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TECHNICAL APPENDIX: Does Neoclassical Theory Account for the Effects of Big Fiscal Shocks? Evidence from World War II*

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1. Introduction

In this technical appendix we describe the details of the computation of the benchmark model (Section 2) and the variable capacity utilization model (Section 3). The Fortran codes that accompany the benchmark model are bench.f90 (used for the perfect foresight examples) and sbench.f90 (used for the stochastic examples). The Fortran codes that accompany the variable capacity utilization model are caputil.f90 (used for the perfect foresight examples) and scaputil.f90 (used for the stochastic examples). These codes along with the data are available at the website of the Minneapolis Federal Reserve and are listed under Staff Report 315.

In Section 4, we discuss detailed results for the variable capacity utilization model, which is only briefly discussed in the paper. In Section 5, we report results of our sensitivity analysis.

2. Benchmark model

In this section we analyze the maximization problem for a stand-in household with civilians and draftees. Total output in the model is the sum of nonmilitary output and military compensation. Nonmilitary output is produced with civilian hours, private capital, and public capital that does not include military equipment or structures. Private and public capital used to produce nonmilitary output are assumed to be perfect substitutes. Here, we do not distinguish the two margins for adjusting the labor input: hours per worker and the fraction employed. Later we will.

2.1. Household problem

The representative household has two types of family members, civilians and draftees. In period t, fraction $1 - a$ are civilians and fraction a are drafted (i.e., in the "army"). Civilians can choose their level of hours but draftees cannot. The problem the household solves is given by

$$
\begin{aligned}
\max_{\{c_{ct},c_{dt},i_{pt},l_{ct}\}} & E\sum_{t=0}^{\infty}\beta^t\ \left\{(1-a_t)U(c_{ct},l_{ct})+a_tU(c_{dt},\bar{l})\right\}(1+g_p)^t \\
\text{subject to} \ & (1-a_t)c_{ct}+a_tc_{dt}+i_t=(1-\tau_{kt})r_tk_{pt}+(1-\tau_{lt})(1-a_t)w_tl_{ct}+\tau_{kt}\delta k_{pt}+T_t \\
& k_{pt+1}=[(1-\delta)k_{pt}+i_{pt}]/(1+g_p) \\
& i_{pt}\geq 0 \quad \text{in all states} \\
& c_{ct}\leq c_{max} \quad \text{in some states} \\
& c_{dt}\leq c_{max} \quad \text{in some states} \\
& l_{ct}\leq l_{max} \quad \text{in some states}
\end{aligned}
$$

with processes for a_t, r_t, w_t, τ_{kt}, τ_{lt}, and T_t given. Quantities are in per-capita terms and the population grows at rate g_p. For now, we assume that civilians choose the total labor input. Later, we will consider choices of both hours per week and employment.

From here on, we will assume that $U(c,l)=\log(c)+V(1-l)$ so that the marginal utilities of civilians and draftees are equated. The Lagrangian for the optimization problem in this case is:

$$
\begin{aligned}
\mathcal{L}=E\sum_t\tilde{\beta}^t\Bigg\{&(1-a_t)[\log(\hat{c}_{ct})+V(1-l_{ct})]+a_t\log(\hat{c}_{dt}) \\
&+\frac{\zeta}{3}\Big[\min(\hat{i}_{pt},0)^3+(1-a_t)\min(l_{max}(s_t)-l_{ct},0)^3 \\
&\quad+(1-a_t)\min(c_{max}(s_t)-\hat{c}_{ct},0)^3+a_t\min(c_{max}(s_t)-\hat{c}_{dt},0)^3\Big] \\
&+\mu_t\Big\{(1-\tau_{kt})r_t\hat{k}_{pt}+(1-\tau_{lt})(1-a_t)\hat{w}_tl_{ct}+\tau_{kt}\delta\hat{k}_{pt}+\hat{T}_t \\
&\quad-(1-a_t)\hat{c}_{ct}-a_t\hat{c}_{dt}-\hat{i}_{pt}\Big\} \\
&+\lambda_t\Big\{(1-\delta)\hat{k}_{pt}+\hat{i}_{pt}-(1+g_p)(1+g_z)\hat{k}_{t+1}\Big\}\Bigg\}
\end{aligned}
$$

where $\tilde{\beta}=(1+g_p)\beta$. Variables that grow over time are detrended and denoted with a hat (e.g., $\hat{c}_{ct}=c_{ct}/(1+g_z)^t$). Note that we have added penalty functions to account for our inequality constraints when computing equilibria.

The first-order conditions are thus:

$$1/\hat{c}_{ct} - \zeta \min(c_{max}(s_t) - \hat{c}_{ct}, 0)^2 = \mu_t$$

$$1/\hat{c}_{dt} - \zeta \min(c_{max}(s_t) - \hat{c}_{dt}, 0)^2 = \mu_t$$

$$V'(1 - l_{ct}) + \zeta \min(l_{max}(s_t) - l_{ct}, 0)^2 = \mu_t(1 - \tau_{lt})\hat{w}_t$$

$$\zeta \min(\hat{i}_{pt}, 0)^2 + \lambda_t = \mu_t$$

$$(1 + g_p)(1 + g_z)\lambda_t = \tilde{\beta} E_t \left[\lambda_{t+1}(1 - \delta) + \mu_{t+1}\left\{(1 - \tau_{kt+1})r_{t+1} + \tau_k \delta\right\}\right].$$

Equilibrium equations for computation can be summarized as follows:

$$\mu_t = 1/\hat{c}_{pt} - \zeta \min(c_{max}(s_t) - \hat{c}_{pt}, 0)^2$$

$$\mu_t - \zeta \min(\hat{i}_{pt}, 0)^2 = \hat{\beta} E_t \left[\mu_{t+1}\left\{(1 - \tau_{kt+1})(r_{t+1} - \delta) + 1\right\} - (1 - \delta)\zeta \min(\hat{i}_{pt+1}, 0)^2\right]$$

$$V'(1 - l_{ct}) + \zeta \min(l_{max}(s_t) - l_{ct})^2 = \mu_t(1 - \tau_{lt})\hat{w}_t$$

where $\hat{\beta} = \beta/(1+g_z)$. Notice that $\hat{c}_{ct} = \hat{c}_{dt}$ in equilibrium. We let $\hat{c}_{pt}$ represent both later.

2.2. Firms

The firm's problem in t is:

$$\max_{\{K_t, L_{pt}\}} \; F(K_t, Z_t L_{pt}) - r_t K_t - w_t L_{pt}$$

where L_p is the total labor input (and equal to the fraction of civilians in the population $(1 - a)$ times the ratio of total civilian hours to total civilians (l_c) times the population ($(1 + g_p)^t$).

In equilibrium, the rental price and the wage rate are given by:

$$r_t = F_1(K_t, Z_t L_{pt})$$

$$w_t = F_2(K_t, Z_t L_{pt}) Z_t.$$

Suppose $F(K, L) = K^\theta L^{1-\theta}$. Then,

$$r_t = \theta K_t^{\theta-1}(Z_t L_{pt})^{1-\theta} = \theta \hat{k}_t^{\theta-1}(z_t l_{ct}(1 - a_t))^{1-\theta}$$

$$w_t = (1 - \theta) K_t^\theta Z_t^{1-\theta} L_{pt}^{-\theta} = (1 + g_z)^t (1 - \theta)\hat{k}_t^\theta (z_t l_{ct}(1 - a_t))^{-\theta}.$$

2.3. The Government

The government's period t budget constraint is given by:

$$C_{gt} + I_{gt} + (1+g_p)^t T_t = \tau_{kt}(r_t - \delta)K_{pt} + \tau_{lt} w_t L_{pt} + r_t K_{gt}$$

where we are assuming that wage payments to soldiers are included in transfers to draftees. Spending is therefore:

$$G_t = C_{gt} + I_{gt} + (1+g_p)^t a_t w_t \bar{l}$$

which when detrended for technological growth is

$$\hat{g}_t = \hat{c}_{gt} + \hat{i}_{gt} + a_t \hat{w}_t \bar{l}.$$

Government capital (which is privately operated) is assumed to have the same rate of depreciation as private capital:

$$K_{gt+1} = (1-\delta)K_{gt} + I_{gt}.$$

When normalized this equation becomes

$$(1+g_p)(1+g_z)\hat{k}_{gt+1} = (1-\delta)\hat{k}_{gt} + \hat{i}_{gt}.$$

2.4. Aggregates

Total private consumption, total private hours (which includes nonmilitary government hours), total private investment, and total private capital are given by:

$$\begin{aligned} C_{pt} &= (1+g_p)^t[(1-a_t)c_{ct} + a_t c_{dt}] = (1+g_p)^t c_{ct} \\ L_{pt} &= (1+g_p)^t(1-a_t)l_{ct} \\ I_{pt} &= (1+g_p)^t i_{pt} \\ K_{pt} &= (1+g_p)^t k_{pt}. \end{aligned}$$

The resource constraint for this economy is:

$$C_{pt} + I_{pt} + C_{gt} + I_{gt} = F(K_t, Z_t L_{pt})$$

or

$$\hat{c}_{pt} + \hat{i}_{pt} + \hat{c}_{gt} + \hat{i}_{gt} = F(\hat{k}_t, z_t l_{pt}).$$

2.5. Exogenous stochastic processes

The exogenous stochastic processes in this economy are $\{a, \hat{c}_g, \hat{i}_g, \tau_k, \tau_l, z\}$. Let s index the state, where s is determined by a nth-order Markov chain. We assume that at time t if the state is s, then $a_t = a(s)$, $\hat{c}_{gt} = \hat{c}_g(s)$, $\hat{i}_{gt} = \hat{i}_g(s)$, $\tau_{kt} = \tau_k(s)$, $\tau_{lt} = \tau_l(s)$, and $z_t = z(s)$. The process for s is intended to capture different stages of war and/or peace and different levels of technology.

2.6. Steady State Equations

We will use the following functional forms for disutility and production:

$$V(1-l) = \psi[(1-l)^{\xi} - 1]/\xi \tag{2.1}$$

$$F(K, L) = K^{\theta} L^{1-\theta}. \tag{2.2}$$

In this case, the steady state solves

$$\begin{aligned}
r &= [(1+g_z)/\beta - 1]/(1-\tau_k) + \delta \\
\hat{k}_g &= \hat{i}_g/[(1+g_p)(1+g_z) - 1 + \delta] \\
\hat{k}_p &= \hat{i}_p/[(1+g_p)(1+g_z) - 1 + \delta] \\
\hat{y} &= (\hat{k}_p + \hat{k}_g)^{\theta}(z(1-a)l_c)^{1-\theta} \\
\theta &= r(\hat{k}_p + \hat{k}_g)/\hat{y} \\
\hat{c}_p &= \hat{y} - \hat{c}_g - \hat{i}_p - \hat{i}_g \\
\psi &= (1-\tau_l)(1-\theta)(1-l_c)^{1-\xi}\hat{y}/[\hat{c}_p(1-a)l_c].
\end{aligned}$$

which is 7 equations in 7 unknowns (r, $\hat{k}_g$, $\hat{k}_p$, $\hat{y}$, $\hat{c}_p$, θ, ψ) with $\hat{c}_g$, $\hat{i}_g$, a, τ_k, τ_l, z, g_p, g_z, δ, β, ξ, i_p, and l_c given.

2.7. Algorithm for Computing the Consumption Function

When writing the codes, we use x for total capital, i as the index for today's exogenous state, j as the index for tomorrow's exogenous state, I as total number of exogenous states, and π_{ij} as probability of transiting from i to j. Here, we describe out we compute the decision function $\hat{c}_p(x,i)$, which is represented as a weighted sum of known basis functions,

$$\hat{c}_p = \sum_{k=1}^{nnodes} \alpha_k^i N_k(x)$$

where the N_k is a "tent" function that takes on nonzero values in 2 elements on the grid over x surrounding node k, that is

$$N_k(x) = \begin{cases} \frac{x-x_{a-1}}{x_a-x_{a-1}} & x_{a-1} \le x \le x_a \\ \frac{x_{a+1}-x}{x_{a+1}-x_a} & x_a \le x \le x_{a+1} \\ 0 & \text{elsewhere.} \end{cases}$$

We apply a finite element method. This means that we find unknown coefficients α_k, $k = 1, \dots nnodes$ that satisfy the following equations: $\int R(x,i;\alpha)N_k(x)\,dx = 0$ for all i and k where

$$R(x,i;\alpha) = \mu - \zeta \min(\hat{i}_p, 0)^2 + \hat{\beta}(1-\delta)\zeta \sum_j \pi_{i,j} \min(\hat{i}'_p, 0)^2$$
$$- \hat{\beta} \sum_j \pi_{i,j}\, \mu' \left\{(1-\tau_k(j))[F_1(x', z(j)(1-a(j))l'_c) - \delta] + 1\right\}$$

and $\alpha = [\alpha_1^1, \dots, \alpha_{nnodes}^1, \dots \alpha_{nnodes}^I]'$. The multipliers μ and μ' are

$$\mu = \left(\sum_k \alpha_k^i N_k(x)\right)^{-1} - \zeta \min(c_{max}(i) - \sum_k \alpha_k^i N_k(x), 0)^2$$

$$\mu' = \left(\sum_k \alpha_k^j N_k(x')\right)^{-1} - \zeta \min(c_{max}(j) - \sum_k \alpha_k^j N_k(x'), 0)^2$$

since $\hat{c}_p = \sum_k \alpha_k^i N_k(x)$. The private investments i_p and i'_p satisfy resource constraints:

$$\hat{i}_p = F(x, z(i)(1-a(i))l_c) - \sum_k \alpha_k^i N_k(x) - \hat{c}_g(i) - \hat{i}_g(i)$$

$$\hat{i}'_p = F(x', z(j)(1-a(j))l'_c) - \sum_k \alpha_k^j N_k(x') - \hat{c}_g(j) - \hat{i}_g(j).$$

The next period capital stock is given by:

$$x' = ((1-\delta)x + (\hat{i}_p + \hat{i}_g(i)))/[(1+g_p)(1+g_z)].$$

The labor inputs l_c and l'_c solve:

$$V'(1-l_c)+\zeta\min(l_{max}(i)-l_c,0)^2=\mu(1-\tau_l(i))F_2(x,z(i)(1-a(i))l_c)z(i)$$

$$V'(1-l'_c)+\zeta\min(l_{max}(j)-l'_c,0)^2=\mu'(1-\tau_l(j))F_2(x',z(j)(1-a(j))l'_c)z(j).$$

Derivatives of the residual equation are as follows:

$$\begin{aligned}\frac{\partial R(x,i;\alpha)}{\partial\alpha_k^i}=&\left(\frac{\partial\mu}{\partial\hat{c}_p}-2\zeta\min(\hat{i}_p,0)\frac{d\hat{i}_p}{d\hat{c}_p}\right)\frac{d\hat{c}_p}{\alpha_k^i}\\&+\hat{\beta}\sum_j\pi_{i,j}\ \{(1-\tau_k(j))[F_1(x',z(j)(1-a(j))l'_c)-\delta]+1\}\frac{d\mu'}{d\hat{c}'_p}\frac{d\hat{c}'_p}{d\alpha_k^i}\\&-\hat{\beta}\sum_j\pi_{i,j}\,\mu'(1-\tau_k(j))\\&\quad\left(F_{11}(x',z(j)(1-a(j))l'_c)\frac{dx'}{d\alpha_k^i}\right.\\&\quad\left.+F_{12}(x',z(j)(1-a(j))l'_c)z(j)(1-a(j))\frac{dl'_c}{d\alpha_k^i}\right)\\&+\hat{\beta}(1-\delta)\zeta\min(\hat{i}_p,0)\frac{d\hat{i}'_p}{d\alpha_k^i}\end{aligned}$$

and for $j\neq i$

$$\begin{aligned}\frac{\partial R(x,i;\alpha)}{\partial\alpha_k^j}=&\hat{\beta}\pi_{i,j}\ \{(1-\tau_k(j))[F_1(x',z(j)(1-a(j))l'_c)-\delta]+1\}\,d\mu'd\hat{c}'_p\frac{d\hat{c}'_p}{d\alpha_k^j}\\&-\hat{\beta}\pi_{i,j}\,\mu'(1-\tau_k(j))F_{12}(x',z(j)(1-a(j))l'_c)z(j)(1-a(j))\frac{dl'_c}{d\alpha_k^j}\\&+\hat{\beta}(1-\delta)\zeta\min(\hat{i}_p,0)\frac{d\hat{i}'_p}{d\alpha_k^j}.\end{aligned}$$

The partial derivative of μ is

$$\frac{d\mu}{d\hat{c}_p}=-\frac{1}{\hat{c}_p^2}+2\zeta\min(c_{max}(i)-\hat{c}_p,0).$$

For the partial derivatives of l_c and l'_c, totally differentiate the static first order condition for the consumer to get:

$$\begin{aligned}&-[V''(1-l_c)+2\zeta\min(l_{max}(i)-l_c,0)]\,dl_c\\&\qquad=d\mu(1-\tau_l)F_2z+\mu(1-\tau_l)\,[F_{12}dx+F_{22}z(1-a)dl_c]\,z(1-a)\end{aligned}$$

where the arguments of F are $(x, z(1-a)l_c)$. For the current period l_c, $dx = 0$ since x is a given state variable. For l'_c, dx' is given by:

$$\begin{aligned} dx' &= d\hat{i}_p/[(1+g_p)(1+g_z)] \\ &= (F_2(x, z(1-a)l_c)z(1-a)dl_c - d\hat{c}_p)/[(1+g_p)(1+g_z)]. \end{aligned}$$

Therefore, we have

$$\frac{dl_c}{d\hat{c}_p} = -\frac{(1-\tau_l)F_2 z}{V'' + 2\zeta \min(l_{max}(i) - l_c, 0) + \mu(1-\tau_l)F_{22}z^2(1-a)^2}\frac{d\mu}{d\hat{c}_p}$$

where the argument of V is $1 - l_c$ and the arguments of F are $(x, z(i)(1-a(i))l_c)$, and in the next period we have

$$dl'_c = \frac{V'\, d\hat{c}'_p - (1-\tau_l(j))F_{12}z(j)(1-a(j))\, dx'}{V''\hat{c}'_p + (1-\tau_l(j)F_{22}z(j)^2(1-a(j))^2}$$

where the argument of V is $1 - l'_c$ and the arguments of F are $(x', z(j)(1-a(j))l'_c)$. Finally we need

$$\frac{d\hat{c}'_p}{d\alpha^i_k} = \left(\sum_l \alpha^j_l \frac{\partial N_k(x')}{\partial x'}\right)\frac{dx'}{d\alpha^i_k}$$

$$\frac{d\hat{c}'_p}{d\alpha^j_k} = N_k(x')$$

$$\begin{aligned} \frac{dx'}{d\alpha^i_k} &= \frac{1}{(1+g_p)(1+g_z)}\left(F_2(x, z(1-a)l_c)z(1-a)\frac{dl_c}{d\alpha^i_k} - \frac{d\hat{c}_p}{d\alpha^i_k}\right) \\ &= \frac{1}{(1+g_p)(1+g_z)}\left(F_2(x, z(1-a)l_c)z(1-a)\frac{dl_c}{d\hat{c}_p} - 1\right)\frac{d\hat{c}_p}{d\alpha^i_k} \\ &= \frac{1}{(1+g_p)(1+g_z)}\left(F_2(x, z(1-a)l_c)z(1-a)\frac{dl_c}{d\hat{c}_p} - 1\right)N_k(x) \end{aligned}$$

$$d\hat{i}'_p = F_1(x', z(j)(1-a(j))l'_c)\, dx' + F_2(x', z(j)(1-a(j))l'_c)z(j)(1-a(j))\, dl'_c - d\hat{c}'_p.$$

3. A Version of the Model with Capacity Utilization

In this section, we consider the extension of the benchmark model considered by Braun and McGrattan (1993) in their Appendix B.

3.1. Household Problem

The maximization problem of the stand-in household is now:

$$\max_{\{c_{1t},c_{0t},c_{dt},i_{pt},n_t,h_t\}} E\sum_{t=0}^{\infty}\beta^t\Big((1-a_t)\{n_t[\log c_{1t}+V(1-h_t)]$$

$$+(1-n_t)[\log c_{0t}+V(1)]-p(n_t)\}$$

$$+a_t[\log c_{dt}+V(1-\bar{h})]\Big)(1+g_p)^t$$

$$\text{subject to}\quad (1-a_t)\{n_tc_{1t}+(1-n_t)c_{0t}\}+a_tc_{dt}+i_{pt}$$

$$=(1-\tau_{kt})r_tk_{pt}+(1-\tau_{lt})(1-a_t)w_t(h_t)n_t$$

$$+\tau_{kt}\delta k_{pt}+T_t$$

$$k_{pt+1}=[(1-\delta)k_{pt}+i_{pt}]/(1+g_p)$$

$$i_{pt}\geq 0\quad\text{in all states}$$

$$c_{1t}\leq c_{max}\quad\text{in some states}$$

$$c_{0t}\leq c_{max}\quad\text{in some states}$$

$$c_{dt}\leq c_{max}\quad\text{in some states}$$

$$n_t\leq n_{max}\quad\text{in some states}$$

with processes for a_t, r_{pt}, $w_t(h_t)$, τ_{kt}, τ_{lt}, and T_t given. Quantities are in per-capita terms. The household chooses consumption of those employed c_{1t}, consumption of those not employed c_{0t}, the consumption of the draftees c_{dt}, the number (or fraction) of those employed n_t, the length of the workweek h_t, and investment i_{pt}.[1] Each term in the budget constraint is in per-capita terms. For example, the first term $(1-a_t)n_tc_{1t}$ is

$$\frac{\text{number of civilians}}{\text{total population}}\times\frac{\text{number of civilian workers}}{\text{number of civilians}}\times\frac{\text{consumption of civilian workers}}{\text{number of civilian workers}}.$$

The function $p(n)$ represents costs—in utility terms—of increasing employment or changing its level. If $p(n)=0$ then the equilibrium workweek is constant. (See Braun and McGrattan (1993).) The disutility of employment captured by $p(n_t)$ ensures that both the intensive margin and the extensive margin are used. The disutility of employment term

[1] Note that we could assume that c_{dt} is determined by the government as Braun and McGrattan (1993) do. Since we work with log preferences, whether it is given or not will not affect our results.

was motivated by Braun and McGrattan (1993) as follows. They assume that individual preferences are given by

$$E\sum_{t=0}^{\infty}\beta^t[U(c_t,1-h_t)-\eta\chi_{\{h_t>0\}}]$$

where η measures the disutility of entering the work force and χ is an indicator function. If the utility costs of entering the workforce vary, then η will have a nondegenerate distribution. If civilians are aligned with points on the interval [0,1-a), then we can construct a cost function. For example, suppose that individuals are aligned so that costs are linear and increasing. Then, in the aggregate the costs of increasing employment are given by

$$-(1-a)\int_0^n(\zeta_0+2\zeta_1x)dx=-(1-a)(\zeta_0n+\zeta_1n^2)$$

with $\zeta_1>0$.

The Lagrangian for the optimization problem in this case is:

$$\begin{aligned}\mathcal{L}=E\sum_t\tilde{\beta}^t\Big\{&(1-a_t)\{n_t[\log(\hat{c}_{1t})+V(1-h_t)]+(1-n_t)[\log\hat{c}_{0t}+V(1)]\}\\&+a_t\log(\hat{c}_{dt})-(1-a_t)p(n_t)\\&+\frac{\zeta}{3}\big[\min(\hat{i}_{pt},0)^3+(1-a_t)\min(n_{max}(s_t)-n_t,0)^3\\&\quad+(1-a_t)n_t\min(c_{max}(s_t)-\hat{c}_{1t},0)^3\\&\quad+(1-a_t)(1-n_t)\min(c_{max}(s_t)-\hat{c}_{0t},0)^3\\&\quad+a_t\min(c_{max}(s_t)-\hat{c}_{dt},0)^3\big]\\&+\mu_t\Big\{(1-\tau_{kt})r_t\hat{k}_{pt}+(1-\tau_{lt})(1-a_t)\hat{w}_t(h_t)n_t+\tau_{kt}\delta\hat{k}_{pt}+\hat{T}_t\\&\quad-(1-a_t)\{n_t\hat{c}_{1t}-(1-n_t)c_{0t}\}-a_t\hat{c}_{dt}-\hat{i}_{pt}\Big\}\\&+\lambda_t\Big\{(1-\delta)\hat{k}_{pt}+\hat{i}_{pt}-(1+g_p)(1+g_z)\hat{k}_{t+1}\Big\}\Big\}\end{aligned}$$

where $\tilde{\beta}=(1+g_p)\beta$. As before, variables that grow over time are detrended and denoted with a hat (e.g., $\hat{c}_{ct}=c_{ct}/(1+g_z)^t$).

The first-order conditions are thus:

$$1/\hat{c}_{1t} - \zeta \min(c_{max}(s_t) - \hat{c}_{1t}, 0)^2 = \mu_t$$

$$1/\hat{c}_{0t} - \zeta \min(c_{max}(s_t) - \hat{c}_{0t}, 0)^2 = \mu_t$$

$$1/\hat{c}_{dt} - \zeta \min(c_{max}(s_t) - \hat{c}_{dt}, 0)^2 = \mu_t$$

$$V'(1-h_t) = \mu_t(1-\tau_{lt})\hat{w}_t'(h_t)$$

$$\begin{aligned} 0 = \log \hat{c}_{1t} + V(1-h_t) - \log \hat{c}_{0t} - V(1) - p'(n_t) - \zeta \min(n_{max}(s_t) - n_t, 0)^2 \\ + \zeta \min(c_{max}(s_t) - \hat{c}_{1t}, 0)^2 - \zeta \min(c_{max}(s_t) - \hat{c}_{0t}, 0)^2 \\ + \mu_t[(1-\tau_{lt})\hat{w}_t(h_t) - \hat{c}_{1t} + \hat{c}_{0t}] \end{aligned}$$

$$\zeta \min(\hat{i}_{pt}, 0)^2 + \lambda_t = \mu_t$$

$$(1+g_p)(1+g_z)\lambda_t = \tilde{\beta} E_t \left[\lambda_{t+1}(1-\delta) + \mu_{t+1}\{(1-\tau_{kt+1})r_{t+1} + \tau_k \delta\}\right].$$

Equilibrium equations for computation can be summarized as follows:

$$\mu_t = 1/\hat{c}_{pt} - \zeta \min(c_{max}(s_t) - \hat{c}_{pt}, 0)^2$$

$$\mu_t - \zeta \min(\hat{i}_{pt}, 0)^2 = \hat{\beta} E_t \left[\mu_{t+1}\{(1-\tau_{kt+1})(r_{t+1} - \delta) + 1\} - (1-\delta)\zeta \min(\hat{i}_{pt+1}, 0)^2\right]$$

$$V'(1-h_t) = \mu_t(1-\tau_{lt})\hat{w}_t'(h_t) \tag{3.1}$$

$$V(1-h_t) - V(1) + V'(1-h_t)\hat{w}_t(h_t)/\hat{w}_t'(h_t) = p'(n_t) + \zeta \min(n_{max}(s_t) - n_t, 0)^2 \tag{3.2}$$

where $\hat{\beta} = \beta/(1+g_z)$ and $\hat{c}_{pt} = \hat{c}_{1t} = \hat{c}_{0t} = \hat{c}_{dt}$.

3.2. Firms

Let's turn next to the production technologies. There are different technologies each defined by length of the workweek, h,

$$Y_t = Z_t^{1-\theta} K_t^{\theta} N_t^{1-\theta} h_t^{\phi} = z_t^{1-\theta} \hat{k}_t^{\theta} (n_t(1-a_t))^{1-\theta} h_t^{\phi}. \tag{3.3}$$

A firm of type h solves the following maximization problem

$$\max_{K_t, N_t} Y_t - r_t K_t - w_t(h_t) N_t$$

subject to (3.3). The rental rate and wage rate is therefore given by

$$r_t = \theta Z_t^{1-\theta} K_t^{\theta-1} N_t^{1-\theta} h_t^{\phi}$$
$$w_t(h_t) = (1-\theta) Z_t^{1-\theta} K_t^{\theta} N_t^{-\theta} h_t^{\phi}.$$

The total capital stock K_t and workforce N_t is

$$K_t = (1+g_p)^t (1+g_z)^t \hat{k}_t$$
$$N_t = (1+g_p)^t n_t (1-a_t).$$

Thus, when we normalize rental rates and wage rates, we have

$$\begin{aligned} r_t &= \theta \hat{k}_t^{\theta-1} (z_t n_t (1-a_t))^{1-\theta} h_t^{\phi} \\ \hat{w}_t(h_t) &= (1-\theta) \hat{k}_t^{\theta} z_t^{1-\theta} (n_t(1-a_t))^{-\theta} h_t^{\phi} = w_t(h_t)(1+g_z)^t \\ \hat{w}_t'(h_t) &= \phi \hat{k}_t^{\theta} z_t^{1-\theta} (n_t(1-a_t))^{-\theta} h_t^{\phi-1} \\ &= \phi \hat{w}_t(h_t) / [(1-\theta) h_t] \end{aligned}$$

and $\hat{w}_t(h_t)/\hat{w}_t'(h_t) = (1-\theta)h_t/\phi$. From the perspective of the household, $\hat{w}_t$ is a function of both h_t which they are choosing and r_t which they are not. To see this, note that

$$\begin{aligned} \hat{w}_t(h_t; r_t) &= (1-\theta) \left(\frac{\hat{k}_t}{z_t n_t (1-a_t)} \right)^{\theta} z_t h_t^{\phi} \\ &= (1-\theta) \left(\frac{r_t}{\theta h_t^{\phi}} \right)^{\frac{\theta}{\theta-1}} z_t h_t^{\phi} \\ &= z_t (1-\theta) \theta^{\frac{\theta}{1-\theta}} r_t^{\frac{\theta}{\theta-1}} h_t^{\frac{\phi}{1-\theta}-1}. \end{aligned}$$

3.3. The Government

The government's period t budget constraint is given by:

$$C_{gt} + I_{gt} + (1+g_p)^t T_t = \tau_{kt}(r_t - \delta) K_{pt} + \tau_{lt} w_t(h_t) N_t + r_t K_{gt}$$

where we are assuming that wage payments to soldiers are included in transfers to draftees. Spending is therefore:

$$G_t = C_{gt} + I_{gt} + \text{military compensation}.$$

3.4. Aggregates

Total private consumption, total private labor input (which includes nonmilitary government), total private investment, and total private capital are given by:

$$
\begin{aligned}
C_{pt} &= (1+g_p)^t[(1-a_t)(n_t c_{1t} + (1-n_t)c_{0t}) + a_t c_{dt}] \\
I_{pt} &= (1+g_p)^t i_{pt} \\
K_{pt} &= (1+g_p)^t k_{pt}.
\end{aligned}
$$

The resource constraint for this economy is:

$$C_{pt} + I_{pt} + C_{gt} + I_{gt} = Z_t^{1-\theta} K_t^{\theta} N_t^{1-\theta} h_t^{\phi}$$

or

$$\hat{c}_{pt} + \hat{i}_{pt} + \hat{c}_{gt} + \hat{i}_{gt} = \hat{k}_t^{\theta}(z_t n_t(1-a_t))^{1-\theta} h_t^{\phi}.$$

3.5. Steady State Equations

We will use the same functional form for V and F as before (see (2.1) and (2.2)), and we will try various functional forms for $p(n)$. At the start, we assume

$$p(n) = \eta[n^{\rho} - 1]/\rho.$$

The steady state for this problem is

$$
\begin{aligned}
r &= [(1+g_z)/\beta - 1]/(1-\tau_k) + \delta \\
\hat{k}_g &= \hat{i}_g/[(1+g_p)(1+g_z) - 1 + \delta] \\
\hat{k}_p &= \hat{i}_p/[(1+g_p)(1+g_z) - 1 + \delta] \\
\hat{y} &= (\hat{k}_p + \hat{k}_g)^{\theta}(z(1-a)n)^{1-\theta} h^{\phi} \\
\hat{c}_p &= \hat{y} - \hat{c}_g - \hat{i}_p - \hat{i}_g \\
\theta &= r(\hat{k}_p + \hat{k}_g)/\hat{y} \\
\hat{w} &= (1-\theta)\hat{y}/(n(1-a)) \\
\psi &= (1-\tau_l)\phi(1-h)^{1-\xi}\hat{w}/[\hat{c}_p h] \\
\eta &= (\psi((1-h)^{\xi} - 1)/\xi + (1-\tau_l)\hat{w}/\hat{c}_p)/n^{\rho-1}
\end{aligned}
$$

which is 9 equations in 9 unknowns (r, $\hat{k}_g$, $\hat{k}_p$, $\hat{y}$, $\hat{c}_p$, θ, $\hat{w}$, ψ, η) with $\hat{c}_g$, $\hat{i}_g$, a, τ_k, τ_l, z, g_p, g_z, δ, β, ϕ, ξ, ρ, i_p, h, and n given.

3.6. Algorithm for Computing the Consumption Function

As before, we are computing $c_p(x, i)$, which is represented as a weighted sum of known basis functions,

$$\hat{c}_p = \sum_{k=1}^{nnodes} \alpha_k^i N_k(x)$$

with known basis functions. The residual in this case is given by:

$$R(x, i; \alpha) = \mu - \zeta \min(\hat{i}_p, 0)^2 + \hat{\beta}(1-\delta)\zeta \sum_j \pi_{i,j} \min(\hat{i}'_p, 0)^2$$

$$- \hat{\beta} \sum_j \pi_{i,j}\, \mu' \left\{(1-\tau_k(j))[\theta(x')^{\theta-1}(z(j)(1-a(j))n')^{1-\theta}(h')^{\phi} - \delta] + 1\right\}$$

and $\alpha = [\alpha_1^1, \ldots, \alpha_{nnodes}^1, \ldots \alpha_{nnodes}^I]'$. The multipliers μ and μ' are

$$\mu = \left(\sum_k \alpha_k^i N_k(x)\right)^{-1} - \zeta \min(c_{max}(i) - \sum_k \alpha_k^i N_k(x), 0)^2$$

$$\mu' = \left(\sum_k \alpha_k^j N_k(x')\right)^{-1} - \zeta \min(c_{max}(j) - \sum_k \alpha_k^j N_k(x'), 0)^2$$

since $\hat{c}_p = \sum_k \alpha_k^i N_k(x)$. The private investments i_p and i'_p satisfy resource constraints:

$$\hat{i}_p = x^{\theta}(z(i)(1-a(i))n)^{1-\theta}h^{\phi} - \sum_k \alpha_k^i N_k(x) - \hat{c}_g(i) - \hat{i}_g(i)$$

$$\hat{i}'_p = (x')^{\theta}(z(j)(1-a(j))n')^{1-\theta}(h')^{\phi} - \sum_k \alpha_k^j N_k(x') - \hat{c}_g(j) - \hat{i}_g(j).$$

The next period capital stock is given by:

$$x' = ((1-\delta)x + (\hat{i}_p + \hat{i}_g(i)))/[(1+g_p)(1+g_z)].$$

If we have μ, x, and the exogenous variables, then we can solve the following two equations for the two unknowns, hours of work h and the employment level n:

$$V'(1-h) = \mu(1-\tau_l(i))\phi(1-\theta)(x/n)^{\theta} z(i)^{1-\theta}(1-a(i))^{-\theta} h^{\phi-1}$$

$$0 = V(1-h) + V'(1-h)h/\phi - p'(n) - \zeta \min(n_{max}(i) - n, 0)^2.$$

Derivatives of the residual equation are given as follows:

$$
\begin{aligned}
\frac{\partial R(x,i;\alpha)}{\partial \alpha_k^i} = & \left(\frac{\partial \mu}{\partial \hat{c}_p} - 2\zeta \min(\hat{i}_p, 0)\frac{d\hat{i}_p}{d\hat{c}_p}\right)\frac{d\hat{c}_p}{\alpha_k^i} \\
& + \hat{\beta}\sum_j \pi_{i,j} \ \left\{(1-\tau_k(j))[\theta(x')^{\theta-1}(z(j)(1-a(j))n')^{1-\theta}(h')^{\phi} - \delta] + 1\right\} \frac{d\mu'}{d\hat{c}'_p}\frac{d\hat{c}'_p}{d\alpha_k^i} \\
& - \hat{\beta}\sum_j \pi_{i,j}\, \mu'(1-\tau_k(j))\theta \\
& \quad \Bigg((\theta-1)(x')^{\theta-2}(z(j)(1-a(j))n')^{1-\theta}(h')^{\phi}\frac{dx'}{d\alpha_k^i} \\
& \quad + (1-\theta)(x')^{\theta-1}(z(j)(1-a(j)))^{1-\theta}(n')^{-\theta}(h')^{\phi}\frac{dn'}{d\alpha_k^i}\Bigg) \\
& \quad + \phi(x')^{\theta-1}(z(j)(1-a(j))n')^{1-\theta}(h')^{\phi-1}\frac{dh'}{d\alpha_k^i}\Bigg) \\
& + \hat{\beta}(1-\delta)\zeta\min(\hat{i}_p, 0)\frac{d\hat{i}'_p}{d\alpha_k^i}
\end{aligned}
$$

and for $j \neq i$

$$
\begin{aligned}
\frac{\partial R(x,i;\alpha)}{\partial \alpha_k^j} = & \hat{\beta}\pi_{i,j} \ \left\{(1-\tau_k(j))[\theta(x')^{\theta-1}(z(j)(1-a(j))n')^{1-\theta}(h')^{\phi} - \delta] + 1\right\} d\mu' d\hat{c}'_p \frac{d\hat{c}'_p}{d\alpha_k^j} \\
& - \hat{\beta}\pi_{i,j}\, \mu'(1-\tau_k(j))\theta(1-\theta)(x')^{\theta-1}(z(j)(1-a(j)))^{1-\theta}(n')^{-\theta}(h')^{\phi}\frac{dn'}{d\alpha_k^j} \\
& - \hat{\beta}\pi_{i,j}\, \mu'(1-\tau_k(j))\theta\phi(x')^{\theta-1}(z(j)(1-a(j))n')^{1-\theta}(h')^{\phi-1}\frac{dh'}{d\alpha_k^j} \\
& + \hat{\beta}(1-\delta)\zeta\min(\hat{i}_p, 0)\frac{d\hat{i}'_p}{d\alpha_k^j}.
\end{aligned}
$$

The partial derivative of μ is

$$\frac{d\mu}{d\hat{c}_p} = -\frac{1}{\hat{c}_p^2} + 2\zeta\min(c_{max}(i) - \hat{c}_p, 0).$$

For the partial derivatives of n, n', h, and h', totally differentiate the static consumer's first order conditions (3.1) and (3.2) to get:

$$
\begin{aligned}
&-V''(1-h)dh = V'(1-h)(d\mu/\mu + \theta dx/x - \theta dn/n + (\phi-1)dh/h) \\
&-V'(1-h)dh + (1-\theta)V'(1-h)/\phi dh - (1-theta)V''(1-h)h/\phi dh \\
&\qquad\qquad = p''(n)dn - 2\zeta \min(n_{max}(i) - n, 0)dn
\end{aligned}
$$

where $\kappa = (1-\tau_l)\phi z^{1-\theta}(1-a)^{-\theta}$. For the current period, $dx = 0$ since x is a given state variable. For the next period, dx' is given by:

$$
\begin{aligned}
dx' &= d\hat{i}_p/[(1+g_p)(1+g_z)] \\
&= (x^\theta(z(1-a)n)^{1-\theta}h^\phi)((1-\theta)dn/n + \phi dh/h) - d\hat{c}_p)/[(1+g_p)(1+g_z)].
\end{aligned}
$$

Finally we need

$$
\frac{d\hat{c}'_p}{d\alpha^i_k} = \left(\sum_l \alpha^j_l \frac{\partial N_k(x')}{\partial x'}\right)\frac{dx'}{d\alpha^i_k}
$$

$$
\frac{d\hat{c}'_p}{d\alpha^j_k} = N_k(x')
$$

$$
\frac{dx'}{d\alpha^i_k} = \frac{1}{(1+g_p)(1+g_z)}\left((x^\theta(z(1-a)n)^{1-\theta}h^\phi\left[\frac{(1-\theta)}{n}\frac{dn}{d\alpha^i_k} + \frac{\phi}{h}\frac{dh}{d\alpha^i_k}\right] - \frac{d\hat{c}_p}{d\alpha^i_k}\right)
$$

$$
d\hat{i}'_p = (x')^\theta(z(j)(1-a(j))n')^{1-\theta}(h')^\phi\ [\theta dx'/x' + (1-\theta)dn'/n' + \phi dh'/h'] - d\hat{c}'_p.
$$

4. Results for the Variable Capacity Utilization Model

Figures A1–A4 summarize the main results for the model with variable capacity utilization.

To generate these pictures, we use the same parameters as in the benchmark model (Table 1 of the paper) with the exception of ψ and ξ; here, we use $\psi = .58$, $\xi = -2$. A lower labor elasticity relative to the benchmark case is chosen in order to generate a quantitatively important role for varying the workweek. Having nontrivial costs $p(n)$ also helps in this dimension so we chose $\eta = 1.3$ and $\rho = 2$. Values of ψ and η imply that the levels for the fraction employed and the hours per worker are consistent with U.S. data. The parameters ξ and ρ govern elasticities of hours and employment. We varied these to see how they affected our results. For technology, we need to also choose ϕ. Here, we set it equal to 1 so that there are no diminishing returns to the workweek.

In the model with variable capacity utilization, we also have a different sequence for TFP than in the benchmark since the production technology is slightly different. However, the procedure for deriving the sequence is the same; we use data on output and capital and labor inputs to determine TFP residually. In the variable capacity utilization case, TFP rises by roughly 10 percent between 1941 and 1944.

The main finding from Figures A1–A3 is that the predictions for GNP, consumption, investment, total hours, nonmilitary hours, the return to capital, and nonmilitary labor productivity are very similar to the benchmark model. This is seen by comparing Figures 5–7 in the paper with Figures A1–A3 in this appendix.

In Figure A4, we see the decomposition of the labor input into hours per worker and the fraction employed. With $p(n) = 0$, the hours per worker is constant. As we increase ρ, and thus the costs to varying employment, we can get a larger response in the hours per worker. However, we were unable to generate as large of a response as is seen in the data (even if we rais ρ from 2 to 10 and adjust the utility parameters to get the levels of n and h consistent with the data). A comparison of the two plots in Figure A4 show that the model is matching up well on the total labor input because we underpredict the change in hours per worker and overpredict the change in employment.

5. Sensitivity Analysis for the Benchmark Model

In this section we describe several computational experiments that we conduct to check the sensitivity of our results. We describe how the results change when we (a) start our simulations in 1939; (b) use an alternative estimate for the tax on labor; and (c) vary the postwar tax rates on labor and capital. Also, given the emphasis in the literature on the household's intratemporal condition, we study the fit of this condition for three time periods. Finally, we discuss model predictions for several alternative specifications of expectations during the war.

5.1. Simulations Starting in 1939

For the stochastic simulations starting in 1941, we took draws from a six-state urn. The first state was associated with the period before war starts. For the stochastic simulations starting in 1939, we used the same transitional probabilities for 1939, 1940, and 1941.

In Figures A5–A7, we display the analogues of the benchmark results (shown Figure 5–7 in the main text). In both cases, predicted is initially high because labor input is high. This is to be expected for two reasons. First, we do not build in counterfactual expectations of low taxes following war. Thus, in the model economy, households view the prewar states as a good time to work. This can be seen in Figure A6. Second, we do not build in the policies in place during the Great Depression that depressed labor inputs.

Overall, starting in 1941 versus 1939 does not have a big effect on the model predictions.

5.2. An Alternative Labor Tax Rate

In the benchmark simulations, we used labor and capital tax rates of Joines (1981). Mulligan (1998) uses the Barro and Sahasakul (1986) tax rate—that mixes taxes on labor and capital—for his tax rate on labor. To see how much of a difference this makes, we rerun our benchmark simulations replacing the Joines tax rate on labor with that of Barro and Sahasakul. These rates are displayed in Figure A8. We also adjust ψ as in the benchmark model in order to match the level of the U.S. per capita hours series for 1946–1960. Specifically, we set $\psi = 2.23$.

The simulation results with the alternative labor tax rate are shown in Figures A9–A11. The main difference between these results and those of the benchmark (Figures 5–7) is the initial labor supply response. With the Barro-Sahasakul rate, predicted hours are significantly above actual hours in 1941. The reason is that the alternative tax rate series rises by more during the war. Joines' labor tax rate is roughly 12 percent in 1941 and rises to a peak of 19 percent in 1945. Barro and Sahasakul's tax rate is roughly 12 percent in 1941 and rises to a peak of 26 percent in 1945. Most of the rise in the Barro-Sahasakul tax rate is between 1941 and 1942 when the United States enters the war. Thus, most of the difference in the predicted labor response is between 1941 and 1942. High tax rates during the war induce households to increase hours before the war starts. The higher the anticipated rise, the higher the jump in hours.[2]

5.3. Alternative Postwar Tax Rates

For our benchmark results, we set the postwar tax rates on labor and capital equal to 18.8 percent and 61.7 percent, respectively. (See Figure 1 of the main text). In this case, the wartime debt is paid off by 1975 in our benchmark numerical simulation. In this section, we discuss the sensitivity of our results to alternative rates (that yield sufficient revenue to retire the accumulated debt).

If we set the labor tax equal to 25 percent for the postwar period, then the debt would have been paid off by 1960. Our predictions for consumption during the war are lower by roughly 1 percentage point and investment in 1943 and 1944 is higher, roughly 7 percent, and closer to the actual investment. Also, the predicted nonmilitary hours series during the war is slightly higher than in the benchmark simulation since the postwar labor tax rate is higher. However, overall the quantitative and qualitative effects are small. If we lower the labor tax to delay paying the debt for 15 additional years (that is, by 1990), we have to set the tax rate slightly above 17 percent. With such a small change in the tax rate, the effects on the wartime series are negliglible.

To pay off the debt by 1960 with higher tax rates on capital (and the labor tax rate at 18.79) would require tax rates above 90 percent. Such an increase has a large effect on the simulations, especially investment and the after-tax return to capital. A delay until 1990, on the other hand, requires a postwar rate around 54 percent. The quantitative effects for this setting are small.

5.4. The Intratemporal Condition

Mulligan (1998) has argued that the neoclassical growth model cannot explain labor supply behavior during WWII. One reason for this is his view that the intratemporal condition does not come close to holding. Here, we use the intratemporal condition along with

[2] If households are not expecting to enter the war, then the jump is not so large. For example compare the perfect foresight case with a complete surprise. In the perfect-foresight case, hours rise 0.55 percent between 1941 and 1942. In the perfect-surprise case, they rise 4.4 percent. In the United States, hours rose 5.8 percent.

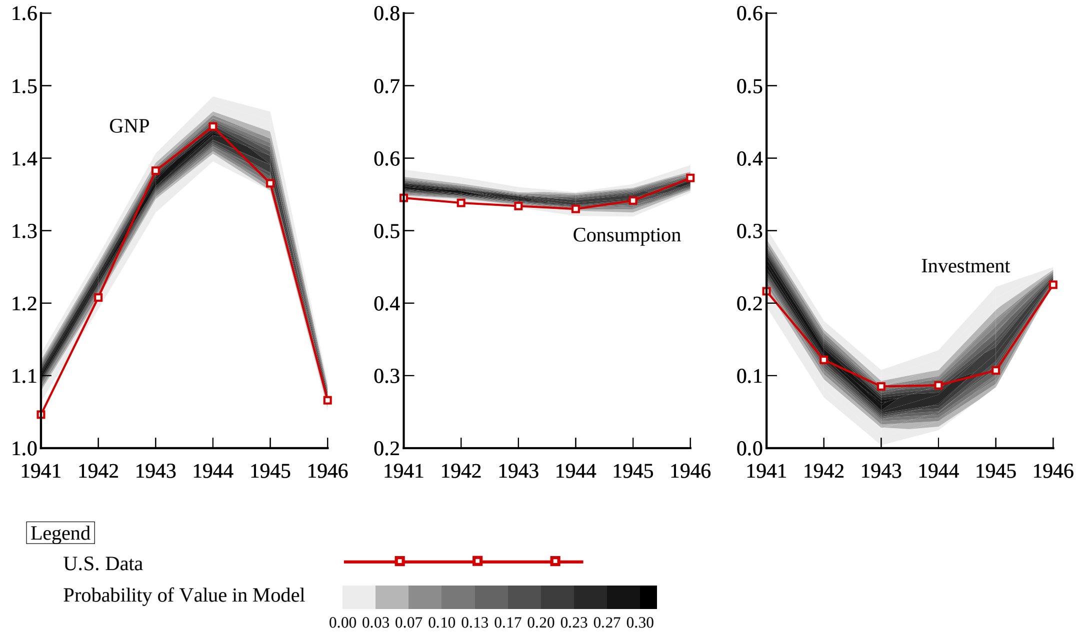
observations on tax rates and the consumption-output ratio to investigate this claim. We also consider two other periods in which hours changed significantly: the 1982 recession and the late 1990s boom.

In Figures A12 and A13, we use data from McGrattan and Prescott (2006) to investigate the recession of 1982 and the 1990s boom. To derive predicted hours, we use U.S. observations on consumption, output, and tax rates and infer hours from the household's static first order conditions. Figure A12 shows that predicted and actual hours are close for 1979–1983 and then start to drift apart. In contrast, the actual and predicted series are never close during the 1990s, a fact that motivated the study by McGrattan and Prescott 2006.

Figure A14 shows results for the same exercise using data for World War II. Notice that there is some deviation in the predicted and actual. Most of the deviation is due to the fact that predicted (nonmilitary) hours are too high initially—a problem we discussed earlier. Changes in actual and predicted after 1941 line up well. In Figure A15, we demonstrate this by changing the index baseyear to 1943.

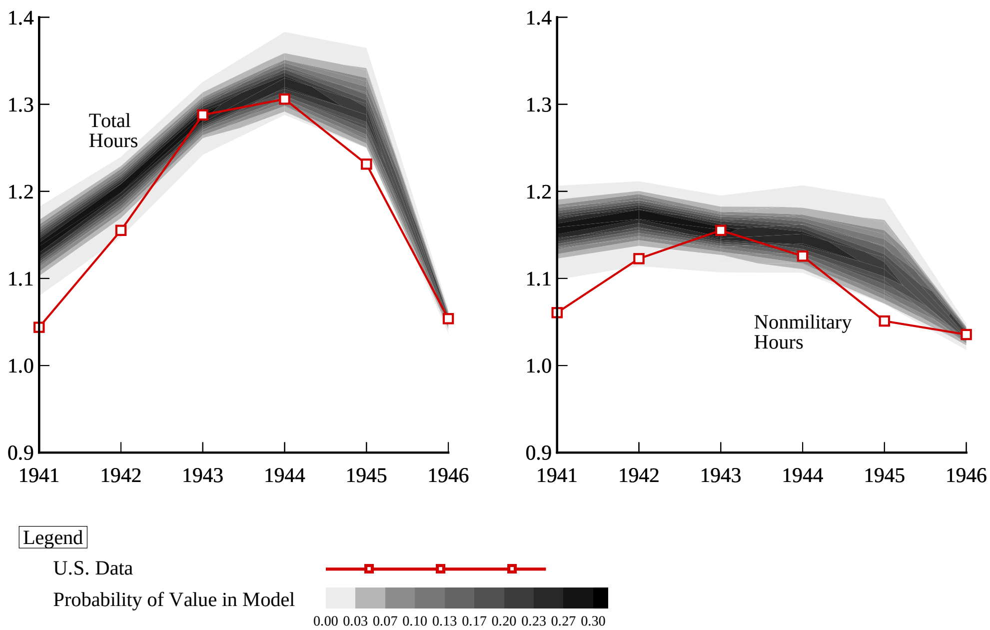
Would these results lead one to conclude that there is a large deviation from theory for the 1940s? Our answer is no.

Figure A1. Real Detrended GNP, Private Consumption, and Private Investment, 1941–1946
(Capacity Utilization Stochastic Model)



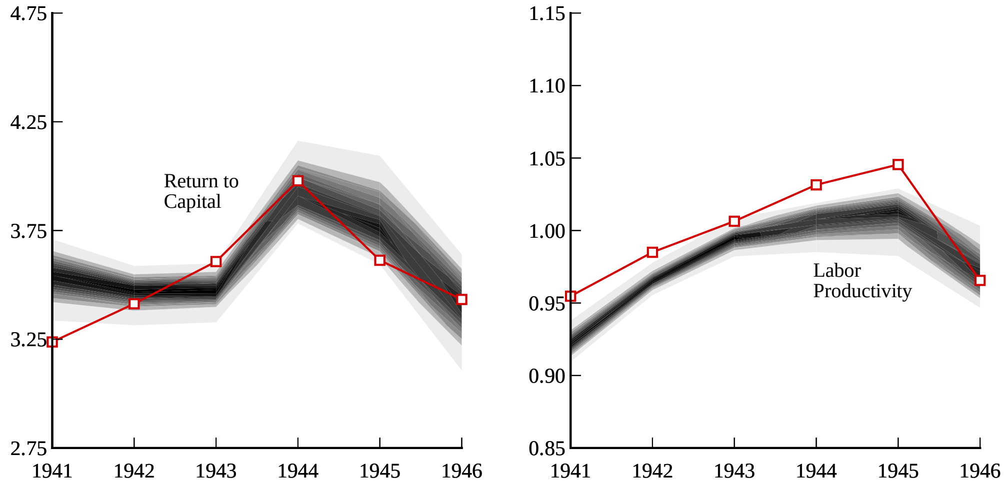
Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.

Figure A2. Per Capita Total and Nonmilitary Hours of Work, 1941–1946
(Capacity Utilization Stochastic Model)



Note: Hours series are divided by the 1946–1960 U.S. averages.

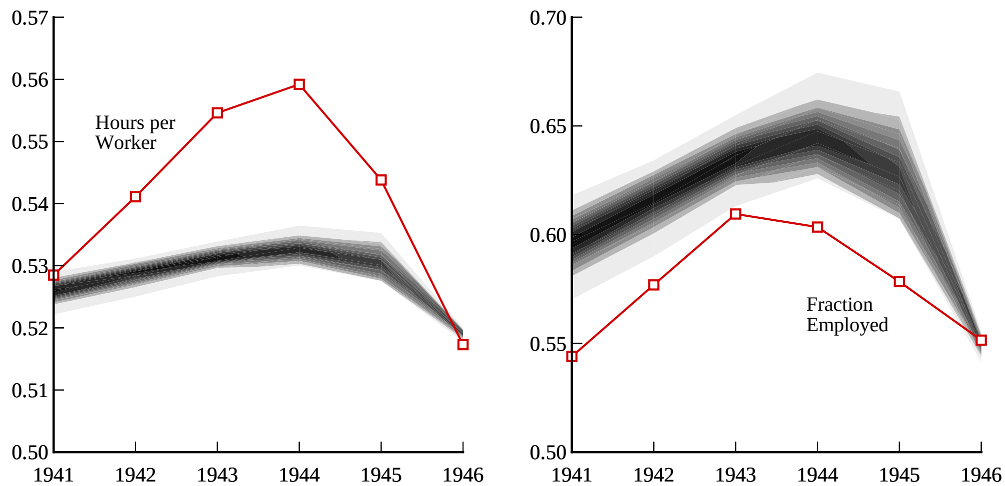
Figure A3. After-tax Return to Capital and Nonmilitary Labor Productivity, 1941–1946
(Capacity Utilization Stochastic Model, All Series Constructed Using Marginal Productivities)



Legend
U.S. Data
Probability of Value in Model
0.00 0.03 0.07 0.10 0.13 0.17 0.20 0.23 0.27 0.30

Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Labor productivity is nonmilitary output divided by hours that are normalized by the 1946–1960 U.S. average.

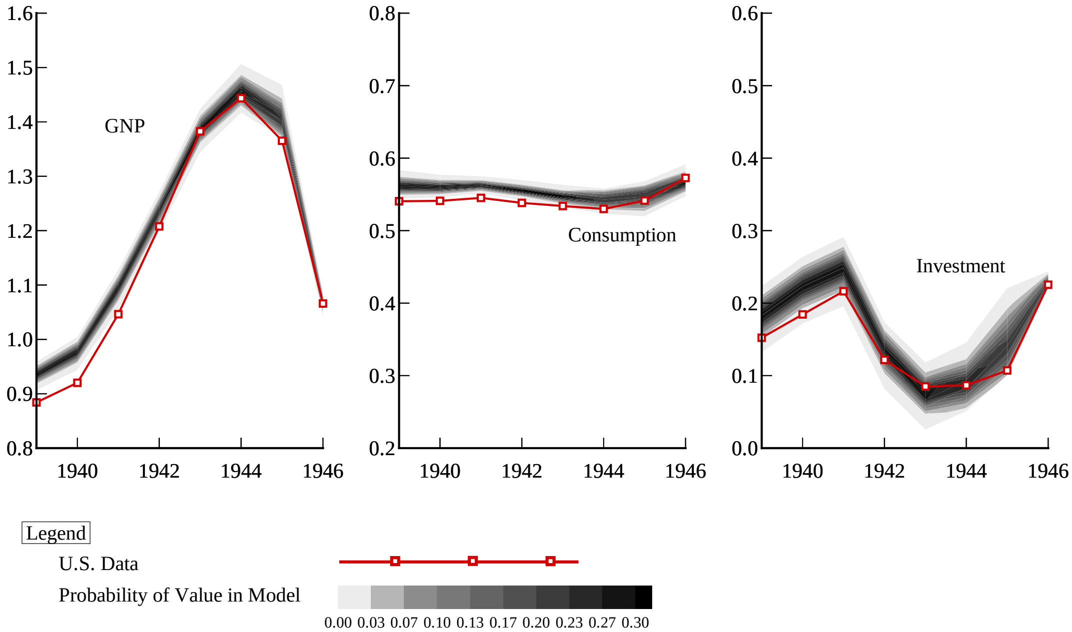
Figure A4. Decomposing Average Hours of Civilians, 1941–1946
(Capacity Utilization Stochastic Model)



Legend
U.S. Data
Probability of Value in Model
0.00 0.03 0.07 0.10 0.13 0.17 0.20 0.23 0.27 0.30

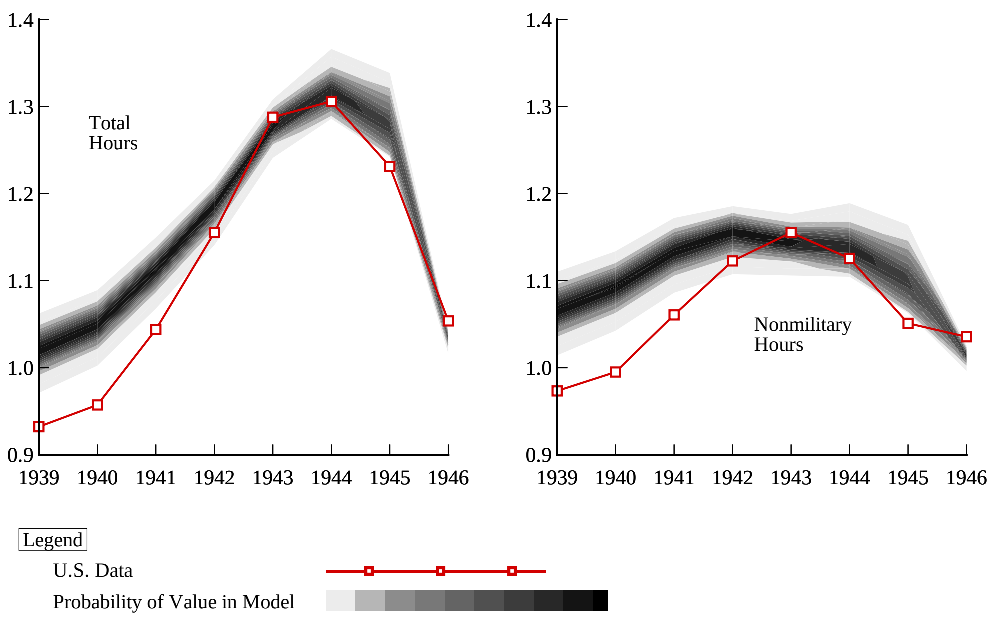
Note: The product of these two series is the civilian labor input.

Figure A5. Real Detrended GNP, Private Consumption, and Private Investment, 1939–1946
(Benchmark Stochastic Model, Starting 1939)



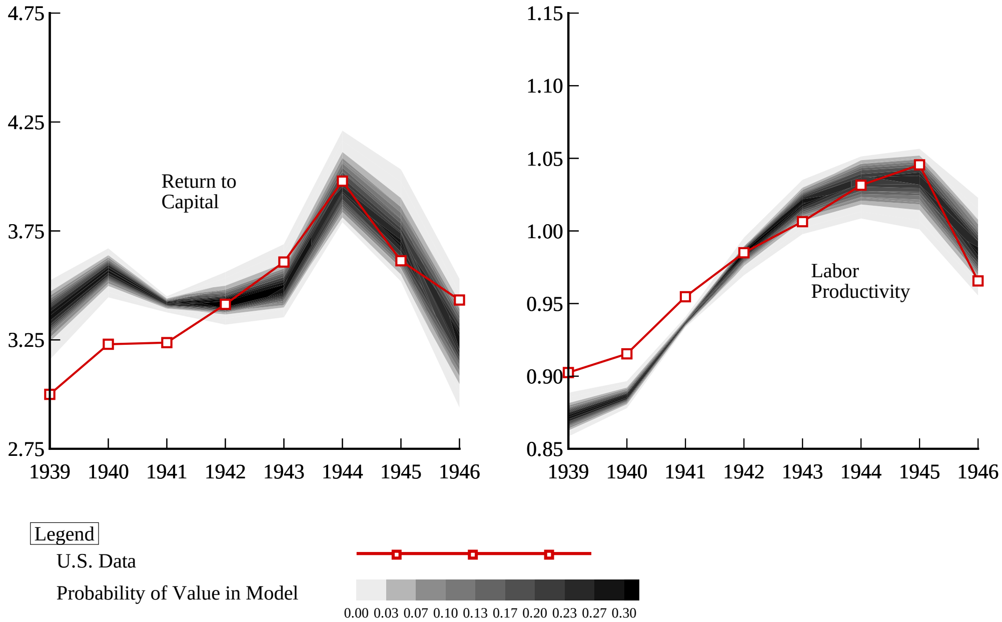
Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.

Figure A6. Per Capita Total and Nonmilitary Hours of Work, 1939–1946
(Benchmark Stochastic Model, Starting 1939)



Note: Hours series are divided by the 1946–1960 U.S. averages.

Figure A7. After-tax Return to Capital and Nonmilitary Labor Productivity, 1939–1946
(Benchmark Stochastic Model, Starting 1939)



Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Labor productivity is nonmilitary output divided by hours that are normalized by the 1946–1960 U.S. average.

Figure A8. Two Measures of the Tax Rate on Labor, 1941–1946

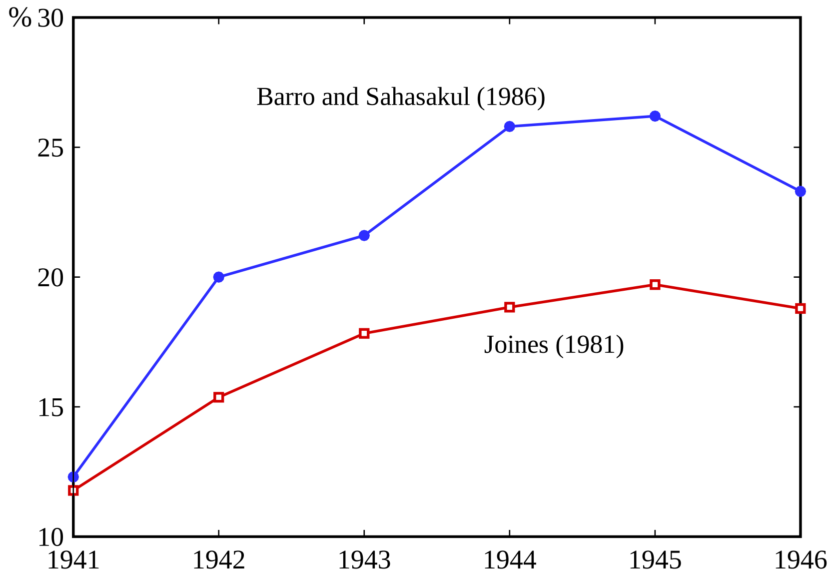
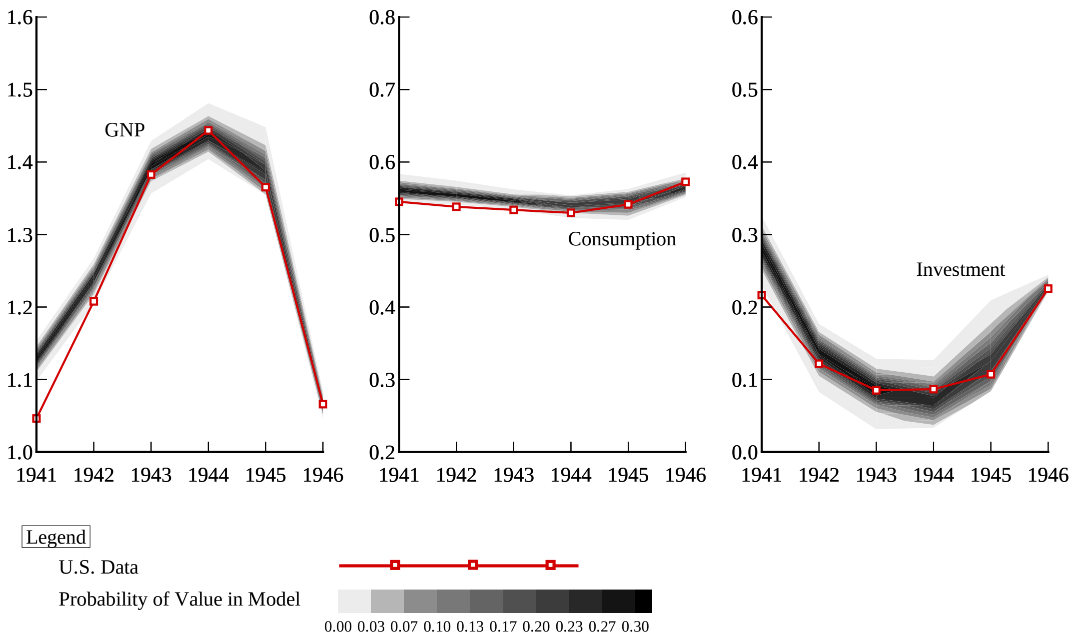
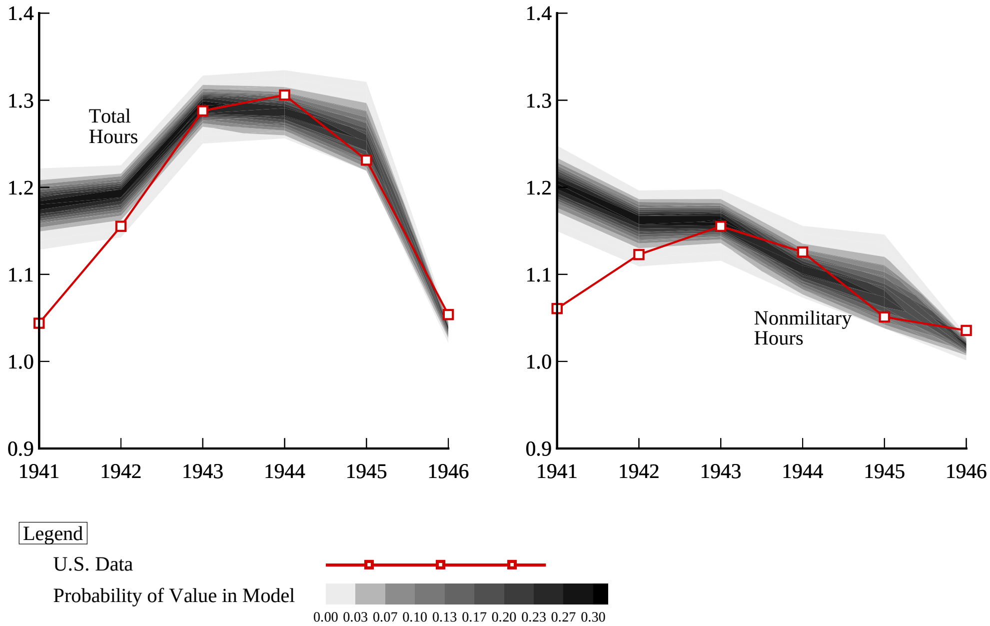


Figure A9. Real Detrended GNP, Private Consumption, and Private Investment, 1941–1946
(Stochastic Model with Alternative Labor Tax Rate)



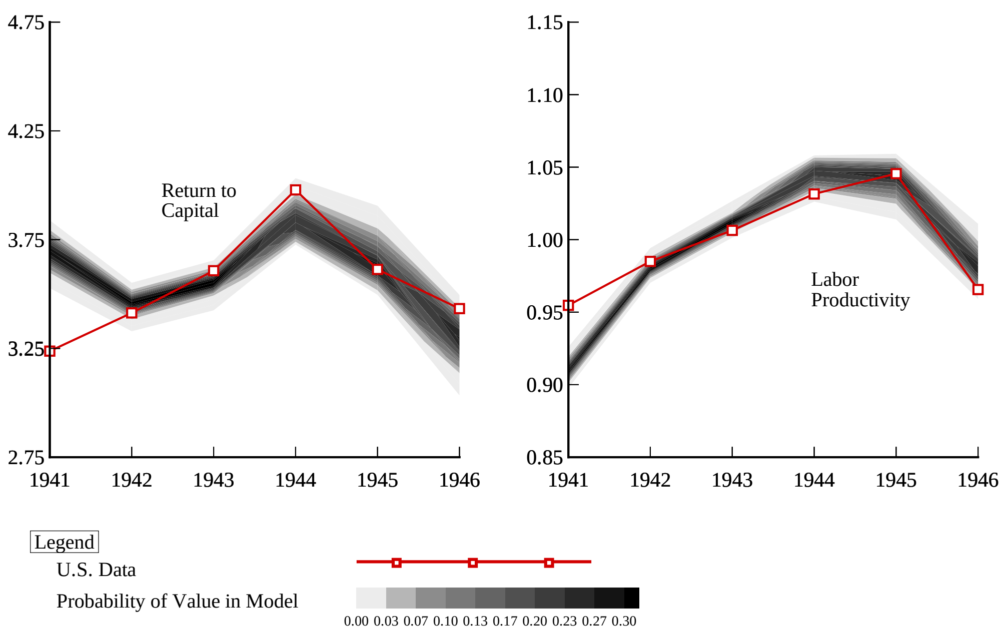
Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.

Figure A10. Per Capita Total and Nonmilitary Hours of Work, 1941–1946
(Stochastic Model with Alternative Labor Tax Rate)



Note: Hours series are divided by the 1946–1960 U.S. averages.

Figure A11. After-tax Return to Capital and Nonmilitary Labor Productivity, 1941–1946
(Stochastic Model with Alternative Labor Tax Rate)



Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Labor productivity is nonmilitary output divided by hours that are normalized by the 1946–1960 U.S. average.

Figure A12. U.S. Per Capita Hours and Prediction Based on the Intratemporal Condition, 1979–1985

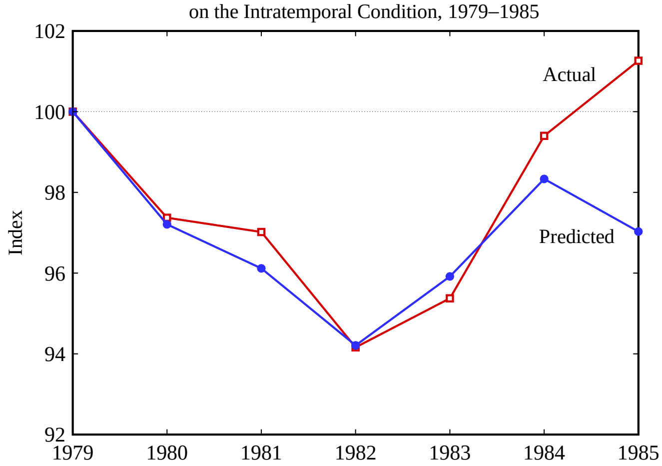


Figure A13. U.S. Per Capita Hours and Prediction Based on the Intratemporal Condition, 1990–2003

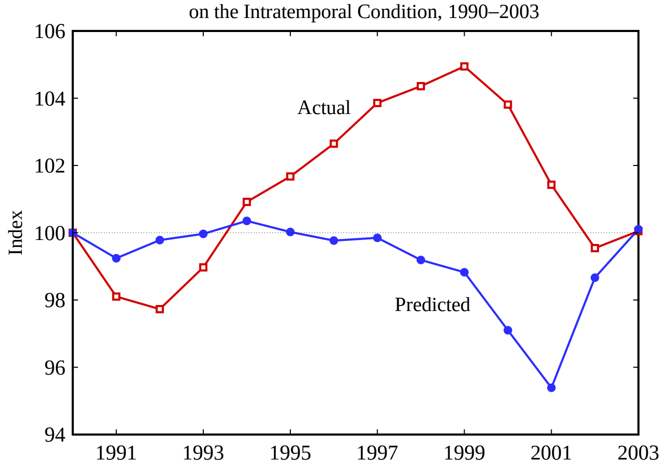


Figure A14. U.S. Civilian Per Capita Hours and Prediction Based on the Intratemporal Condition, 1941–1946

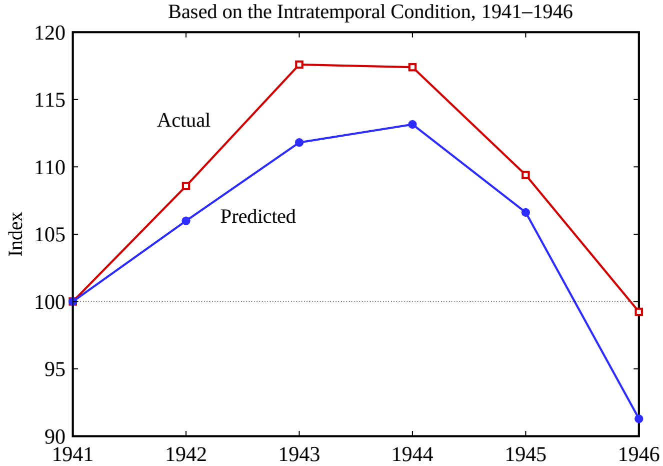


Figure A15. U.S. Civilian Per Capita Hours and Prediction Based on the Intratemporal Condition, 1941–1946, 1943=100