would not satisfy either criterion. The theory would not satisfy the input justification criterion because it predicts a huge boom in R&D and other expensed investments ( $q_t x_{it}$ ) when there was not a shred of micro evidence for it—in fact, the true economy being

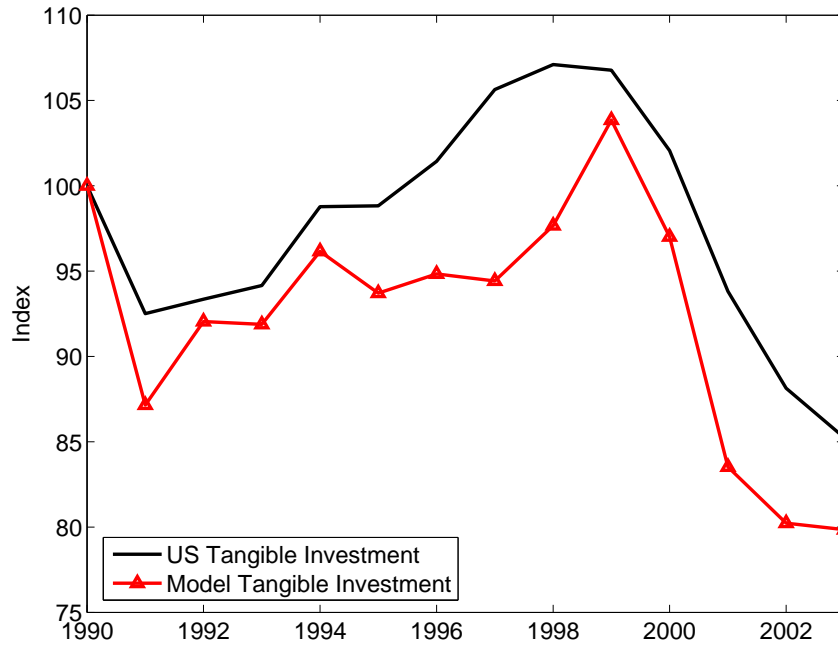


FIGURE 29. U.S. PER CAPITA REAL INVESTMENT AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$   
(Investment wedge constant)

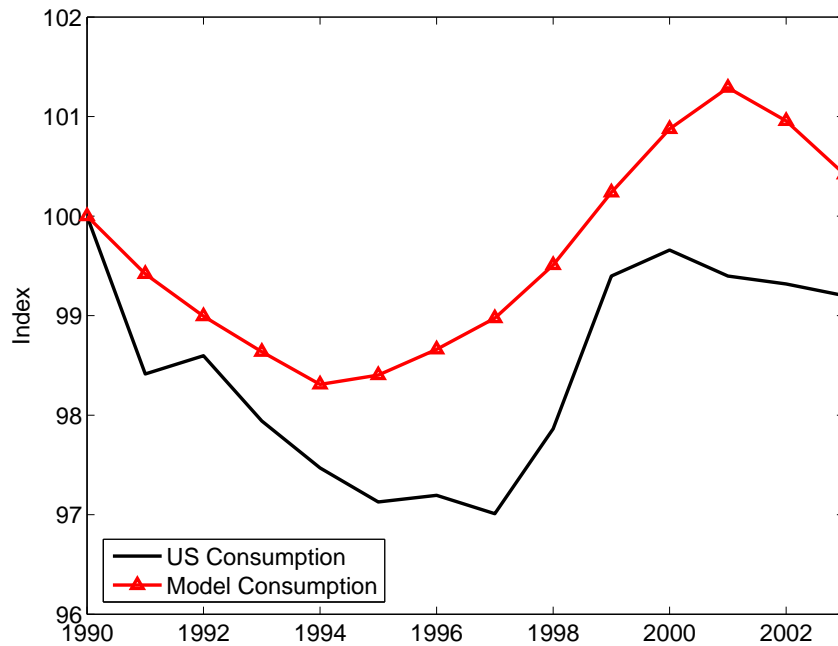


FIGURE 30. U.S. PER CAPITA REAL CONSUMPTION AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$   
(Investment wedge constant)

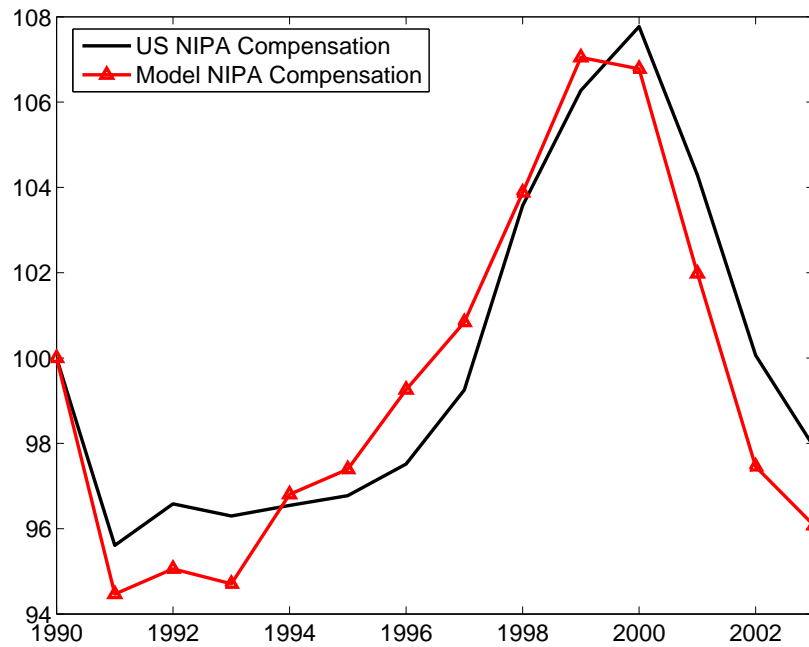


FIGURE 31. U.S. PER CAPITA REAL BUSINESS COMPENSATION AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$   
(Investment wedge constant)

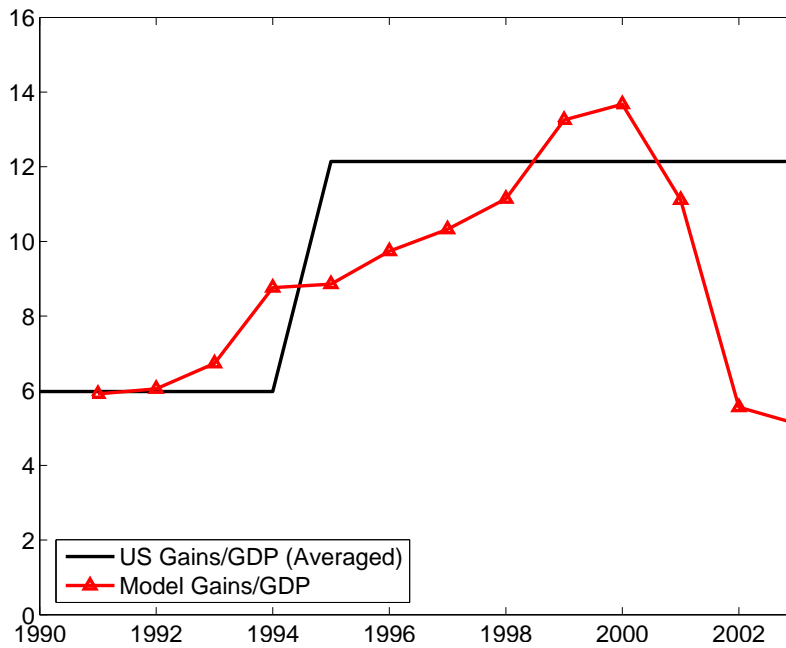


FIGURE 32. U.S. REAL HOLDING GAINS AS % OF GDP AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY  
(Investment wedge constant)

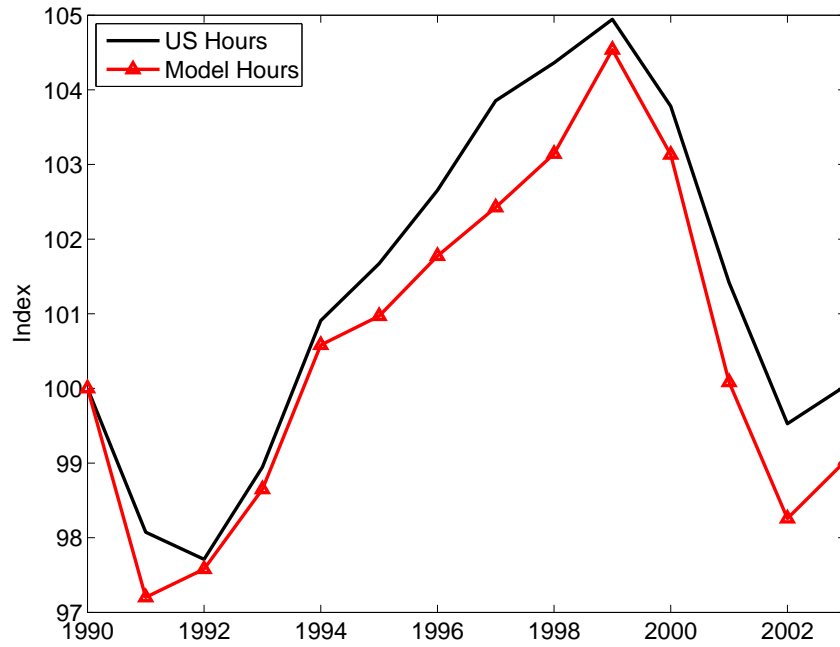


FIGURE 33. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY (Business software excluded from GDP)

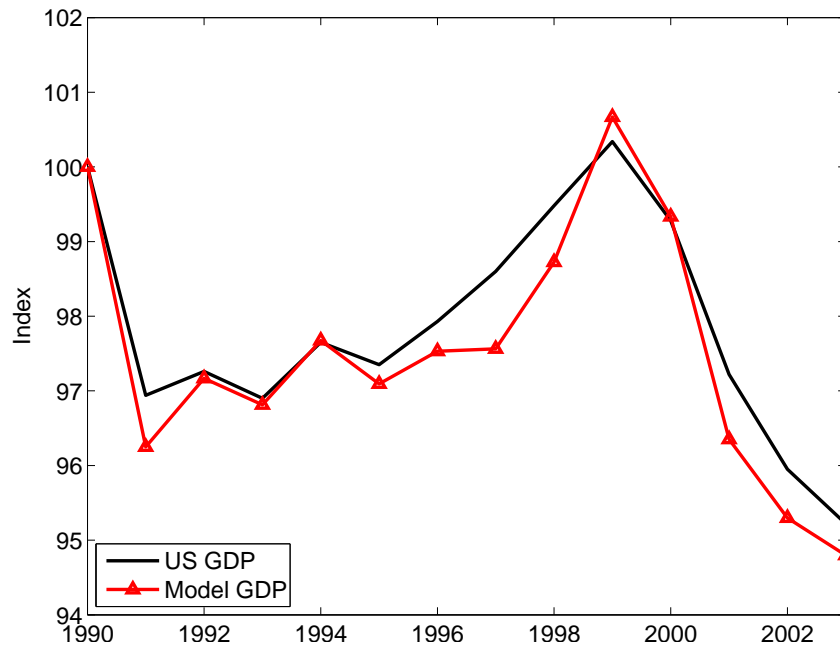


FIGURE 34. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Business software excluded from GDP)

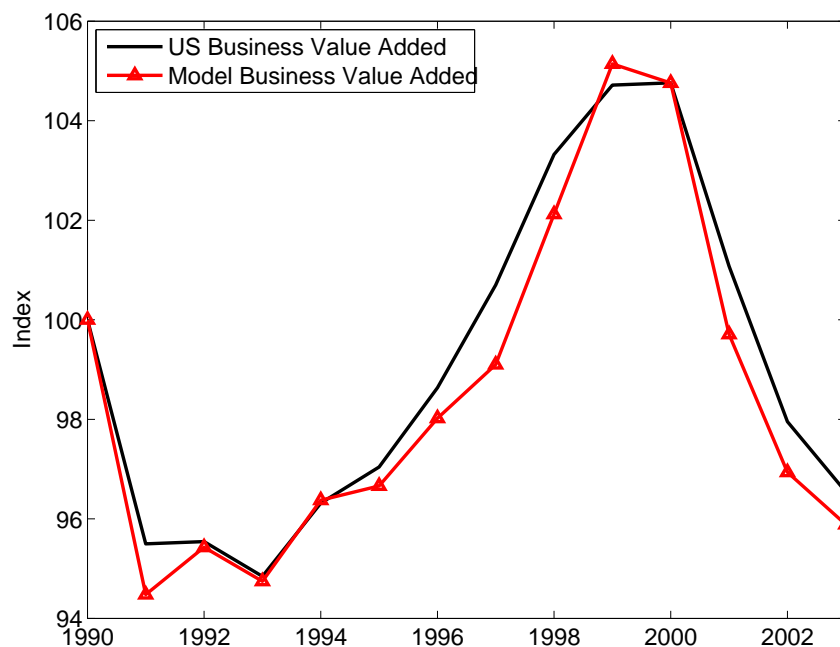


FIGURE 35. U.S. PER CAPITA REAL BUSINESS VA AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Business software excluded from GDP)

analyzed has *no intangible capital at all*. Even if there was a sizable intangible capital stock, we would face the same problem if it was not in fact abnormally high during the period of interest. With our model, we would infer an abnormally high intangible stock, but would find no micro evidence for the rise. We would also predict a big temporary shift in employment in certain occupations ( $h_{2t}$  high relative to  $h_{1t}$ ) but would find no evidence of this given that the changes in the true economy are neutral with respect to employment sectors. The model would not satisfy the prediction criterion because the capital gains in the true economy barely changed, whereas we would be predicting a huge boom.

### Summary

In this chapter, we assessed three theories designed to generate equilibrium paths for GDP, consumption, investment, and hours that matched the U.S. time series exactly.



Despite the perfect fit, only one theory satisfied our criteria for a successful theory. We also showed that this was not a foregone conclusion just because intangible investment was included in the model.

## Chapter 3.

### Accounting for Business Cycles During 1960–1989

In our paper we focus on the 1990s as a period during which a huge unexplained deviation from theory occurred. Many people have noted that the baseline model has difficulties in replicating the unfiltered and TFP-detrended macroeconomic time series not only in the 1990s, but generally. In this chapter, we demonstrate that the basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity, and taxes are incorporated.

The view that the “basic” neoclassical growth model does poorly in general is largely due to key missing factors in the basic model. For example, to account for movements in hours and labor productivity, one must account for key distortions to the labor market, in particular tax rates on labor. If we compare the predictions of Uhlig’s (2003) real business cycle model and Chen et al.’s (2007) real business cycle model, we reach a different conclusion about the performance of the “basic” neoclassical growth model driven by real factors.

Figure 36 shows Uhlig’s (2003) prediction for the log deviation in business along with the U.S. measure. In Uhlig’s model, fluctuations are driven by changes in TFP, government spending, and population growth. Periods in his model are quarters. The figure is reproduced from Uhlig’s Table 6, which has the caption “uncomfortably big gaps are visible.” In terms of hours, the most uncomfortable gaps are during the recessions of the 1970s and 1980s and especially during the boom of the 1990s. The model gets 1/3 of

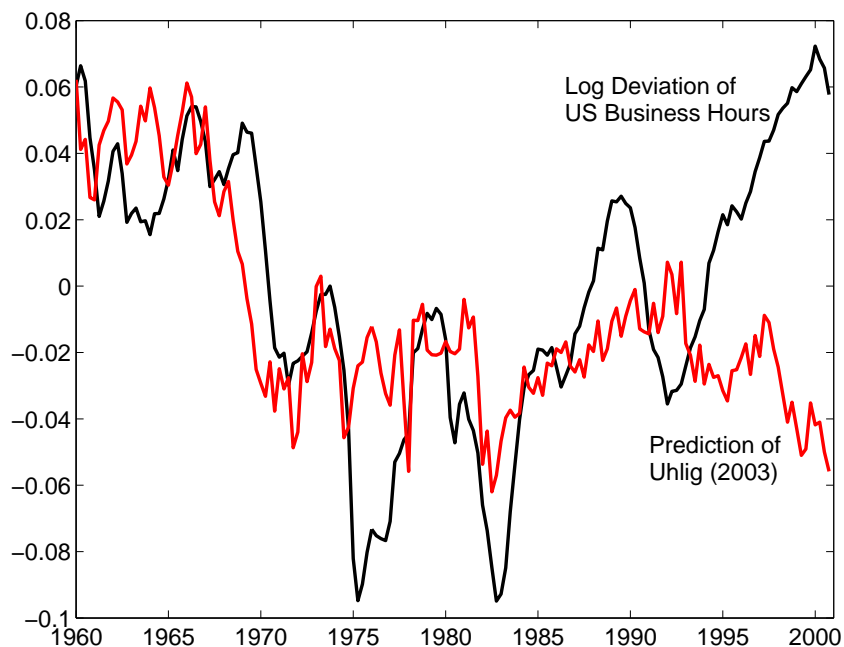


FIGURE 36. DEVIATION OF LOGARITHM OF U.S. PER CAPITA BUSINESS HOURS AND PREDICTION OF UHLIG (2003, TABLE 6) MODEL, 1960:1–2000:4

the 1970s decline in hours, 1/2 of the 1980s decline, and predicts a depression in the 1990s when in fact there was a boom.

Figure 37 shows Chen et al.’s prediction of total hours along with the U.S. measure (based on the establishment survey). This figure is reproduced from their paper (page 15). In Chen et al.’s model, fluctuations are driven by changes in TFP, labor tax rates, capital tax rates, depreciation rates, government spending, and population growth. Periods in their model are years. Interestingly, if we focus on the cyclical movements of hours prior to the 1990s, the model does quite well. To see this better, we apply a Hodrick-Prescott (1997) filter (with the smoothing parameter set equal to 100). Figure 38 shows the filtered series for 1960–1989. The cyclical predictions are extremely good until the late 1980s. In contrast to Uhlig’s model, this model accounts for most of the decline in the 1970s and all of the decline in the 1980s. An important difference between Uhlig (2003) and Chen et

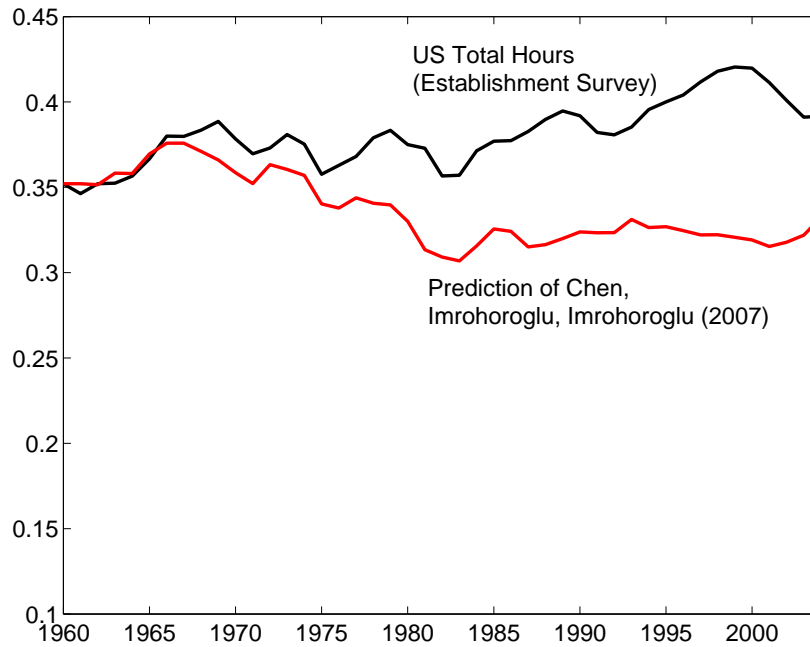


FIGURE 37. U.S. PER CAPITA TOTAL HOURS AND PREDICTION OF CHEN ET AL. (2007, P.15) MODEL, 1960–2004

al. (2007)—especially in the case of hours worked—is that Chen et al. include variations in taxes, whereas Uhlig does not.

However, during the 1990s, both the Uhlig model and the Chen et al. model do poorly. Both predict depressions when in fact there was a boom. Figure 39—which displays the filtered series of Chen et al. for 1990–2004—is a dramatic contrast to Figure 38 for the earlier period. Clearly, there is something missing. This was our starting point. This is what puzzled us for many years.

An open question remains about the basic model’s (of Chen et al.) predictive ability for secular trends. Some of the deviation between theory and data in Figure 37 may be due to measurement. For example, the aggregate CPS-based hours series that we use does not rise as much as the aggregate establishment-based hours series that Chen et al. use. Some of the deviation between theory and data may be due to treatment of households as

uni-sex. Work is beginning on this important topic. In our opinion, the deviation in the 1990s was a big and important puzzle, so we started there.

One final note concerns the role of intangible capital in the pre-1990 period. If technological change is neutral, the predictions of our model *with intangible capital* is the same as Chen et al.'s model *without intangible capital* for the relevant set of (measured) variables. In particular, we too would generate results like that in Figure 39 as long as we allow for variations in TFP, tax rates, government spending, population, and depreciation as in Chen et al. (2007).

### *Summary*

In this chapter, we displayed the results of Chen et al. (2007) to motivate our claim that the basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity, and taxes are incorporated. We contrasted these results with the model in Uhlig (2003), which does not include variation in tax rates and, therefore, does poorly in accounting for movements in hours and productivity.

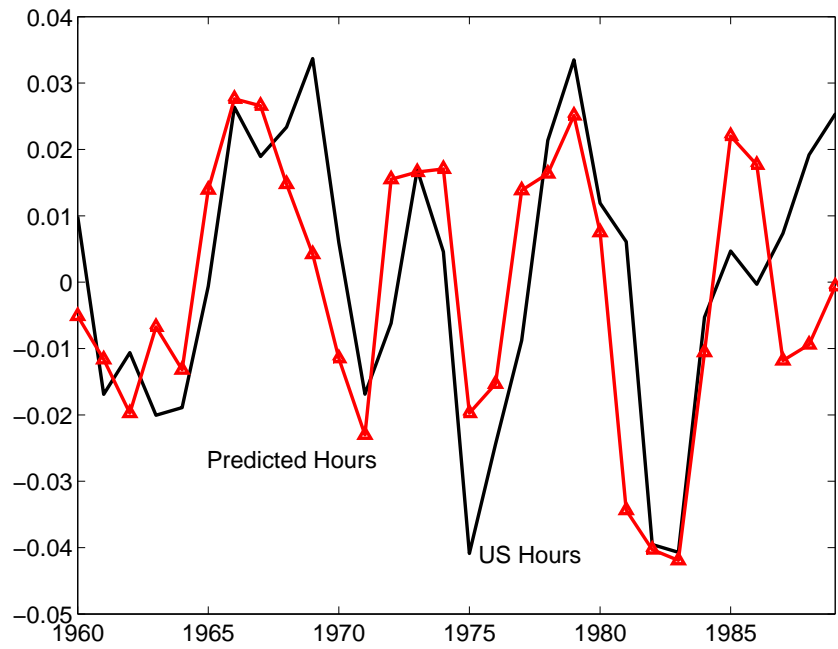


FIGURE 38. FILTERED U.S. PER CAPITA HOURS AND PREDICTION OF CHEN ET AL. (2007) MODEL, 1960–1989

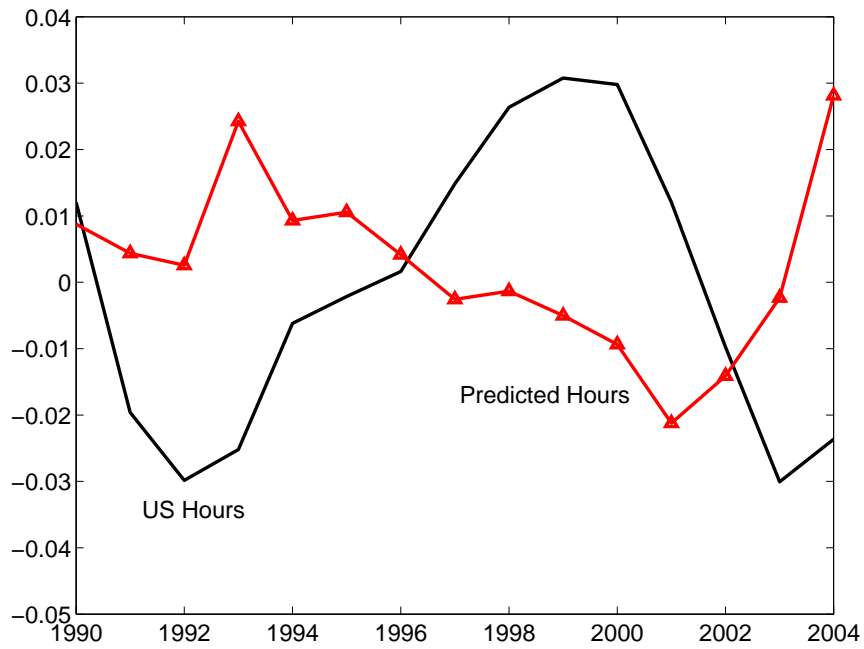


FIGURE 39. FILTERED U.S. PER CAPITA HOURS AND PREDICTION OF CHEN ET AL. (2007) MODEL, 1990–2004

## Chapter 4.

### Assessing New Keynesian Business Cycle Theory

One reasonable reaction to predictions such as those shown in Figures 1–19 is to abandon the standard neoclassical theory and replace it with something else. That has happened to some extent in the macro literature as many researchers have switched to analyzing and using new Keynesian theories. In this chapter, we briefly describe a typical model in this class that Smets and Wouters (2007) analyzed. We then ask, in the context of their model, Why did hours boom in the 1990s? The answer is the same one given in Section 2.1: because a labor wedge boom occurred. The Smets-Wouters model says that there was a shock to wage markups and point out that “alternatively, we could interpret this disturbance as a labour supply disturbance coming from changes in preferences for leisure” (p. 15). Here, we report on some key predictions of the Smets-Wouters model and compare them to the same predictions of the model of Section 2.1.

The new Keynesian models are more complicated than the models worked out in Chapter 2 because they typically have many nominal and real “rigidities” intended to help propagate shocks. Here, we will report results based on the Smets-Wouters (2007) model, which has sticky wages and prices, habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. Fluctuations are driven by seven exogenous variables (to account perfectly for seven observables). Five of the exogenous stochastic variables are modeled as AR(1) processes, and two are modeled as ARMA(1,1) processes. None of the exogenous variables are taken from outside data sources or estimated outside the model (e.g., tax rates or government spending). (See Smets and Wouters 2007 for details.)

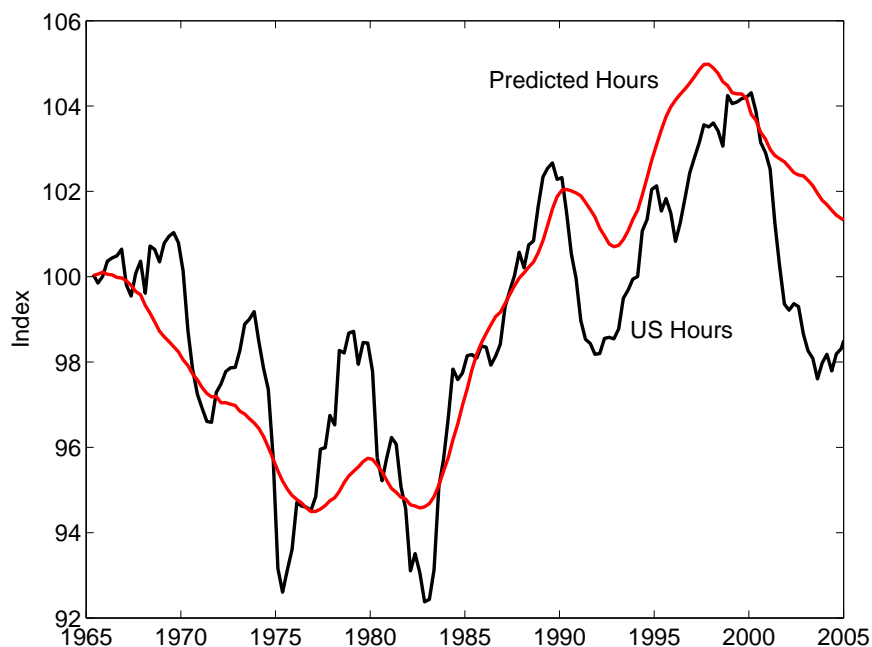


FIGURE 40. U.S. PER CAPITA BUSINESS HOURS AND SMOOTHED PREDICTION FOR SMETS-WOUTERS (2007) MODEL (Preference shock to leisure only, 1965:1–2004:4)

Smets and Wouters (2007) estimate the parameters of their model and use the estimates to compute a variance decomposition for the observed variables. For business hours, they find that 67 percent of variation in the 1965–2005 period is due to shocks to preferences for leisure—what we call the labor wedge. In Figure 40, we plot the series for business hours that they use (which is based on establishment data) along with the model’s prediction of hours. It is not a perfect fit because we shut off all shocks except the shock to preferences for leisure. (Note that if we turn on all shocks, the model’s prediction will be the same as the data by construction.) The figure shows why most of the variation in hours is being attributed to the labor wedge.

If we zoom in on the figure in the 1990s, we get Figure 41. We can compare this Smets-Wouters prediction for the path of business hours due to the labor wedge with the prediction of standard theory (of Section 2.1) for business hours that is shown in Figure



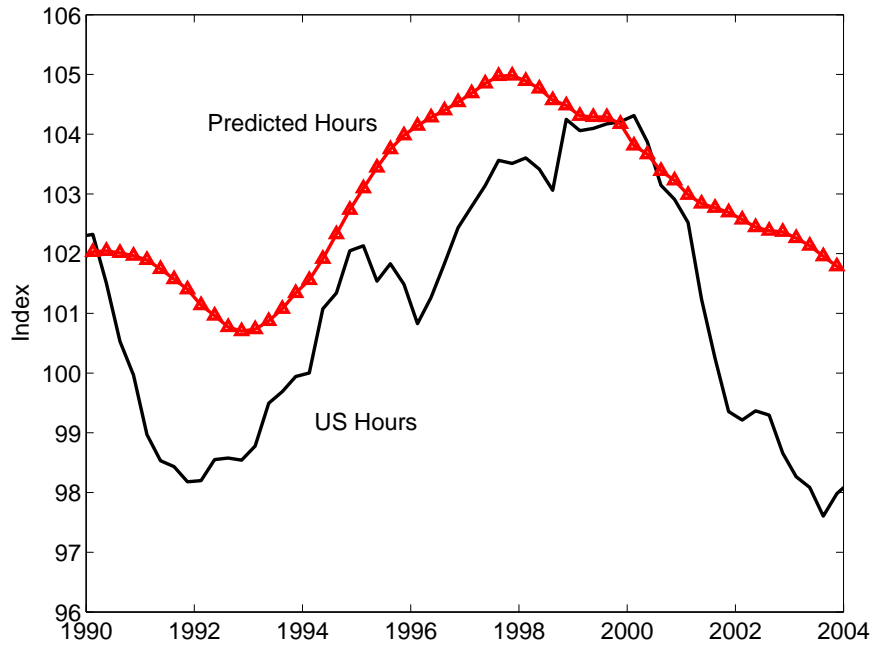


FIGURE 41. U.S. PER CAPITA BUSINESS HOURS AND SMOOTHED PREDICTION FOR SMETS-WOUTERS (2007) MODEL (Preference shock to leisure only, 1990:1–2003:4)

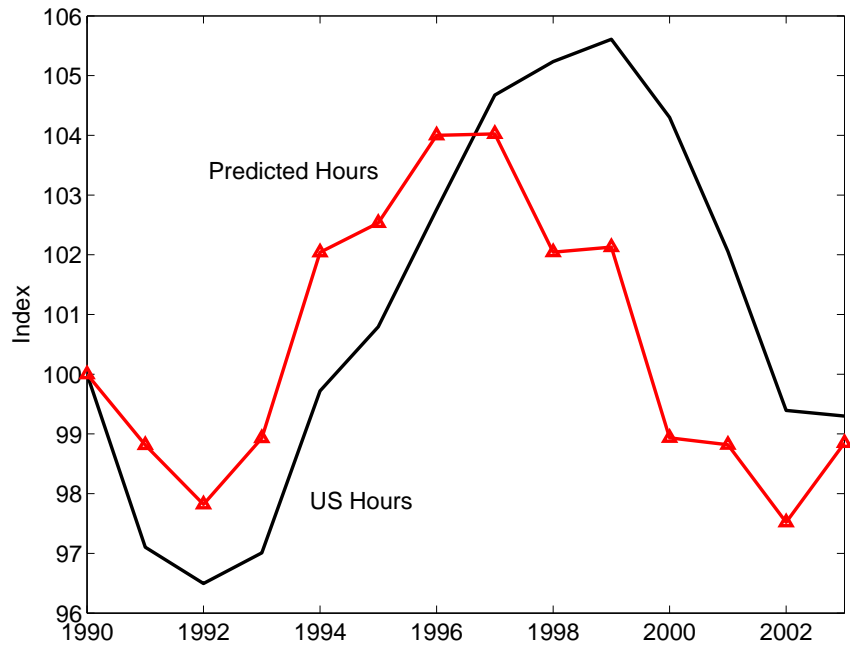


FIGURE 42. U.S. PER CAPITA BUSINESS HOURS AND PREDICTION FOR MODEL WITHOUT INTANGIBLE CAPITAL (TFP and investment wedge constant)

42. We find them similar. Both models attribute the rise in hours to this wedge. In neither model is the boom due to TFP or to monetary shocks. So, what is the implication of this new line of research? It is that the hours boom of the 1990s is primarily due to a preference shock for leisure, which is of the same order of magnitude as the neoclassical model requires to generate the observed hours boom. In building in monetary factors necessary to evaluate the effects of monetary policy, a key puzzle has not been resolved.

### *Summary*

Without evidence or theory for labor wedges, new Keynesian theory does not provide any better understanding of the 1990s than the neoclassical model without intangible capital and non-neutral technological change.

## Chapter 5.

### Checking the Sensitivity of Our Results

In this section, we report on our sensitivity analysis for the model with intangible capital and non-neutral technology. In Section 5.1, we examine the sensitivity of our findings to the values of the parameters and find that the results are robust to their specification. Consequently, the parameters are not being selected to fit the episode. In Section 5.2, we establish that the expectation assumption with regard to the total factor productivity parameters is of little consequence for the realized path of the economy.

#### 5.1. Varying Parameters of the Model

Here, we describe how our results are affected as we vary factor share parameters  $(\theta_i, \phi_i)$ , the depreciation rate on intangible capital  $(\delta_I)$ , the share of intangible investment financed by shareholders  $(\chi)$ , and tax rates on labor and profits  $(\tau_{ht}, \tau_{pt})$ .

We do not have enough data to separately identify the capital shares in the production of intangible investment  $\theta_2, \phi_2$  (assuming they differ from the capital shares in the production of final goods and services), the depreciation rate on intangible capital  $\delta_I$ , and the share of intangible investment financed by shareholders  $\chi$ . To make this determination, we would need measures of all intangible investment, the total stock of intangible capital, and the amount of expensing done by owners of capital and labor. We do not have these data, but we do have data on business valuations and a range of estimates of the value of the intangible capital stock. Neither the data nor the estimates are perfect. Business valuations fluctuate a lot, and the (direct) estimates of the intangible capital stock are

based on a subset of investments. But both sets of data do provide some guidance when choosing *combinations* of the parameters  $\theta_2$ ,  $\phi_2$ ,  $\delta_I$ , and  $\chi$ .

To see how, we next consider combinations of these parameters that imply the same initial 1990 business valuation and NIPA incomes as observed in the U.S. data (and, by construction, for our benchmark parameterization). We'll start by assuming that capital shares are equated across sectors,  $\theta = \theta_1 = \theta_2$  and  $\phi = \phi_1 = \phi_2$ . (We'll relax these restrictions later.) In the benchmark parameterization, the capital shares are equal to  $\theta = 0.269$ ,  $\phi = 0.076$ , the depreciation rate for intangible capital is  $\delta_I = 0$ , and the fraction of intangible investment financed by shareholders is  $\chi = 0.5$ . As we showed earlier (in equations (2.3.13) through (2.3.17)), these parameter estimates are consistent with the 1990 NIPA incomes. In particular, recall from (2.3.14) that we chose parameters so that the initial stock of intangible capital satisfied

$$\hat{k}_I = \frac{\hat{y}_b - r_T \hat{k}_T - \text{1990 NIPA business compensation}}{r_I - \chi q [(1 + \gamma)(1 + \eta) - 1 + \delta_I]},$$

and thus we impose on our parameterization that the 1990 business compensation was equal in the model and data if measured in the same way as it is in the U.S. national accounts.<sup>13</sup> By construction, the NIPA profits also line up in the data and model for 1990. Furthermore, the benchmark parameterization implies that the holding gains in 1990 in the model are consistent with the Flow of Funds average for the period 1953 to 1994. (See Figure 32.)

Consider varying the choice of  $\delta_I$  and choosing the capital shares as before using equations (2.3.16) and (2.3.17). Notice that the derivations also require us to choose a

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<sup>13</sup> In earlier work, we abstracted from sweat equity ( $\chi = 1$ ). In that case, the estimates of the intangible capital stock and the market value of the businesses do not depend on the rate of depreciation of intangible capital.

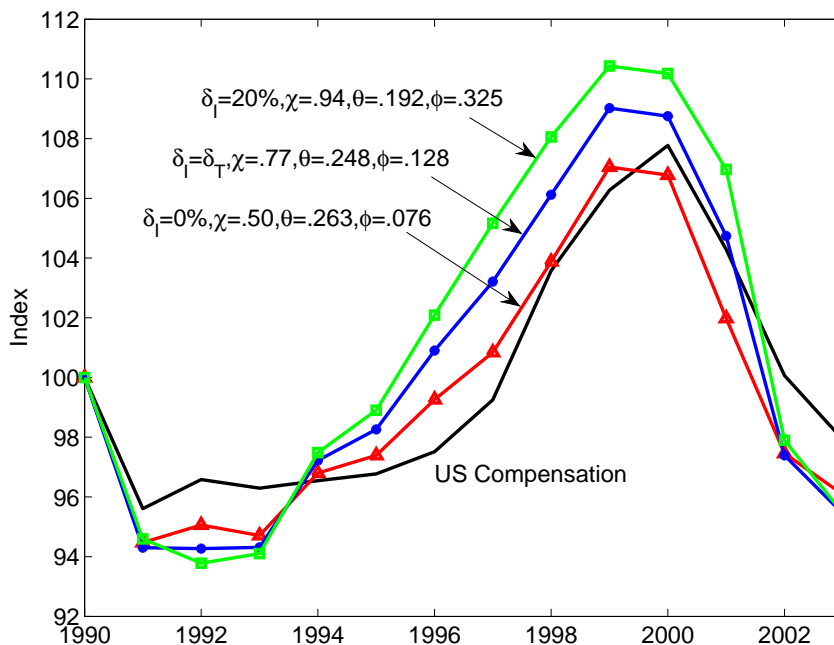


FIGURE 43. U.S. PER CAPITA NIPA COMPENSATION AND PREDICTIONS OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Parameters Chosen to Match 1990 Business Valuation and Incomes)

value for  $\chi$ . We'll choose  $\chi$  so that the resulting initial holding gains line up in the model and data as before. If we use the same algorithm to identify the technology parameters  $A_t^1$  and  $A_t^2$ , we find that the quantitative predictions for hours, output, business value added, GDP per hour, business value added per hour, intangible investment, consumption, and capital gains are so close as to not be distinguishable from the equilibrium paths shown in Figures 23, 25 through 30, and 32.

Two equilibrium paths that do change are compensation, measured as in NIPA, and intangible investment. In Figure 43, we plot U.S. compensation along with predictions for three different choices of  $\delta_I$ . The path marked  $\delta_I = 0\%$  is the benchmark in which  $\theta = 0.263$ ,  $\phi = 0.076$ , and  $\chi = 0.5$ . The path marked  $\delta_I = \delta_T$  assumes that the depreciation rates of intangible and tangible capital are equal. In this case, we chose  $\theta = 0.248$ ,

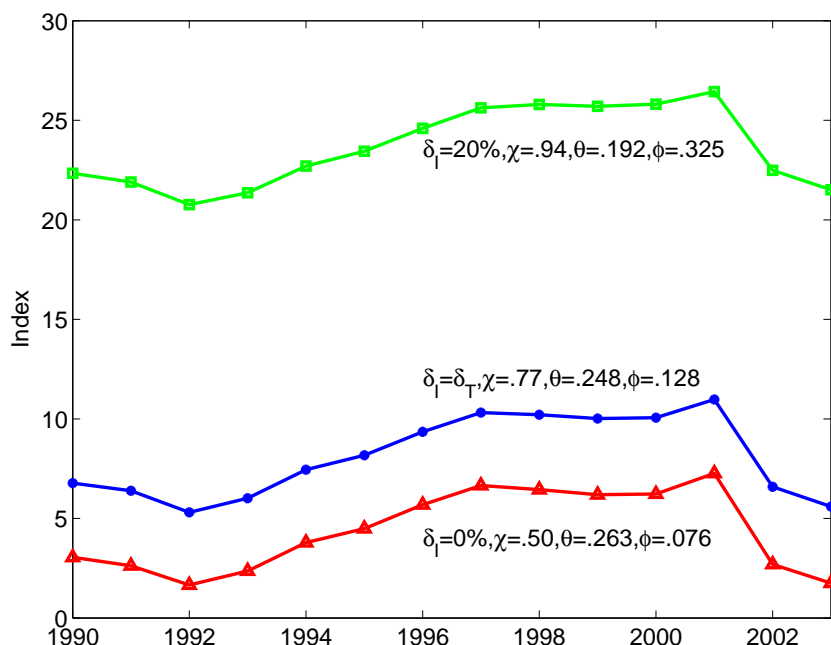


FIGURE 44. PREDICTED INTANGIBLE SHARE OF OUTPUT IN MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Parameters Chosen to Match 1990 Business Valuation and Incomes)

$\phi = 0.128$ , and  $\chi = 0.77$  so as to match the initial business valuation and NIPA incomes. The path marked  $\delta_I = 20\%$  assumes a much larger rate of depreciation. In this case, we chose  $\theta = 0.192$ ,  $\phi = 0.325$ , and  $\chi = 0.94$  in order to match the initial business valuation and NIPA incomes. From Figure 43, we see that the predicted increase in NIPA compensation is higher as we raise  $\delta_I$ . This occurs because we require a higher value of  $\chi$  to match the initial incomes. But values of  $\chi$  that are too close to 1 (as in the experiment with  $\delta_I = 20\%$ ) lead us to overestimate the increase in labor compensation in the late 1990s. On the other hand, values for  $\chi$  in the range of 0.5 to 0.75 are consistent with the pattern of U.S. compensation.

In Figure 44, we plot the model predictions for the share of intangible investment in total output. In the benchmark case, the investment share rises to about 7 percent. If the

depreciation rates of intangible and tangible capital are equal, the investment share rises to about 11 percent. For a depreciation rate of 20 percent, the share rises to about 27 percent, which is much higher than any direct estimates of intangible investment. Interestingly, the different choices for  $\delta_I$  imply very different levels of investment but have little impact on the change in investment. In other words, the predictions for net investment are quite robust across the experiments.

Thus far, we chose the capital shares for the two production functions to be equal. With our specification, this means that intangible capital is used in a nonrival way to produce both final goods and services and new intangible capital goods. We next ask, How do the results change if the capital shares differ? The specific case we consider assumes that intangible capital has a much smaller share in the second activity ( $\phi_1 = .076$ ,  $\phi_2 = .01$ ) and that the labor shares are equal ( $1 - \theta_1 - \phi_1 = 1 - \theta_2 - \phi_2$ ) in the two activities. We also chose  $\chi$  and  $\theta_1$  in a way that maintains consistency between the model and U.S. 1990 incomes and valuations as before. With these restrictions, the implied tangible capital shares are  $\theta_1 = 0.260$  and  $\theta_2 = 0.327$ , and the financing share is  $\chi = .457$ .

In Figure 45, we plot hours of work in the benchmark economy and the alternative economy along with the U.S. data. As the figure shows, there is only a small difference in the predictions. In Figure 46, we plot per capita real GDP for the different experiments. Here, we see that the benchmark parameterization does a better job overall in mimicking the pattern of GDP, but both simulations capture the late 1990s boom. We prefer the benchmark parameterization not only because it does a better job on some dimensions, but also because it seems more plausible to us that existing brands and patents are used in the development of new brands and new R&D.

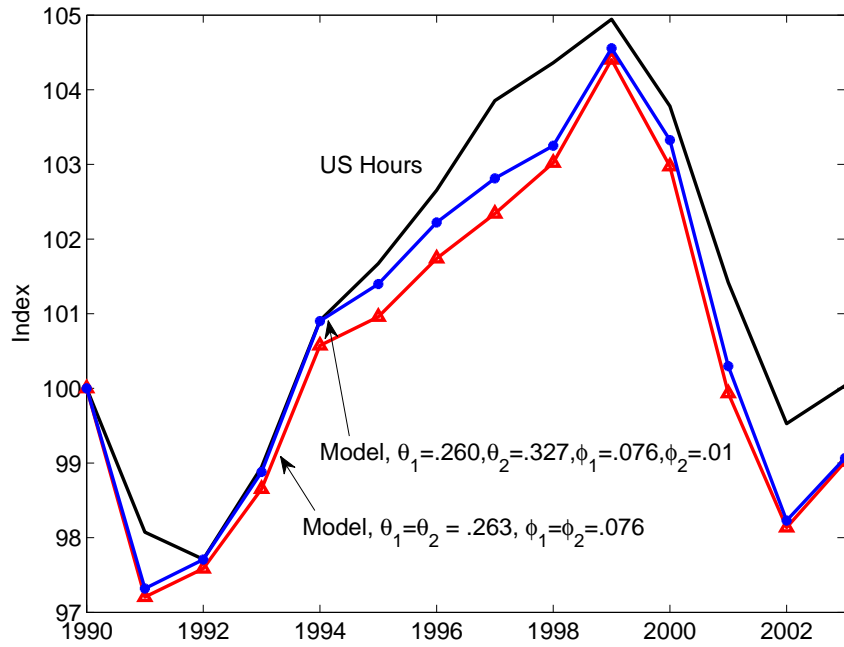


FIGURE 45. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY (Parameters Chosen to Match 1990 Business Valuation and Incomes)

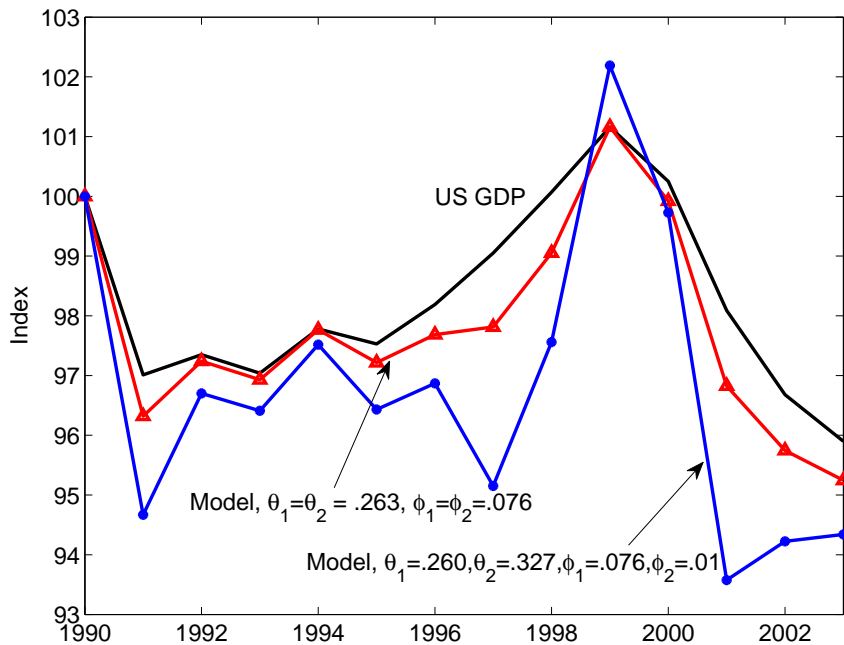


FIGURE 46. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Parameters Chosen to Match 1990 Business Valuation and Incomes)



We turn next to the tax rates. Given the fact that most current models rely on large preference shocks to leisure—which are isomorphic to large declines in the tax on labor—we thought it important to try other measures of the labor tax. In Figures 47 and 48, we display the predictions for per capita hours and real GDP for the baseline rate  $\tau_{ht}$  in Table 4 and the estimates of the NBER TAXSIM model available at [www.nber.org](http://www.nber.org). (See Feenberg and Coutts 1993 for details.) It is hard to distinguish the predictions for these alternative measures in the figures.

We also examined how our results change if we relax the assumption of a constant profit tax rate. We use the following measure for  $\tau_{pt}$ : NIPA corporate tax liabilities divided by corporate income (with Federal Reserve profits subtracted from both the numerator and the denominator). The difference in the hours predictions for this case and the baseline case is hard to see in Figure 47. There is some noticeable difference in the predictions for GDP because the constructed  $\tau_{pt}$  series has some effect on our prediction for tangible capital.

## 5.2. Varying Expectations

In this section, we demonstrate that the perfect-foresight assumption is innocuous. We do this by comparing results for two versions of our model that differ only in the assumption of expectations.

To simplify our analysis, we set all tax rates and non-business variables equal to their 1990 levels and ignore the investment wedge. Another simplification involves the TFP processes. We want to generate a non-neutrality in  $A_t^2$  relative to  $A_t^1$ . We do this by

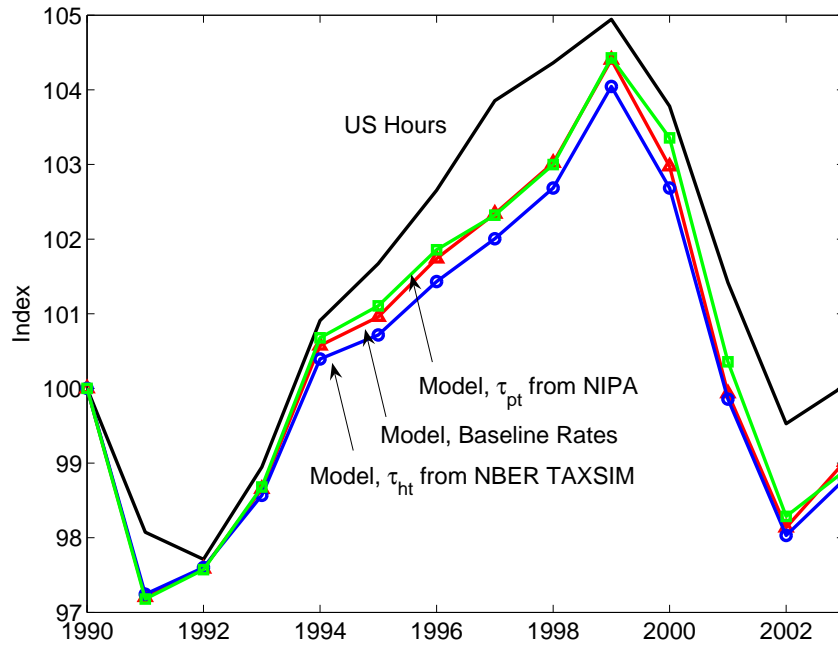


FIGURE 47. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY (Investment wedge constant)

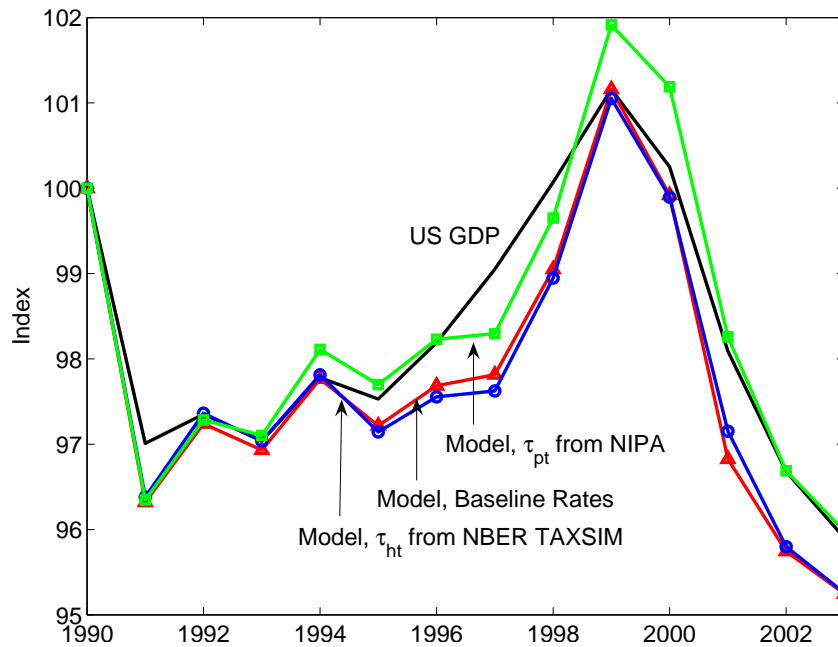


FIGURE 48. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY  $1.02^t$  (Investment wedge constant)

modeling the two TFPs as functions of the same AR(1) process. Thus, the problem boils down to one with a one-dimensional shock process.

More specifically, we use Tauchen's (1986) discrete approximation for AR processes to construct an 11-state Markov chain for an AR(1) process  $\ln s_{t+1} = \rho \ln s_t + \epsilon_t$  with  $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ . We set  $\ln A_t^1$  equal to its steady-state value plus  $\ln s_t$ . We set  $\ln A_t^2$  equal to its steady-state value plus two times  $\ln s_t$ . The transition matrix is the matrix computed using Tauchen's method for a specific value of  $\rho$ .

Given the processes for the TFP parameters, we find an equilibrium stochastic process for the economy. We pick a particular realization of the exogenous stochastic TFPs, making sure that our choice implies increases in hours and output that are of similar magnitude as their observed counterparts. We then find the equilibrium realization of the endogenous variables for the realization of exogenous stochastic elements of the economy. Finally, we compare this to the realization if households had perfect foresight about the paths of the TFP parameters.

In Figure 49, we display the realizations of the paths of TFP in final production  $A_t^1$  and TFP in intangible production  $A_t^2$  that we use in our experiment. These are not the same paths used in generating the results of the paper because of the restrictions on the exogenous inputs. However, these values of TFPs do imply a technology boom of the same magnitude as we saw in the United States during the 1990s.

In Figure 50, we display the household's realized hours in the case that TFPs are governed by an AR(1) process and in the perfect-foresight case. For the AR(1) processes, we show several values of  $\rho$  between 0 and 0.95<sup>4</sup>.<sup>14</sup> There is a modest difference in predicted

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<sup>14</sup> Typical values in the business cycle literature for  $\rho$  are in the range of 0.9 to 0.95 for quarterly data.

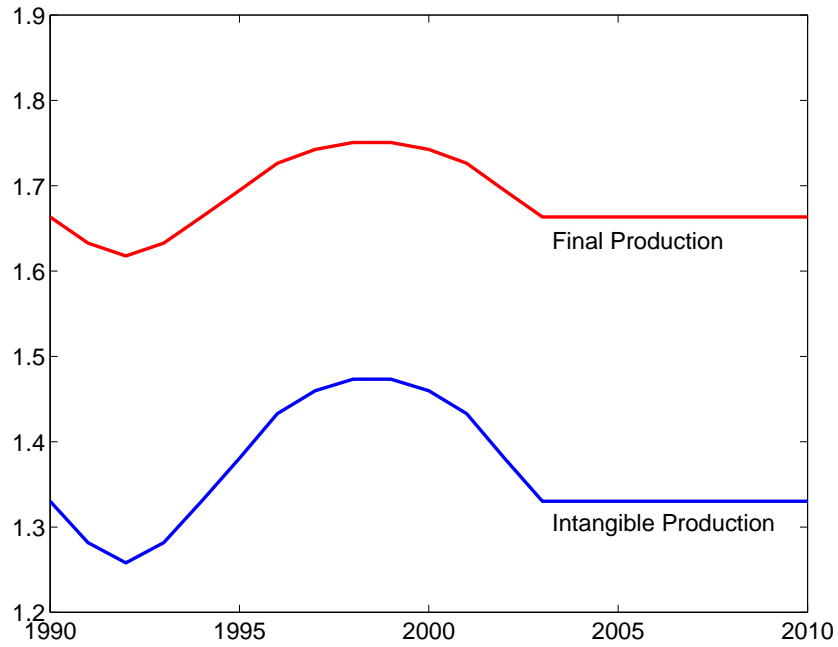


FIGURE 49. REALIZATIONS OF TFP INPUTS IN STOCHASTIC MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY  
(All other exogenous inputs constant)

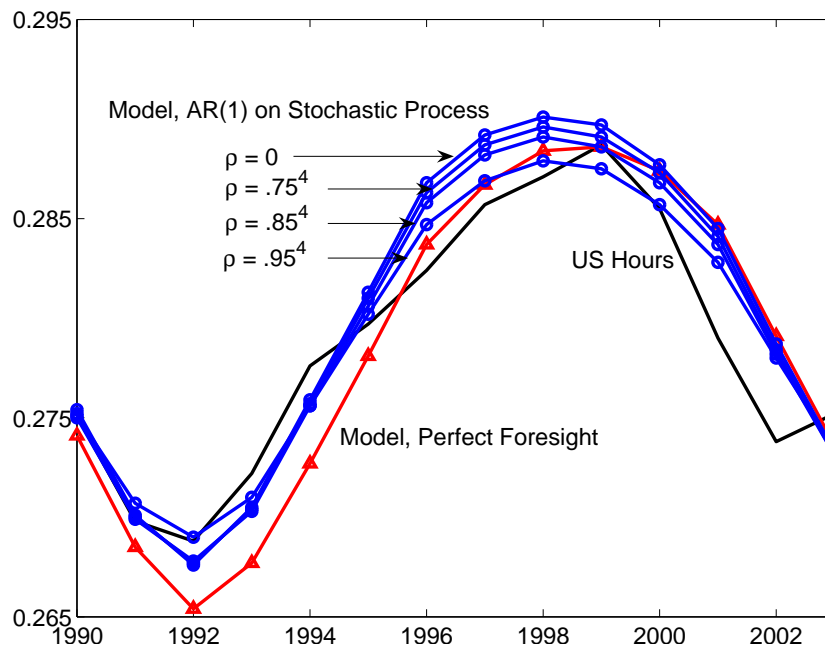


FIGURE 50. U.S. PER CAPITA HOURS AND PREDICTIONS OF STOCHASTIC MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY  
(All other exogenous inputs constant)

Here, we raise the values to the power 4 since we are working with annual data. Higher values of  $\rho$  yield levels of persistence that are much higher than observed in U.S. time series.

paths at the beginning of the decade, but the hours boom in the second half of the 1990s is similar in all experiments. The difference in terms of other endogenous variables is similarly modest. The reason is that the realizations of  $A_t^1$  and  $A_t^2$  are what is important, not the choice of household expectations. Also plotted are U.S. hours. As we noted earlier, this model does generate a boom of the right magnitude.

### *Summary*

We find that our conclusions are robust to plausible variations in parameters and expectations.

## Chapter 6.

### Reviewing the Micro Evidence

In this chapter, we provide more details on the micro evidence discussed in our paper. We use data from the Bureau of Labor Statistics (BLS) to show that the large increase in hours during the 1990s can be traced to a subset of occupations and industries engaged in producing and using information technology. We then use data from the BEA R&D satellite accounts and the Federal Reserve's Survey of Consumer Finances (SCF) to link these IT-related activities to part of intangible investment—namely R&D—and intangible income in the form of business capital gains.

#### 6.1. Bureau of Labor Statistics

In this section, we report statistics constructed by the Economics and Statistics Administration and statistics that we construct from microsamples of the Current Population Survey (March supplement). The first set of statistics shows that there was large growth in IT-related employment which was not accompanied by above-average growth in IT-related weekly earnings. This evidence suggests that IT-related work was compensated in another way. The second set of statistics decomposes the rise in total hours into that attributable to educated workers in IT-related activities and all other workers. We show that the first group is relatively small but is responsible for a significant fraction of the hours boom. The latter data allow us to connect our macro data to the micro evidence. Furthermore, we can do the same exercise later with data from the Survey of Consumer Finances and make a further connection to micro evidence on capital gains.

### 6.1.1. Economics and Statistics Administration Reports

There is a consensus among analysts of U.S. productivity growth that the rapid growth of information technology (IT) was a primary engine of growth in the 1990s. It is not surprising, then, that U.S. statistical agencies such as the Department of Commerce's Economics and Statistics Administration (ESA) have devoted resources to tracking changes in IT-related industries and occupations.

Figure 51, which is reproduced from the ESA's *Digital Economy 2003*, shows the employment trends in IT-producing industries versus all private industries over the 1990s based on data from the BLS.<sup>15</sup> Between 1993 and 2000, employment in IT-producing industries rose 52 percent, while employment in all private industries rose 21 percent (with no correction for trend in population). Figure 52, which is also reproduced from *Digital Economy 2003*, shows the levels of employment in the IT-producing sector. In 1993, there were 3.54 million employed in these industries. This accounts for 3.9 percent of all employed in private industries. By 2000, the employment had grown to 5.38 million, roughly 4.8 percent of all employed. Figures 51 and 52 also show a large drop in employment in IT-producing industries between 2000 and 2002 as these industries contracted.

Despite strong growth in IT employment during the 1990s, median weekly earnings in core IT occupations were not above average. In the *Digital Economy 2000*, the ESA reports that the average growth rate in the employment of computer scientists, computer engineers, and systems analysts was 15 percent per year between 1992 and 1998, much faster than the average rate of 2.5 percent for all occupations. The average growth rates in median earnings, however, were the same for these IT workers and all workers: 2.7 percent

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<sup>15</sup> Appendix Table A-1.1 of *Digital Economy 2003* contains the list of industries (six-digit, 1997 NAICS) categorized as IT-producing. The industries fall into four main categories: computer hardware, communications equipment, software and computer services, and communications services.

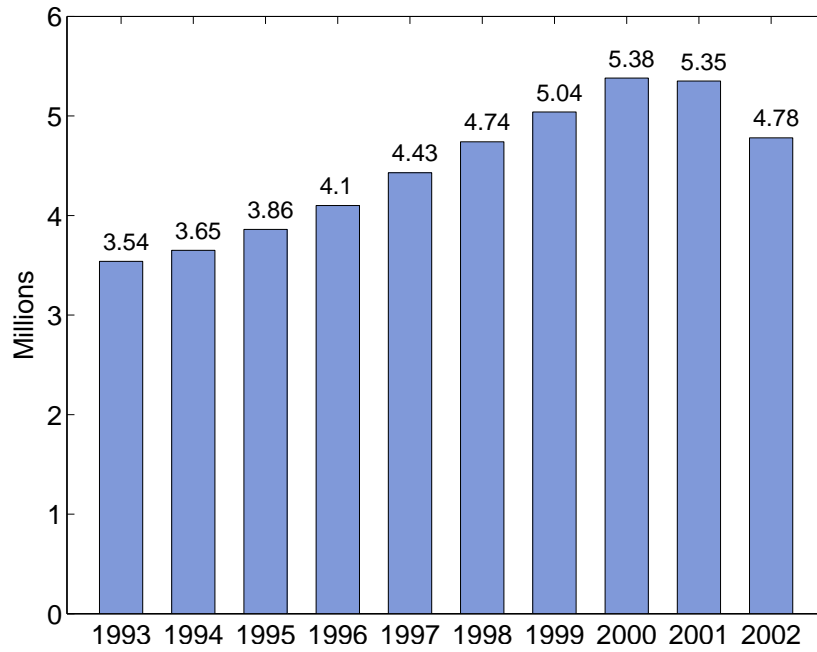


FIGURE 51. IT-PRODUCING INDUSTRY EMPLOYMENT LEVELS

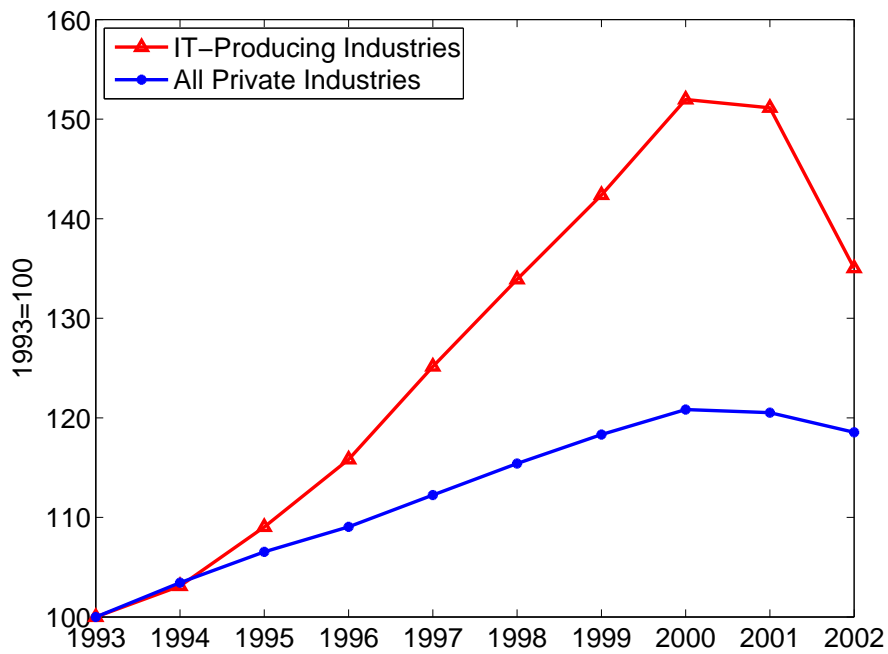


FIGURE 52. EMPLOYMENT INDICES, IT-PRODUCING AND ALL PRIVATE INDUSTRIES

per year between 1992 and 1998. One possible explanation put forth by the ESA is that businesses use non-wage benefits such as stock options to attract employees. Stock options



show up as compensation later when options are exercised. Another explanation that we explore below is that these workers owned shares in the business and earned capital gains.

We turn next to evidence on individual hours from the Current Population Survey.

### **6.1.2. Current Population Survey**

Our study uses total hours as the labor input in production. The aggregate hours series that we use (see Figure 1) is total hours taken from the Current Population Survey divided by the noninstitutional population aged 16 to 64. Here, we examine a microsample of individual data from the same survey and focus in on particular subsamples of workers. We show that the rise in hours was disproportionately accounted for by educated workers in IT-producing industries, IT-using industries, and IT-related occupations.

There is an issue we face when comparing statistics across datasets: the industry and occupation classification codes are not the same. For industries, we use correspondence tables from the U.S. Census Bureau to match up 1997 NAICS categories with 1987 SIC categories and then 1987 SIC categories with CPS categories (IPUMs variable IND for the period 1992–2002). In Table 5, we report our list of IT-producing industries and the corresponding industry classification codes for the CPS data. In Table 6, we report codes for IT-using industries if they are not already included in Table 5. Analysts of U.S. productivity point to capital deepening related to information technology as another source of abnormally high productivity in the 1990s. This list is consistent with Oliner, Sichel, and Stiroh’s (2007) classification of IT-using industry. (See their Table A-1.)

To categorize IT-related occupations of high-skill workers, we use two methods. First, we match up IT-related occupations for high skill level as defined in the ESA’s *Digital*

321	Office and accounting machines
322	Electronic computing equipment
331	Machinery except electrical
341	Radio, television and communication equipment
342	Electrical machinery, equipment and supplies
371	Scientific and controlling equipments
440	Radio broadcasting and television
441	Telephone
442	Telegraph
510	Professional and commercial equipment
732	Computer and data programming service
741	Business service n.e.c.
752	Electrical repair shops

TABLE 5. CPS CODES FOR IT-PRODUCING INDUSTRIES

*Economy 2003*, Box 2.1, with the CPS occupation classification (IPUMs variable OCC for the period 1992–2002). This procedure gives the first five occupations listed in Table 7. Second, we found the occupations that were most common among workers with at least one year of college (IPUMs variable EDUCREC with values 8 or higher) and in an IT-producing industry (Table 5). The frequency cutoff that we used was 2 percent of the occupations. So, for example, conditioning on education and industry, each of the occupations in Table 7 accounted for at least 2 percent of the occupations. From the list, we dropped secretaries and telephone installers, which we view as lower-skilled occupations.

Given filters for industries and occupations, we next decompose total hours into those worked by the “high-skill” and “high-tech” and the remainder; total hours per person is therefore equal to the sum of hours of high-skill, high-tech workers divided by the number of persons and hours of workers who are not high-skill or high-tech divided by the number of persons. In Table 8, we show the results. First, if we sum up hours for all individuals in 2000, we find that the total hours is 6.5 percent higher in 2000 than

060	Construction
172	Printing, publishing, and allied industries
180–192	Chemicals and allied products
310–332	Machinery and computing equipment
340–350	Electrical machinery, equipment, and supplies
351–370	Transportation equipment
391	Miscellaneous manufacturing industries
420–422	Water, air, and pipeline transportation
440–442	Broadcasting and telecommunications
550–571	Wholesale trade
700–711	Finance and insurance
730	Commercial research, development and testing labs
841	Legal services
891-893	Professional, scientific, and technical services

TABLE 6. CPS CODES FOR IT-USING INDUSTRIES

in 1992 (which is normalized to 100). If the high-skill, high-tech group is defined to be workers in IT-producing industries with at least one year of college—which we refer to as “educated”—then we find that they contributed 4.8 percent of the hours worked in 1992. Workers not in this category supply the remaining 95.2 percent of hours worked. By 2000, educated workers in the IT-producing industries were contributing 6.2 percent of the hours. Thus, a group that is 2.9 percent of the population in 1992 was responsible for 22 percent ( $=1.4/6.5$ ) of the increase in total hours.

If we use educated workers in IT-related occupations as our measure of high-skill and high-tech, then we find a similar result, namely that a small fraction of the population is contributing disproportionately to the rise in hours. The third row of Table 8 shows that workers with occupations listed in Table 7 were a little more than 5 percent of the 1992 population but added 2.5 hours or 38 percent ( $=2.5/6.5$ ) to the hours increase between 1992 and 2000. If we add workers that are not in an IT-related occupation but work in an IT-producing industry, the contribution rises to 41 percent ( $=2.7/6.5$ ).

022	Managers and administrators n.e.c.
055	Engineers, electrical
064	Computer system analysts
213	Electrical and electronic technicians
229	Computer programmers
013	Managers, marketing, advertising
023	Accountants and auditors
185	Designers
257	Sales occupations and other business services
259	Sales reps

TABLE 7. CPS CODES FOR IT-RELATED OCCUPATIONS OF HIGH-SKILLED

If we expand our list of industries to include both IT-producing industries and IT-using industries (which are summarized in Tables 5 and 6) and again include only workers with at least one year of college, then we can account for 51 percent ( $=3.3/6.5$ ) of the rise in hours. If we add workers in non-IT industries who have IT-related occupations, then we account for 57 percent ( $=3.7/6.5$ ) of the rise even though this group accounts for only 12.6 percent of the population in 1992.

To summarize, we have used BLS data to show that the large rise in hours was concentrated in certain activities, namely those that are IT-related. In the next two sections, we link these activities to increased investment in a particular type of intangible capital, namely R&D, and to increased income from intangible investment in the form of business capital gains.

## 6.2. Bureau of Economic Analysis Satellite Accounts

Universe (Share of 1992 Population)	1992	2000	Chg.	% Chg.
All workers (67%)	100.0	106.5	6.5	6.5
Educated workers in				
IT-producing industries (2.9%)	4.8	6.2	1.4	30.2
IT-related occupations (5.1%)	8.8	11.2	2.5	28.0
IT-producing industries or IT-related occupations (6.6%)	11.2	13.9	2.7	24.2
IT-producing industries or IT-using industries (10.7%)	17.3	20.6	3.3	19.0
All IT industries/occupations (12.6%)	20.6	24.3	3.7	18.0

TABLE 8. HOURS PER NONINSTITUTIONAL POPULATION AGE 16–64  
(1992 TOTAL NORMALIZED TO 100)

Next, we use data from the BEA R&D satellite accounts to show that certain directly measured intangible investment, namely R&D, grew dramatically for IT industries.<sup>16</sup>

In Figure 53, we plot the share of private business R&D funded by the main IT-producing industries, namely computer and electronic product manufacturing, software publishers, and computer systems design and related services (with 2002 NAICS codes 3341, 3342, 3344, 3345, 3343, 3346, 5112, 5415). Between 1990 and 2002, the share of R&D of these industries doubled, rising from 20 percent of total private business R&D to 40 percent.

Data for the period 1959–2004 are available for industries in computer and electronic product manufacturing. Figure 54 shows R&D funded by all IT-producing for the period 1987–2004 as a share of GDP along with the subcomponent for computer and electronic

<sup>16</sup> The source of the BEA R&D expenditure estimates is the National Science Foundation.

product manufacturing (NAICS 334). The figure shows that IT services contributed significantly to the 1990s rise in IT R&D.

We turn next to evidence of income to intangible investment.

### **6.3. Federal Reserve Statistics**

We noted above that weekly earnings for certain core IT occupations did not rise proportionately with hours. This may be due to the fact that their non-wage compensation rose. In this section, we examine the rise in business capital gains.

#### **6.3.1. Flow of Funds**

In Figure 55, we plot the real holding gains (excluding real estate) as a function of GDP along with averages for two subperiods. The gains relative to GDP averaged 6 percent over the period 1953–1994 and slightly over 12 percent over the 1995–2003 period. The picture clearly shows a break between 1994 and 1995, and we find that this break is statistically significant.

#### **6.3.2. Survey of Consumer Finances**

We use the Federal Reserve’s Survey of Consumer Finances (SCF) to determine the characteristics of households responsible for most of the increases in hours and business capital gains during the 1990s. We find two characteristics to be important for both hours and capital gains: education level and occupational group.

Although the SCF public datasets do not report the same occupational and industry

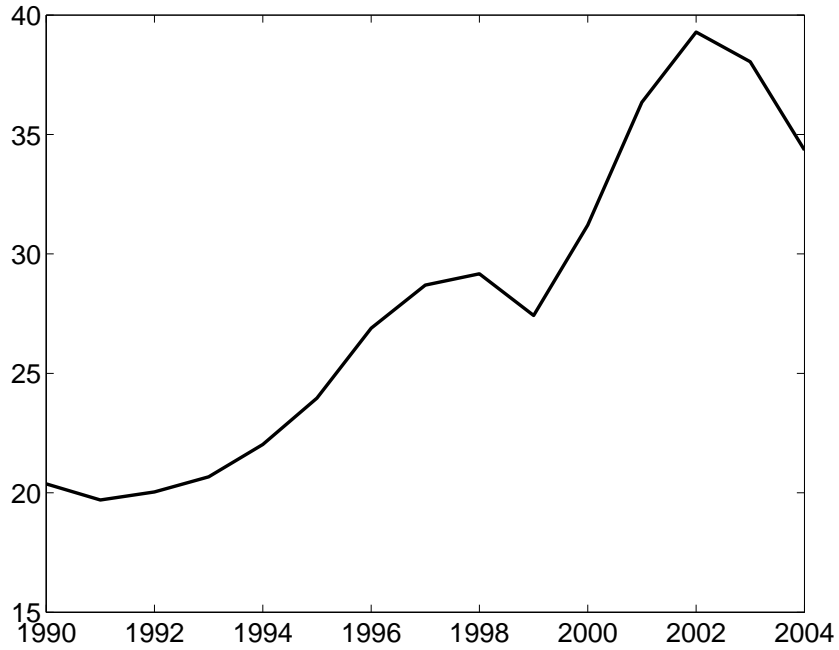


FIGURE 53. SHARE OF PRIVATE R&D FUNDED BY 8 4-DIGIT IT INDUSTRIES

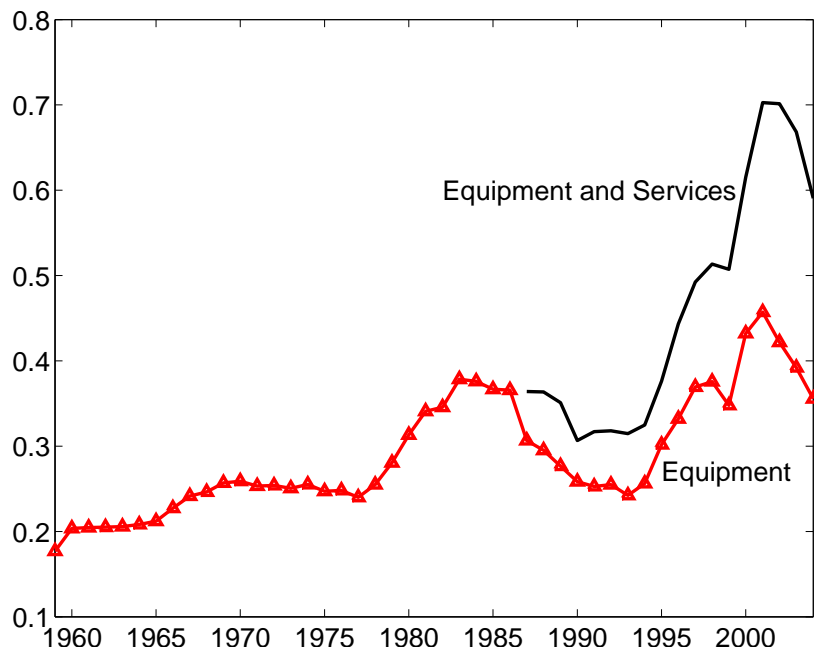


FIGURE 54. PRIVATE R&D FUNDED IT INDUSTRIES RELATIVE TO GDP

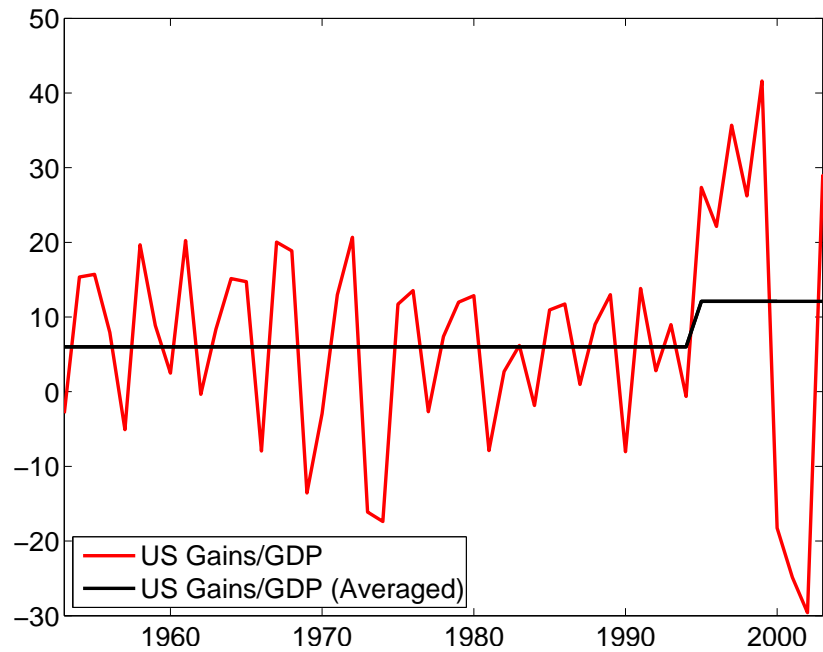


FIGURE 55. U.S. REAL HOLDING GAINS AS % OF GDP

detail as the CPS, we do know from the CPS survey that many of the workers in IT-related occupations fall in the SCF occupational category “managerial and professional.” If we expand the SCF universe to include “technical, sales, and services” occupations, then we cover all IT-related occupations.<sup>17</sup> With industries, the SCF classifications are too broad and most include IT-related industries. Therefore, we work with occupations rather than industries and refer to “high-tech” workers as those in “managerial and professional” occupations or, if we use the broader definition, we include “technical, sales, and service” as well.

Each record in the SCF public dataset includes data for a household. The survey asks respondents to report hours in a normal week and weeks in a normal year for their main job and, if applicable, a second job. If the respondent has a spouse, hours and weeks of work are provided for the spouse as well. The survey also asks about their educational

<sup>17</sup> The CPS codes for “managerial and professional” are 3 to 199 and “technical, sales, and service” are 200 to 469.



achievement. As in the case of the CPS, we define someone as “educated” if they have completed at least one year of college.

Using the hours and educational responses, we construct individual-level statistics for annual hours and capital gains for two groups: (1) those that are educated and in a high-tech occupation and (2) all others. For hours, the data are provided at the individual level. For capital gains, the data are provided at the household level. Thus, we divide the household gains between the respondent and the spouse (if there is one) in proportion to the number of hours that they work.<sup>18</sup>

As before, we find that a small fraction of workers are responsible for most of the rise in hours of work. If we restrict “high-tech” occupations to be those in managerial and professional (CPS codes 3–199), then we find that this group, which is 17 percent of the population, contributed 30 percent of the total hours in 1992. By 2001, this group’s hours rose 22 percent and they accounted for 78 percent of the change in total hours. More importantly, this group accounted for 68 percent of the change in business capital gains between 1992 and 2001. If we expand the notion of “high-tech” occupations to include technical, sales, and service, then we find that the group accounts for 53 percent of the change in hours and 73 percent of the change in capital gains, despite the fact that they are only 29 percent of the population in 1992.

In summary, the micro evidence suggests that the hours boom was concentrated in certain occupations and industries that can be linked to sizable intangible expensed and sweat investments. While the evidence indicates that intangible capital played a critical role in accounting for the puzzling 1990s boom, the data are not sufficiently detailed to

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<sup>18</sup> We do this because we do not have sufficient information to determine which hours of a household are linked to the household’s capital gains.

allow us to split our total hours series into the two components dictated by theory: hours used in producing final goods and services and hours used in producing new intangible investments. However, our theory does allow us to infer this split and make predictions about how much productivity and investment were understated in the 1990s due to intangible investment not being measured.

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