Federal Reserve Bank of Minneapolis
Research Department Staff Report 277

Revised July 2002

Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?

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ABSTRACT

The central puzzle in international business cycles is that fluctuations in real exchange rates are volatile and persistent. We quantify the popular story for real exchange rate fluctuations: they are generated by monetary shocks interacting with sticky goods prices. If prices are held fixed for at least one year, risk aversion is high, and preferences are separable in leisure, then real exchange rates generated by the model are as volatile as in the data and quite persistent, but less so than in the data. The main discrepancy between the model and the data, the *consumption–real exchange rate anomaly*, is that the model generates a high correlation between real exchange rates and the ratio of consumption across countries, while the data show no clear pattern between these variables.

*This paper is a revised version of our 1996 paper "Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business Cycles." A technical appendix, computer codes, and the data used in this paper are available at http://minneapolisfed.org/research/sr/sr277.html. Chari, Kehoe, and McGrattan thank the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

The central puzzle in international business cycles is that fluctuations in real exchange rates are volatile and persistent. Since the work of Dornbusch (1976), the most popular story to explain exchange rate fluctuations is that they result from the interaction of monetary shocks and sticky prices. So far, however, few researchers have attempted to develop quantitative general equilibrium models of this story. Here, we do that, with some success.

In our general equilibrium monetary model with sticky prices, if risk aversion is high and preferences are separable in leisure, then the model can account for the volatility of real exchange rates. With price-stickiness of at least one year, the model also produces real exchange rates that are quite persistent, but less so than in the data. If monetary shocks are correlated across countries, then the model's comovements in aggregates across countries are broadly consistent with those in the data. The main discrepancy between the model and the data is that the model generates a high correlation between real exchange rates and the ratio of consumption across countries (*relative consumption*), while the data show no clear pattern of correlation between these variables.

In constructing our model, we need to choose the source of real exchange rate fluctuations: deviations from the law of one price for traded goods across countries or fluctuations in the relative prices of nontraded to traded goods across countries or both. We choose to abstract from nontraded goods and focus on fluctuations in real exchange rates arising solely from deviations from the law of one price for traded goods. This focus is guided by the data. We present evidence that fluctuations in the relative prices of nontraded to traded goods across countries account for essentially none of the volatility of real exchange rates. Using data for the United States and an aggregate of Europe (and our admittedly imperfect measures), we find that only about 2% of the variance of real exchange rates is due to fluctuations in the relative prices of nontraded to traded goods. This evidence is consistent with studies which document that even at a very disaggregated level, the relative price of traded goods has large and persistent fluctuations. (See, for example, the work of Engel (1993, 1999) and Knetter (1993).)

Our two-country model is a version of Svensson and van Wijnbergen's (1989) model modified to allow for several features that we expect will help the model produce patterns like those in the data. We introduce price-discriminating monopolists in order to get fluctuations in real exchange rates from fluctuations in the relative price of traded goods. (See the work of Dornbusch (1987), Krugman (1987), Knetter (1989), Marston (1990), and Goldberg and Knetter (1997).) We introduce sticky prices in order to get volatility and staggered price-setting in order to get persistence in real exchange rates. We introduce capital accumulation in order to generate the relative volatility of consumption and output observed in the data. In our model, this relative volatility is closely connected to the volatility of the real exchange rate relative to that of output.

In this benchmark model, the real exchange rate is the ratio of the marginal utilities of consumption of households in the two countries. Since the utility function is separable in leisure, the volatility of real exchange rates is essentially determined by the risk aversion parameter and the volatility of consumption, while the persistence of real exchange rates is essentially determined by the persistence of consumption. More precisely, we show that the volatility of real exchange rates is approximately equal to the product of the risk aversion parameter and the volatility of relative consumption in the two countries. We show that this calculation implies that a risk aversion parameter of about 5 produces the real exchange rate volatility in the data.

We also show that the persistence of real exchange rates is approximately equal to the autocorrelation of relative consumption in the two countries. If prices are set for a substantial length of time, then monetary shocks lead to persistent fluctuations in consumption and, hence, in real exchange rates. In our quantitative analysis, we assume that prices are set for one year at a time along the lines of the evidence summarized by Taylor (1999). We find that with this amount of price-stickiness, real exchange rates are persistent in our model, but somewhat less so than in the data. We refer to this discrepancy as the *persistence anomaly.*

To address the persistence anomaly, we replace the model's frictionless labor markets with sticky wages. The idea is that with sticky wages, nominal marginal costs respond less to monetary shocks, so prices do too, thereby increasing persistence. While this avenue is conceptually promising, it does little to increase persistence.

Our benchmark model also displays a more troublesome anomaly. In the model, the correlation between the real exchange rate and relative consumption is high and positive. Yet for the United States and Europe, this correlation is somewhat negative while for other country pairs it ranges between small and positive to somewhat negative. We refer to this discrepancy between the model and the data as the *consumption–real exchange rate anomaly.*

In our model, the correlation between the real exchange rate and relative consumption is high because the real exchange rate is proportional to the ratio of the marginal utilities of consumption. This proportionality follows from our assumption that asset markets are complete. We make this assumption because we want to isolate the role of a particular type of goods market friction, namely, price-stickiness. Hence, we abstract from asset market frictions. We emphasize that this proportionality between the real exchange rate and marginal utilities holds in *any* model with complete asset markets, regardless of the frictions in the goods and labor markets like sticky prices, sticky wages, shipping costs, and so on.

This anomaly leads us to consider two asset market frictions to try to weaken the link between the real exchange rate and relative consumption. We begin by replacing the model's complete international asset markets with incomplete markets that allow for trade in only an uncontingent nominal bond. This avenue is conceptually promising because it breaks the link between real exchange rates and the marginal utilities of consumption. However, the anomaly turns out to be as severe in the incomplete markets model as it is in the benchmark model.

We then explore whether habit persistence in consumption can address this anomaly. This specification is appealing because habit persistence has been found to solve other anomalies in asset markets. (See the work of Jermann (1998), Campbell and Cochrane (1999), and Boldrin, Christiano, and Fisher (2001).) However, we use some simple calculations to show that adding habit persistence to the model is unlikely to eliminate the consumption–real exchange rate anomaly.

Many researchers have investigated the economic effects of sticky prices. For some early work in a closed-economy setting, see the studies by Svensson (1986), Blanchard and Kiyotaki (1987), and Ball and Romer (1989). The international literature on sticky prices has three branches. The pioneering work laying out the general theoretical framework is by Svensson and van Wijnbergen (1989) and Obstfeld and Rogoff (1995). (See also the recent work by Corsetti, Pesenti, Roubini, and Tille (2000).) More closely related to our paper is the work of Betts and Devereux (2000) and Kollmann (2001), who consider economies with price-discriminating monopolists who set prices, as in the work of Calvo (1983). Betts and Devereux are primarily interested in replicating the vector autoregression evidence on monetary policy shocks and exchange rates. Kollmann considers a semi-small open-economy model in which both prices and wages are sticky; he shows that the model generates volatile exchange rates. An early paper on the consumption–real exchange rate anomaly is that of Backus and Smith (1993). They develop a model with nontraded goods in which there is a close relation between relative consumption and real exchange rates and show there is little evidence for this relation in the data. Finally, for some other work on the implications of sticky prices for monetary policy under fixed exchange rates, see the work of Ohanian and Stockman (1997).

1. DATA

Here we document properties of measures of bilateral exchange rates between the United States and individual European countries and a European aggregate. The series are constructed from data for individual countries collected by the International Monetary Fund (IMF) and the Organisation for Economic Co-operation and Development (OECD). (For details on our data series, see the files accompanying Chari, Kehoe, McGrattan (2001).) Our data are quarterly and cover the period from 1973:1 through 2000:1. We show that the data clearly support the notion that real exchange rates between the United States and Europe are volatile and persistent. We then demonstrate, using disaggregated price data, that very little—about 2%—of the volatility in real exchange rates arises from fluctuations in the relative prices of nontraded to traded goods across countries. This observation motivates our decision to exclude nontraded goods from the model.

A. *Volatility and persistence of exchange rates*

Our measure of the nominal exchange rate e_t between the United States and Europe is a trade-weighted average of the bilateral nominal exchange rates between the United States and individual European countries.[1] We construct a price index for all consumer goods in the European countries, denoted P_t^*, in an analogous way, using each country's consumer price index (CPI). The U.S. real exchange rate with Europe is denoted $q_t = e_t P_t^* / P_t$, where P_t is the price index for the United States.

In Figure 1, we plot the quarterly U.S. nominal and real exchange rates with Europe and the quarterly ratio of the CPI for Europe to that for the United States for the period 1973–2000. Our aggregate of Europe consists of the 11 countries for which we could get complete data: Austria, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Spain, Switzerland, and the United Kingdom. Clearly, both the nominal and real exchange rates are highly volatile, especially when compared to the relative price level. The exchange rates are also highly persistent. (For an earlier analysis emphasizing these features of the data, see the study by Mussa (1986).)

In Table 1, we present some statistics for exchange rates and CPIs for the United States and the European aggregate and for the 11 individual European countries for the period 1973:1–2000:1. (The data reported in the table are logged and Hodrick-Prescott (H-P) filtered.) The standard deviation of the real exchange rate between the United States and Europe is 7.52.[2] That is about 4.6 times the volatility of U.S. output over the same time period (which has a standard deviation of only 1.64%). Clearly, real exchange rates are very volatile.

We also see in Table 1 that the ratios of both nominal and real exchange rates between the United States and Europe are highly persistent, with autocorrelations of .85 and .83, respectively, and nominal and real exchange rates are very highly correlated with each other, with a cross-correlation of .99. These patterns are also evident in bilateral comparisons between each European country and the United States.

B. *Decomposing real exchange rate fluctuations*

Theoretically, movements in real exchange rates can arise from two sources:[3] deviations from the law of one price for traded goods across countries and movements in the relative prices of nontraded to traded goods across countries. To investigate the relative magnitudes of these sources in the data, define the traded goods real exchange rate as $q_T = eP_T^*/P_T$, where P_T and P_T^* are traded goods price indices in the two countries. Let $p = q/q_T$, where q is the all-goods real exchange rate.

We refer to p as the *nontraded goods relative price.* To see why, suppose, as an approximation, that the price indices in the two countries are given by $P = (P_T)^{1-\alpha}(P_N)^\alpha$ and $P^* = (P_T^*)^{1-\gamma}(P_N^*)^\gamma$, where P_N and P_N^* are nontraded goods price indices, and α and γ are the consumption shares of nontraded goods. Then p is equal to $(P_N^*/P_T^*)^\gamma/(P_N/P_T)^\alpha$, and its value depends on the relative prices of nontraded to traded goods in the two countries. Notice that if the law of one price holds, then q_T is constant and all the variance in q is attributable to the relative prices of nontraded to traded goods.

Here, we use several measures of disaggregated price data to construct this decomposition. One measure uses disaggregated CPI data. The OECD reports price index data in its *Main Economic Indicators,* where it disaggregates the consumer price index for all items into indices for food, all goods less food, rent, and services less rent. We construct a price index for traded goods as a weighted average of the price indices for food and for all goods less food. Since data on expenditure shares among traded goods by country are not readily available, we use U.S. weights obtained from the U.S. Department of Labor (1992) to construct this price index for each country in Europe which has disaggregated price data. These countries are France, Italy, the Netherlands, and Norway. The period is 1973:1–1998:4. For the European aggregate, we use the trade-weighting procedure described above.

Figure 2 plots quarterly values of the all-goods real exchange rate q, the traded goods real exchange rate q_T, and the nontraded goods relative price p for the period 1973–98. This figure shows that virtually none of the movement in the all-goods real exchange rate is due to fluctuations in the relative price of nontraded to traded goods across countries. The variance of the real exchange rate can be decomposed as $\mathrm{var}(\log q) = \mathrm{var}(\log q_T) + \mathrm{var}(\log p) + 2\mathrm{cov}(\log q_T, \log p)$. In the data, the variance decomposition becomes $3.64 = 4.10 + .076 - .54$. Since the covariance between the two components is negative, the maximum portion of the variance of the real exchange rate attributable to variability in the nontraded goods relative price is only about 2%. (More precisely, the portion is $2.09\% = (.076/3.64) \times 100\%$.)

C. *Alternative decompositions*

Table 2 gives some additional statistics on relative prices and nominal and real exchange rates for individual European countries as well as for the aggregate. These are quarterly data for the period 1973:1–1998:4. Here, although there is some heterogeneity in the individual country statistics, the bilateral comparisons have the same basic patterns as the comparison of aggregates. For our European aggregate, the correlation between the traded goods real exchange rate and the all-goods real exchange rate is about 1. In other respects, the statistics in this table are similar to those in Table 1.

These measures provide evidence that the relative price of traded goods varies a great deal across countries. Since these measures are constructed from broad aggregates, the law of one price may hold for each traded good, and the volatility of the traded goods real exchange rate may arise from compositional effects among traded goods. But we doubt that composition effects account for much of the volatility of real exchange rates: European countries have consumption baskets similar to that of the United States, and these consumption baskets do not change much over time.

The OECD also reports nominal and real consumption expenditures for four categories: durable goods, semi-durable goods, nondurable goods, and services. We used these data to construct traded and nontraded goods price indices and got similar results. (For details, see Chari, Kehoe, and McGrattan (1998).)

Our measures of the price of traded goods are clearly imperfect in another way, as well. They measure the price paid by the final user of the goods and, hence, incorporate the value of intermediate nontraded services, such as distribution and retailing. Thus, if the value of such nontraded services is volatile, we would expect the real exchange rate for traded goods to be volatile even if the law of one price held for goods net of the value of the nontraded services.

One way to measure the volatility induced by distribution and retailing services is to examine wholesale price indices (WPIs). These data reflect prices received by producers and thus do not include many distribution and retailing costs. These data do, however, include the prices of exported goods and exclude the prices of imported goods; thus, they are imperfect measures of the real exchange rate. We report in Table 3 relative prices and exchange rates constructed using WPIs. The procedure we used to construct these indices is the same as that for the measures in Tables 1 and 2. For the period 1973:1–2000:1, WPI data are available for the nine countries listed in Table 3. For the European aggregate relative to the United States, the standard deviation of the real exchange rate constructed using WPIs is 7.30, very close to the 7.52 standard deviation found using CPIs (Table 1). The closeness of these measures suggests that volatile distribution costs are unlikely to be a significant source of real exchange rate volatility.

2. THE WORLD ECONOMY

Now we develop a two-country model with infinitely lived consumers that we will use to confront the observations just documented on exchange rates in Europe and the United States. In our model, competitive final goods producers in each country purchase intermediate goods from monopolistically competitive intermediate goods producers. Each intermediate goods producer can price-discriminate across countries and must set prices in the currency of the local market. Once prices are set, each intermediate goods producer must satisfy the forthcoming demand. The intermediate goods producers set prices in a staggered fashion.

Specifically, consider a two-country world economy consisting of a *home* country and a *foreign* country. Each country is populated by a large number of identical, infinitely lived consumers. In each period of time t, the economy experiences one of finitely many states, or events, s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given.

In each period t, the commodities in this economy are labor, a consumption-capital good, money, a continuum of intermediate goods indexed by $i \in [0,1]$ produced in the home country, and a continuum of intermediate goods indexed by $i \in [0,1]$ produced in the foreign country. In this economy, the intermediate goods are combined to form final goods which are country-specific. All trade between the countries is in intermediate goods that are produced by monopolists who can charge different prices in the two countries. We assume that all intermediate goods producers have the exclusive right to sell their own goods in the two countries. Thus, price differences in intermediate goods cannot be arbitraged away.

In terms of notation, goods produced in the home country are subscripted with an H, while those produced in the foreign country are subscripted with an F. In the home country, final goods are produced from intermediate goods according to a production function that combines features from the industrial organization literature (Dixit and Stiglitz (1977)) and the trade literature (Armington (1969)):

$$y(s^t) = \left[a_1 \left(\int_0^1 y_H(i, s^t)^\theta di\right)^{\rho/\theta} + a_2 \left(\int_0^1 y_F(i, s^t)^\theta di\right)^{\rho/\theta}\right]^{1/\rho}, \tag{1}$$

where $y(s^t)$ is the final goods produced and $y_H(i, s^t)$ and $y_F(i, s^t)$ are the intermediate goods produced in the home and foreign countries, respectively. This specification of technology will allow our model to be consistent with three features of the data. The parameter θ will determine the markup of price over marginal cost. The parameter ρ, along with θ, will determine the elasticity of substitution between home and foreign goods. And the parameters a_1 and a_2, together with ρ and θ, will determine the ratio of imports to output.

Final goods producers in our economy behave competitively. In the home country, in each period t, producers choose inputs $y_H(i,s^t)$ for $i \in [0,1]$ and $y_F(i,s^t)$ for $i \in [0,1]$ and output $y(s^t)$ to maximize profits given by

$$\max \; P(s^t)y(s^t) - \int_0^1 P_H(i,s^{t-1})y_H(i,s^t)\,di - \int_0^1 P_F(i,s^{t-1})y_F(i,s^t)\,di \tag{2}$$

subject to the production function (1), where $P(s^t)$ is the price of the final good in period t, $P_H(i,s^{t-1})$ is the price of the home intermediate good i in period t, and $P_F(i,s^{t-1})$ is the price of the foreign intermediate good i in period t. These prices are in units of the domestic currency. The intermediate goods prices can, at most, depend on s^{t-1} because producers set prices before the realization of the period t shocks. Solving the problem in (2) gives the input demand functions

$$y_H^d(i,s^t) = \frac{[a_1 P(s^t)]^{\frac{1}{1-\rho}}\,\bar{P}_H(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}}}{P_H(i,s^{t-1})^{\frac{1}{1-\theta}}}y(s^t) \tag{3}$$

$$y_F^d(i,s^t) = \frac{[a_2 P(s^t)]^{\frac{1}{1-\rho}}\,\bar{P}_F(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}}}{P_F(i,s^{t-1})^{\frac{1}{1-\theta}}}y(s^t), \tag{4}$$

where $\bar{P}_H(s^{t-1}) = \left(\int_0^1 P_H(i,s^{t-1})^{\frac{\theta}{\theta-1}}di\right)^{\frac{\theta-1}{\theta}}$ and $\bar{P}_F(s^{t-1}) = \left(\int_0^1 P_F(i,s^{t-1})^{\frac{\theta}{\theta-1}}di\right)^{\frac{\theta-1}{\theta}}$. Using the zero-profit condition, we have

$$P(s^t) = \left(a_1^{\frac{1}{1-\rho}}\bar{P}_H(s^{t-1})^{\frac{\rho}{\rho-1}} + a_2^{\frac{1}{1-\rho}}\bar{P}_F(s^{t-1})^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}.$$

Thus, in equilibrium, the price of the final good in period t does not depend on the period t shock.

The technology for producing each intermediate good i is a standard constant returns to scale production function

$$y_H(i,s^t) + y_H^*(i,s^t) = F(k(i,s^{t-1}), l(i,s^t)), \tag{5}$$

where $k(i,s^{t-1})$ and $l(i,s^t)$ are the inputs of capital and labor, respectively, and $y_H(i,s^t)$ and $y_H^*(i,s^t)$ are the amounts of this intermediate good used in home and foreign production of the final good, respectively. The capital used in producing good i is augmented by investment of final goods $x(i, s^t)$ and is subject to adjustment costs. The law of motion for such capital is given by

$$k(i, s^t) = (1-\delta)k(i, s^{t-1}) + x(i, s^t) - \phi\left(\frac{x(i, s^t)}{k(i, s^{t-1})}\right)k(i, s^{t-1}), \tag{6}$$

where δ is the depreciation rate of capital and where the adjustment cost function ϕ is convex and satisfies $\phi(\delta) = 0$ and $\phi'(\delta) = 0$.

Sticky prices are introduced by having intermediate goods producers who behave as monopolistic competitors. They set prices for N periods in a staggered way. In particular, in each period t, a fraction $1/N$ of the home country producers choose a home currency price $P_H(i, s^{t-1})$ for the home market and a foreign currency price $P_H^*(i, s^{t-1})$ for the foreign market before the realization of the event s_t. These prices are set for N periods, so for this group of intermediate goods producers, $P_H(i, s^{t+\tau-1}) = P_H(i, s^{t-1})$ and $P_H^*(i, s^{t+\tau-1}) = P_H^*(i, s^{t-1})$ for $\tau = 0, \ldots, N-1$. The intermediate goods producers are indexed so that those with $i \in [0, 1/N]$ set new prices in 0, N, $2N$, and so on, while those with $i \in [1/N, 2/N]$ set new prices in 1, $N+1$, $2N+1$, and so on, for the N cohorts of intermediate goods producers.

Consider, for example, intermediate goods producers in a particular cohort, namely, $i \in [0, 1/N]$. Let $Q(s^t)$ be the price of one unit of home currency in s^t in an abstract unit of account, $e(s^t)$ be the nominal exchange rate, and $w(s^t)$ be the real wage. The intermediate goods producers in a particular cohort choose prices $P_H(i, s^{t-1}), P_H^*(i, s^{t-1})$, inputs of labor $l(i, s^t)$, capital $k(i, s^t)$, and investment $x(i, s^t)$ to solve

$$\begin{aligned} \max \ \textstyle\sum_{t=0}^{\infty}\sum_{s^t} Q(s^t)[&P_H(i, s^{t-1})y_H(i, s^t) + e(s^t)P_H^*(i, s^{t-1})y_H^*(i, s^t) \\ &-P(s^t)w(s^t)l(i, s^t) - P(s^t)x(i, s^t)] \end{aligned} \tag{7}$$

subject to (5), (6), and the constraints that their supplies to the home and foreign markets $y_H(i, s^t)$ and $y_H^*(i, s^t)$ must equal the amount demanded by home and foreign final goods producers, $y_H^d(i, s^t)$ from (3) and its analogue. In addition, the constraints that prices are set for N periods are $P_H(i, s^{t-1}) = P_H(i, s^{-1})$ for $t = 0, \ldots, N-1$ and $P_H(i, s^{t-1}) = P_H(i, s^{N-1})$ for $t = N, \ldots, 2N-1$ and so on, with similar constraints for $P_H^*(i, s^{t-1})$. The initial capital stock $k(i, s^{-1})$ is given and is the same for all producers in this cohort.

The optimal prices for $t = 0,\ N,\ 2N$ are

$$P_H(i, s^{t-1}) = \frac{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau) P(s^\tau) v(i, s^\tau) \Lambda_H(s^\tau)}{\theta \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau) \Lambda_H(s^\tau)}$$

$$P_H^*(i, s^{t-1}) = \frac{\sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau) P(s^\tau) v(i, s^\tau) \Lambda_H^*(s^\tau)}{\theta \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q(s^\tau) e(s^\tau) \Lambda_H^*(s^\tau)},$$

where $v(i, s^t)$ is the real unit cost which is equal to the wage rate divided by the marginal product of labor, $w(s^t)/F_l(i, s^t)$, $\Lambda_H(s^t) = [a_1 P(s^t)]^{\frac{1}{1-\rho}} \bar{P}_H(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}} y(s^t)$, and $\Lambda_H^*(s^t) = [a_2 P^*(s^t)]^{\frac{1}{1-\rho}} \bar{P}_H^*(s^{t-1})^{\frac{\rho-\theta}{(1-\rho)(\theta-1)}} y^*(s^t)$. Here, $F_l(i, s^t)$ denotes the derivative of the production function with respect to l. We use similar notation throughout the paper.

In a symmetric steady state, the real unit costs are equal across firms. Hence, in this steady state, these formulas reduce to $P_H(i) = eP_H^*(i) = Pv/\theta$, so that the law of one price holds for each good and prices are set as a markup $(1/\theta)$ over nominal costs Pv. Thus, in this model, all deviations from the law of one price are due to shocks which keep the economy out of the deterministic steady state.

In this economy, the markets for state-contingent money claims are complete. We represent the asset structure by having complete, contingent, one-period nominal bonds denominated in the home currency. We let $B(s^t, s_{t+1})$ denote the home consumers' holdings of such a bond purchased in period t and state s^t with payoffs contingent on some particular state s_{t+1} at $t+1$. Let $B^*(s^t, s_{t+1})$ denote the foreign consumers' holdings of this bond. One unit of this bond pays one unit of the home currency in period $t+1$ if the particular state s_{t+1} occurs and 0 otherwise. Let $Q(s^{t+1}|s^t)$ denote the price of this bond in units of the home currency in period t and state s^t. Clearly $Q(s^{t+1}|s^t) = Q(s^{t+1})/Q(s^t)$. (Including bonds denominated in the foreign currency would be redundant.) For notational simplicity, we assume that claims to the ownership of firms in each country are held by the residents of that country and cannot be traded.

In each period $t = 0, 1, \ \ldots$, consumers choose their period t allocations after the realization of the event s_t. Consumers in the home country face the sequence of budget constraints

$$
\begin{aligned}
&P(s^t)c(s^t) + M(s^t) + \textstyle\sum_{s_{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}) \\
&\qquad \leq P(s^t)w(s^t)l(s^t) + M(s^{t-1}) + B(s^t) + \Pi(s^t) + T(s^t)
\end{aligned}
\tag{8}
$$

and a borrowing constraint $B(s^{t+1}) \geq -P(s^t)\bar{b}$, where $c(s^t)$, $l(s^t)$, and $M(s^t)$ are consumption, labor, and nominal money balances, respectively; $s^{t+1} = (s^t, s_{t+1})$; $\Pi(s^t)$ is the profits of the home country intermediate goods producers; and $T(s^t)$ is transfers of home currency. The positive constant $\bar{b}$ constrains the amount of real borrowing of the consumer. The initial conditions $M(s^{-1})$ and $B(s^0)$ are given.

Home consumers choose consumption, labor, money balances, and bond holdings to maximize their utility:

$$
\sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t) U\big(c(s^t), l(s^t), M(s^t)/P(s^t)\big) \tag{9}
$$

subject to the consumer budget constraints. Here β is the discount factor. The first-order conditions for the consumer can be written as

$$
-\frac{U_l(s^t)}{U_c(s^t)} = w(s^t), \tag{10}
$$

$$
\frac{U_m(s^t)}{P(s^t)} - \frac{U_c(s^t)}{P(s^t)} + \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\frac{U_c(s^{t+1})}{P(s^{t+1})} = 0, \tag{11}
$$

$$
Q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1})\frac{U_c(s^t)}{U_c(s^{t-1})}\frac{P(s^{t-1})}{P(s^t)}. \tag{12}
$$

Here $U_c(s^t)$, $U_l(s^t)$, and $U_m(s^t)$ are the derivatives of the utility function with respect to its arguments, and $\pi(s^t|s^{t-1}) = \pi(s^t)/\pi(s^{t-1})$ is the conditional probability of s^t given s^{t-1}.

The problems of the final goods producers, the intermediate goods producers, and the consumers in the foreign country are analogous to these problems. Allocations and prices in the foreign country are denoted with an asterisk.

Now let's develop a relationship between the real exchange rate and the marginal utilities of consumption of the consumers in the two countries, which is implied by arbitrage. The budget constraint of a consumer in the foreign country is given by

$$
\begin{aligned}
&P^*(s^t)c^*(s^t) + M^*(s^t) + \textstyle\sum_{s_{t+1}} Q(s^{t+1}|s^t)B^*(s^{t+1})/e(s^t) \\
&\qquad \leq P^*(s^t)w^*(s^t)l^*(s^t) + M^*(s^{t-1}) + B^*(s^t)/e(s^t) + \Pi^*(s^t) + T^*(s^t),
\end{aligned}
\tag{13}
$$

where $B^*(s^t)$ denotes the foreign consumer's holdings of the home country bonds at s^t. The first-order condition with respect to bond holdings for a foreign consumer is

$$Q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1})\frac{U_c^*(s^t)}{U_c^*(s^{t-1})}\frac{e(s^{t-1})}{e(s^t)}\frac{P^*(s^{t-1})}{P^*(s^t)}.$$

Substituting for the bond price in this equation from (12) and iterating, we obtain

$$\frac{U_c(s^t)}{U_c(s^0)}\frac{P(s^0)}{P(s^t)} = \frac{U_c^*(s^t)}{U_c^*(s^0)}\frac{e(s^0)}{e(s^t)}\frac{P^*(s^0)}{P^*(s^t)}.$$

Defining the real exchange rate as $q(s^t) = e(s^t)P^*(s^t)/P(s^t)$, we obtain

$$q(s^t) = \kappa\frac{U_c^*(s^t)}{U_c(s^t)}, \tag{14}$$

where the constant $\kappa = e(s^0)U_c(s^0)P^*(s^0)/U_c^*(s^0)P(s^0)$. We use this relationship between real exchange rates and marginal rates of substitution to develop intuition for our quantitative results.

The money supply processes in the home and foreign countries are given by $M(s^t) = \mu(s^t)M(s^{t-1})$ and $M^*(s^t) = \mu^*(s^t)M^*(s^{t-1})$, where $\mu(s^t)$ and $\mu^*(s^t)$ are stochastic processes and $M(s^{-1})$ and $M^*(s^{-1})$ are given. New money balances of the home currency are distributed to consumers in the home country in a lump-sum fashion by having transfers satisfy $T(s^t) = M(s^t) - M(s^{t-1})$. Likewise, transfers of foreign currency to foreign consumers satisfy $T^*(s^t) = M^*(s^t) - M^*(s^{t-1})$.

An equilibrium requires several market-clearing conditions. The resource constraint in the home country is given by

$$y(s^t) = c(s^t) + \int_0^1 x(i,s^t)\,di,$$

and the labor market-clearing condition is $l(s^t) = \int l(i,s^t)\,di$. Similar conditions hold for the foreign country. The market-clearing condition for contingent bonds is $B(s^t) + B^*(s^t) = 0$.

An *equilibrium* for this economy is a collection of allocations for home consumers $c(s^t)$, $l(s^t)$, $M(s^t)$, $B(s^{t+1})$; allocations for foreign consumers $c^*(s^t)$, $l^*(s^t)$, $M^*(s^t)$, $B^*(s^{t+1})$; allocations and prices for home intermediate goods producers $y_H(i,s^t)$, $y_H^*(i,s^t), l(i,s^t)$, $x(i,s^t)$, and $P_H(i,s^{t-1})$, $P_H^*(i,s^{t-1})$ for $i \in [0,1]$; allocations and prices for foreign intermediate goods producers $y_F(i,s^t)$, $y_F^*(i,s^t), l^*(i,s^t)$, $x^*(i,s^t)$, and $P_F(i,s^{t-1})$, $P_F^*(i,s^{t-1})$ for $i \in [0,1]$; and allocations for home and foreign final goods producers $y(s^t), y^*(s^t)$, final good prices $P(s^t)$, $P^*(s^t)$, real wages $w(s^t)$, $w^*(s^t)$, and bond prices $Q(s^{t+1}|s^t)$ that satisfy the following five conditions: (i) the consumer allocations solve the consumers' problem; (ii) the prices of intermediate goods producers solve their maximization problem; (iii) the final goods producers' allocations solve their problem; (iv) the market-clearing conditions hold; and (v) the money supply processes and transfers satisfy the specifications above.

We are interested in a stationary equilibrium and thus restrict the stochastic processes for the growth rates of the money supplies to be Markovian. To make the economy stationary, we deflate all nominal variables by the level of the relevant money supply. A stationary equilibrium for this economy consists of stationary decision rules and pricing rules that are functions of the state of the economy. The state of the economy when monopolists make their pricing decisions (that is, before the event s_t is realized) must record the capital stocks for a representative monopolist in each cohort in the two countries, the prices set by the other $N-1$ cohorts in the two countries, and the period $t-1$ monetary shocks.

The shocks from period $t-1$ are needed because they help forecast the shocks in period t. The current shocks are also included in the state of the economy when the rest of the decisions are made (that is, after the event s_t is realized).

We compute the equilibrium using standard methods to obtain linear decision rules (Chari, Kehoe, and McGrattan (2001)). For the benchmark preferences with one-quarter price-stickiness and $N = 2$, we checked the accuracy of the linear decision rules against nonlinear decision rules obtained by the finite element method. (For an introduction to the finite element method, see McGrattan (1996).)

3. PARAMETERIZATION

In this section, we describe how we choose functional forms and benchmark parameter values. We report our choices in the top panel of Table 4. (In later sections, we do extensive sensitivity analyses of our model. We report the parameter values for the variations in the bottom panel of the table.)

We consider a benchmark utility function of the form

$$U(c,l,M/P) = \frac{1}{1-\sigma}\left[\left(\omega c^{\frac{\eta-1}{\eta}} + (1-\omega)\left(\frac{M}{P}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right]^{1-\sigma} + \psi\frac{(1-l)^{(1-\gamma)}}{1-\gamma} \tag{15}$$

and an intermediate goods production function of the form $F(k,l) = k^{\alpha}l^{1-\alpha}$. Notice that the utility function is separable between a consumption-money aggregate and leisure.

Consider first the preference parameters. The discount factor β is set to give an annual real return to capital of 4%. The literature has a wide range of estimates for the curvature parameter σ, which determines the level of risk aversion. We set σ to 5 and show later that this value is critical for generating the right volatility in the real exchange rate. Balanced growth considerations lead us to set $\gamma = \sigma$.[4] We set ψ so that households devote one-quarter of their time to market activities. With these choices for σ, γ, and ψ, the elasticity of labor supply, with marginal utility held constant, is 1/2.

To obtain η and ω, we draw on the money demand literature. Our model can be used to price a variety of assets, including a nominal bond which costs one dollar at s^t and pays $R(s^t)$ dollars in all states s^{t+1}. The first-order condition for this asset can be written as $U_m(s^t) = U_c(s^t)[R(s^t)-1]/R(s^t)$. When we use our benchmark specification of utility, the first-order condition can be rewritten as

$$\log\frac{M(s^t)}{P(s^t)} = -\eta\log\frac{\omega}{1-\omega} + \log c(s^t) - \eta\log\left(\frac{R(s^t)-1}{R(s^t)}\right), \tag{16}$$

which has the form of a standard money demand function with consumption and interest rates. To obtain the interest elasticity η, we ran a quarterly regression from 1960 to 1995 (inclusive) in which we used M1 for money; the deflator of the gross domestic product (GDP) for P; consumption of durables, nondurables, and services for c; and the three-month U.S. Treasury bill rate for R. Our estimate of the interest elasticity is $\eta = .39$, and the implied value for ω is .94.

Consider next the final goods technology parameters. In our model, the elasticity of substitution between home goods and foreign goods is $1/(1-\rho)$. Studies have estimated quite a range for this parameter. The most reliable studies seem to indicate that for the United States the elasticity is between 1 and 2, and values in this range are generally used in empirical trade models. (See, for example, the survey by Stern, Francis, and Schumacher (1976).)) We follow the work of Backus, Kehoe, and Kydland (1994) and use an elasticity of 1.5, so that $\rho = 1/3$. To set a_1 and a_2, note that in a symmetric steady state, $y_H/y_F = [a_1/a_2]^{\frac{1}{1-\rho}}$. In U.S. data, imports from Europe are roughly 1.6% of GDP. This implies that $y_H/y_F = .984/.016$. Together with our normalization, this gives the values of a_1 and a_2 reported for the benchmark model in Table 4.

For the intermediate goods technology parameters, we set the capital share parameter $\alpha = 1/3$ and the depreciation rate $\delta = .021$. The latter implies an annual depreciation rate of 10%. These are typical estimates for U.S. data. Based on the work of Basu and Fernald (1994, 1995), Basu (1996), and Basu and Kimball (1997), we choose $\theta = .9$, which implies a markup of 11% and an elasticity of demand of 10. We set $N = 4$, so that prices are set for four quarters based on the empirical studies summarized by Taylor (1999).

We consider an adjustment function of the form $\phi(x/k) = b(x/k - \delta)^2/2$. Notice that with this specification at the steady state, both the total and marginal costs of adjustment are 0. Uncertainty about the size of these adjustment costs is high. In all of our experiments, we choose the parameter b so that the standard deviation of consumption relative to the standard deviation of output is equal to that in the data. One measure of the adjustment costs is the resources used up in adjusting capital relative to investment given by $\phi(x/k)/x$. For our benchmark economy, the average resource cost in adjusting capital is .19% of investment.

The details of the monetary rules followed in the United States and Europe are extensively debated. For the benchmark economy, we assume that all the monetary authorities follow a simple rule, namely, that the growth rates of the money stocks for both areas follow a process of the form

$$\begin{aligned} \log \mu_t &= \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu t} \\ \log \mu_t^* &= \rho_\mu \log \mu_{t-1}^* + \varepsilon_{\mu t}^*, \end{aligned} \tag{17}$$

where $(\varepsilon_\mu, \varepsilon_\mu^*)$ is a normally distributed, mean-zero shock. (Notice that each period now has a continuum of states. Our earlier analysis with a finite number of states extends immediately to this case.) Each shock has a standard deviation of σ_μ, and the shocks have a positive cross-correlation. The stochastic process for money in the foreign country is the same. We choose $\rho_\mu = .68$ from the data by running a regression of the form (17) on quarterly U.S. data for M1 from 1959:2 through 2001:1, obtained from the Board of Governors of the Federal Reserve System.

In our experiments, we choose the standard deviation of the shocks so that the volatility of output is the same in the model as in the U.S. data. (For example, in the benchmark model we choose the standard deviation of log μ to be 2.3% in order to produce a standard deviation of output of 1.82%. In the data, the standard deviation of log μ is 1.15%.) We also choose the cross-correlation of these shocks to produce a cross-correlation for output that is similar to that in the data. We choose the standard deviation and the cross-correlation of these shocks in this way because we want to investigate whether a model in which monetary shocks account for the observed movements in outputs can also account for the observed movements in exchange rates and other macroeconomic variables.

4. FINDINGS

We report on the H-P–filtered statistics for the data, the benchmark economy, and some variations on that economy in Tables 5 and 6. The statistics for the data are all computed with the United States as the home country and the aggregate of Europe as the foreign country for the period 1973:1–1994:4.[5] In these tables, net exports are measured by bilateral real net exports from the United States to Europe. Unless otherwise noted, all other variables are from U.S. data.

Overall, we find that the benchmark model generates nominal and real exchange rates that match the data qualitatively: they are volatile, persistent, and highly cross-correlated. However, quantitatively, along some dimensions, the model does less well: while its volatility of exchange rates is about right, it generates too little persistence in exchange rates, too high a correlation between real exchange rates and relative consumption, too much volatility in the price ratio and employment, and too little volatility in investment.

In Table 5, we see that in the benchmark model, compared to output, the nominal exchange rate is 4.32 times as variable and the real exchange rate is 4.27 times as variable. These values are close to those in the data (4.67 and 4.36). The benchmark model also produces substantial persistence (autocorrelations) of nominal and real exchange rates (.69 and .62), but this persistence is less than that in the data (.86 and .83). Because these differences are substantial, we refer to them as the *persistence anomaly.* We will explore ways to eliminate this anomaly in Section 6A.

The high volatility of real exchange rates comes from our choice of a high curvature parameter σ, which corresponds to a choice of high risk aversion. To see the connection between volatility and σ, log-linearize the expression for real exchange rates, (14), to obtain

$$\hat{q} = A(\hat{c} - \hat{c}^*) + B(\hat{m} - \hat{m}^*) + D(\hat{l} - \hat{l}^*), \tag{18}$$

where a caret denotes the deviation from the steady state of the log of the variable and m, m^* denote real money balances. The coefficients A, B, and D are given by

$$A = -\frac{cU_{cc}}{U_c},\ B = -\frac{mU_{cm}}{U_c},\ D = -\frac{lU_{cl}}{U_c}$$

evaluated at the steady state. For preferences of the form (15), the coefficient of relative risk aversion A is approximately equal to the curvature parameter $\sigma = 5$, B is unimportant, and $D = 0$. (The actual values are $A = 4.96$ and $B = .04$. Notice that A is only approximately equal to σ, because of the nonseparability between consumption and money balances.) Thus, for our preferences,

$$\frac{\text{std}(\hat{q})}{\text{std}(\hat{y})} \cong \sigma \frac{\text{std}(\hat{c} - \hat{c}^*)}{\text{std}(\hat{y})}.$$

In Figure 3 we graph the benchmark model's volatility of real exchange rates against the curvature parameter σ, where this volatility is measured as in Table 5. As we vary σ, we alter the adjustment cost parameter b to keep roughly unchanged the standard deviation of consumption relative to that of output.[6] We see that a curvature parameter of about 5 is needed to reproduce the data's volatility of real exchange rates relative to output (4.36). Note also in Figure 3 that as σ is varied, the autocorrelation of real exchange rates is essentially unchanged.

In terms of the persistence of real exchange rates, for our preferences the autocorrelation of real exchange rates can be written as

$$\mathrm{corr}(\hat{q}, \hat{q}_{-1}) \cong \mathrm{corr}(\hat{c} - \hat{c}^*, \hat{c}_{-1} - \hat{c}^*_{-1}).$$

This expression suggests that the autocorrelation of real exchange rates is essentially determined by the autocorrelation of consumption. In Table 6, we see that the autocorrelation of consumption is high in the model (.61), but not as high as in the data (.89), which mirrors the feature (from Table 5) that the autocorrelation of real exchange rates is high in the model but lower than that in the data.

Without substantial price-stickiness, neither consumption nor real exchange rates would have much persistence. To see this, consider Figure 4 in which we graph the autocorrelation of consumption, the autocorrelations of real and nominal exchange rates, and the volatility of the price ratio relative to the volatility of output against the number of periods that prices are held fixed, N. Notice that the autocorrelations of consumption and the real exchange rate match almost exactly. When $N = 1$, both of these autocorrelations are negative; as N increases, so do the autocorrelations. Notice also that as the number of periods of price-stickiness increases, the volatility of the price ratio relative to that of output declines and the behavior of the real exchange rate comes to mirror that of the nominal exchange rate.

Our model has a tight link between real exchange rates and the ratio of marginal utilities given by (18). This link implies a high correlation between real exchange rates and relative consumption. In Table 6, we see that in the data this correlation is $-.35$ while in the model it is 1. We refer to this large discrepancy as the *consumption–real exchange rate anomaly.* We investigate this anomaly in Section 6B.

Consider now the rest of the statistics for the benchmark economy in Tables 5 and 6. In Table 5, we see that the price ratio is substantially more volatile in the model (3.00) than in the data (.71) while real and nominal exchange rates are less correlated in the model (.76) than in the data (.99). These differences occur because prices move to offset nominal exchange rate movements more in the model than in the data. In Table 6, we see that real exchange rates and output are more correlated in the model than in the data (.51 vs. .08), while real exchange rates and net exports are slightly negatively correlated in the model ($-.04$) and slightly positively correlated in the data (.14).[7]

In Table 6, we see that investment is a little more than half as volatile in the model as in the data (1.59 vs. 2.78), while employment is more than twice as volatile in the model as in the data (1.51 vs. .67). Investment is less volatile in the model because when $\sigma = 5$, a relatively high adjustment cost parameter is needed to make consumption have the right volatility. With that level of adjustment costs, investment is not very volatile. If a sufficiently low adjustment cost parameter were used, then investment would be as volatile as in the data, but consumption would be significantly less volatile. (For example, when the adjustment cost parameter is set at a level to make investment have a volatility of about 2.78, as in the data, the volatility of consumption is only .50 while in the data it is .83.)

Employment is more rather than less volatile than output in the model because almost all of the movement in output comes from variations in the labor input. Specifically, note that log-deviations in output can be written as $\hat{y} = \alpha\hat{k} + (1-\alpha)\hat{l}$. Since investment is only a small percentage of the capital stock, this stock moves only a small amount at business cycle frequencies, and we roughly have that $\text{std}(\hat{y}) \cong (1-\alpha)\text{std}(\hat{l})$. With $\alpha = 1/3$, this gives $\text{std}(\hat{l})/\text{std}(\hat{y}) \cong 1.5$. So, in a sticky price model like ours, we should expect employment to be much more volatile than output. This feature does not arise in standard real business cycle models because in them the technology shock accounts for much of the movement in output.[8] (A related problem of sticky price models more generally is that labor productivity is countercyclical in the model but procyclical in the data.)

In Table 6, we also see that in the model, the cross-country correlation of output is the same as that of consumption (.49 in both) while in the data, the cross-correlation of output is higher than that of consumption (.60 vs. .38). While the cross-correlation of consumption in our model is higher than that in the data, the model does much better on this dimension than does the standard real business cycle model (Backus, Kehoe, and Kydland (1994)). In the standard real business cycle model, the law of one price holds for all traded goods, and the real exchange rate does not vary as much as it does in our model. Since an equation like (14) holds in both models, the lower variability of real exchange rates in the real business cycle model leads to a higher correlation of the marginal utilities of consumption and, thus, to a higher cross-country correlation of consumption. A minor discrepancy between the benchmark model and the data is that in the data, net exports are somewhat countercyclical ($-.41$) while in the model they are essentially acyclical (.04).

5. SENSITIVITY ANALYSIS

Here we examine the sensitivity of our findings for the benchmark model by varying assumptions about four of the benchmark model's features. We raise the export share of output and find little change. We consider nonseparable preferences and find a dramatic reduction in the volatility of real exchange rates. We add technology and government consumption shocks and find little change. Finally, we model monetary policy as an interest rate rule and with alternative rules find a reduction in the persistence of real exchange rates.

A. *High exports*

For the benchmark economy, we have chosen parameters so that the export share of output is 1.6%, which is similar to the share that the United States has in its bilateral trade with Europe. More open economies have much larger shares than this. To see what difference a larger share might make, we consider a variation of the model with an export share of 15% instead of 1.6% (by adjusting a_1 and a_2 accordingly). (See the bottom panel of Table 4.) To put this value in perspective, note that it is similar to the share that the United States has with the rest of the world.

In Tables 5 and 6, the columns labeled "High Exports" list the model's predictions with the 15% export share. Raising the export share worsens the model's predictions for net exports by making this variable more procyclical and by slightly lowering its correlation with real exchange rates. But raising the export share produces little change overall.

B. *Nonseparable preferences*

Now we consider what would happen to the benchmark model's predictions if we make a change in the form of preferences.

One concern with our benchmark preferences is that balanced growth considerations impose a very tight restriction between γ, one of the parameters that determine the labor supply elasticity, and σ, a parameter that determines risk aversion. If these parameters do not satisfy this restriction, then growth in the model is unbalanced. However, a commonly used class of preferences yields balanced growth with no such restrictions. A typical specification of such preferences is

$$U(c, l, M/P) = \left[\left(\omega c^{\frac{\eta-1}{\eta}} + (1-\omega)(M/P)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} (1-l)^{\xi}\right]^{1-\sigma} /(1-\sigma). \tag{19}$$

It is easy to verify that these preferences are consistent with balanced growth.

Unfortunately, they do not generate volatile exchange rates. We determine this by setting the parameters η, σ, and ω as in the benchmark model, but setting the new parameter $\xi = 2.25$, as is typical in the business cycle literature (for example, Chari, Christiano, and Kehoe (1991)). We display the resulting statistics in the columns labeled "Nonseparable Preferences" in Tables 5 and 6. Now real exchange rates vary hardly at all.

To understand this finding, recall that the real exchange rate is the ratio of the marginal utility of consumption in the two countries. In this model, monetary shocks lead to small changes in the marginal utility of consumption because movements in employment tend to offset the effects of movements in consumption. Here an increase in the money growth rate in the home country increases both consumption and employment in that country. The increase in consumption decreases the marginal utility of home consumption. With our nonseparable preferences and $\sigma > 1, U_{cl} > 0$, so that the increase in employment increases the marginal utility of home consumption and, hence, tends to offset the effects of marginal utility arising from consumption. This offsetting effect does not occur when preferences are separable in leisure since $U_{cl} = 0$.

To get a feel for the magnitude of the offsetting effects from employment, consider the expression for log-linearized real exchange rates

$$\hat{q} = A(\hat{c} - \hat{c}^*) + B(\hat{m} - \hat{m}^*) + D(\hat{l} - \hat{l}^*), \tag{20}$$

where the coefficients are $A = -cU_{cc}/U_c$, $B = -mU_{cm}/U_c$, and $D = -lU_{cl}/U_c$. In our quantitative model $A = 4.96$, $B = .04$, and $D = -3.02$. Since B is essentially zero, the fluctuations in real balances are quantitatively unimportant in determining the variance of real exchange rates. To gain intuition, suppose that B is exactly zero so that

$$\begin{array}{lcl} \dfrac{\operatorname{var} \hat{q}}{\operatorname{var} \hat{y}} & \cong & A^2 \dfrac{\operatorname{var}(\hat{c} - \hat{c}^*)}{\operatorname{var} \hat{y}} + D^2 \dfrac{\operatorname{var}(\hat{l} - \hat{l}^*)}{\operatorname{var} \hat{y}} + 2AD \dfrac{\operatorname{cov}(\hat{c} - \hat{c}^*, \hat{l} - \hat{l}^*)}{\operatorname{var} \hat{y}}, \\ & & \quad (21.16) \qquad\quad + \qquad\quad (20.98) \qquad\quad - \qquad\quad (42.24) \end{array} \tag{21}$$

where the numbers below the equation are the values of each of the three terms for the model economy with nonseparable preferences. Here we have used the values for the model's statistics given by $\operatorname{var}(\hat{c} - \hat{c}^*)/\operatorname{var} \hat{y} = .86$, $\operatorname{var}(\hat{l} - \hat{l}^*)/\operatorname{var} \hat{y} = 2.30$, and $\operatorname{cov}(\hat{c} - \hat{c}^*, \hat{l} - \hat{l}^*)/\operatorname{var} \hat{y} = 1.41$.

The numbers below (21) demonstrate the importance of the covariance between relative consumption and relative employment in the two countries. Notice that if D were equal to zero, then the variance of the real exchange relative to output would be simply the first term in (21), and the standard deviation relative to output would be 4.6 ($\approx (21.16)^{1/2}$). Thus, if it were not for the offsetting effects from employment, the model with nonseparable preferences could easily generate the volatility of exchange rates seen in the data.

Note that the offsetting effects from employment are higher in the model than in the data. In the data, $\operatorname{var}(\hat{c}-\hat{c}^*)/\operatorname{var}\hat{y} = .62$, $\operatorname{var}(\hat{l}-\hat{l}^*)/\operatorname{var}\hat{y} = .40$, and $\operatorname{cov}(\hat{c}-\hat{c}^*, \hat{l}-\hat{l}^*)/\operatorname{var}\hat{y} = .37$. Substituting these values into (21) and using $A = 4.96$ and $D = -3.02$ as above yields (std $\hat{q}$)/ (std $\hat{y}$) = 2.8. Thus, the main problem with nonseparable preferences is that the covariance between relative consumption and relative employment is much larger in the model than in the data (1.41 vs. .37). Of course, if the model could generate the type of comovements between relative consumption and relative employment in the data, it would also generate a substantial amount of the variability of real exchange rates in the data.

In our benchmark model, we assume that preferences are separable between consumption and leisure. Thus, $D = 0$, there are no offsetting effects from employment, and the model can generate the volatility of real exchange rates in the data if A is sufficiently large, regardless of the comovements between relative consumption and relative employment. Indeed, for the benchmark model, $\operatorname{cov}(\hat{c}-\hat{c}^*, \hat{l}-\hat{l}^*)/\operatorname{var}\hat{y} = 1.33$, so that the model misses the data in terms of this statistic, but with $D = 0$, that problem does not affect the volatility of real exchange rates.

C. *Real shocks*

So far, the only shocks in the model are monetary shocks. Now we add real shocks of two types: shocks to technology and to government consumption. Here we primarily want to examine whether adding these shocks improves the model's performance on business cycle statistics. As noted above, employment is too volatile in our model because variation in labor input is the primary source of variation in output at business cycle frequencies. Adding other shocks will add other sources of variation in output. Unfortunately, this change changes the model's predictions little.

We allow for country-specific technology shocks which are common across all intermediate goods producers. The technology for producing intermediate goods in the home country and foreign countries is now $F(k_t, A_t l_t)$ and $F(k_t^*, A_t^* l_t^*)$. Here the technology shocks A_t and A_t^* are common across all intermediate goods and follow a stochastic process given by

$\log A_{t+1} = \rho_A \log A_t + \varepsilon_{At+1}$ and $\log A^*_{t+1} = \rho_A \log A_t + \varepsilon^*_{At+1}$, where the technology innovations ε_A and ε^*_A have zero means, are serially uncorrelated, and are uncorrelated with other types of shocks. We follow Kehoe and Perri (2002) and use $\rho_A = .95$, $\mathrm{var}(\varepsilon_A) = \mathrm{var}(\varepsilon^*_A) = (.007)^2$, and $\mathrm{corr}(\varepsilon_A, \varepsilon^*_A) = .25$.

We add government consumption shocks as follows. The final good is now used for government consumption as well as private consumption and investment. The resource constraint for the home country is now

$$y_t = c_t + g_t + \int_0^1 x_t(i)\, di,$$

where home government consumption g_t follows a stochastic process $\log g_{t+1} = (1 - \rho_g)\mu_g + \rho_g \log g_t + \varepsilon_{gt+1}$. To obtain estimates for this autoregressive process, we ran a regression with data on real government purchases for the United States over the period 1947:1 through 1998:4. Our estimates from this regression are as follows: $\mu_g = .13$, $\rho_g = .97$, and $\mathrm{var}(\varepsilon_g) = (.01)^2$. We assume that the shock ε_g is serially uncorrelated and uncorrelated with shocks to money and technology or to the shock to government consumption in the foreign country. We model government consumption in the foreign country symmetrically.

In each period, first the technology and government consumption shocks occur, then prices are set, and then the monetary shock occurs. (Alternative timing assumptions lead to similar results except for the volatility of investment, which increases as we would expect.)

We report the results for this economy in the columns labeled "Real Shocks" in Tables 5 and 6. Again, most of the statistics change little. The main exception is that the relative volatility of investment increases to be closer to that in the data (from 1.59 in the benchmark model to 2.34 in the model with real shocks). This change is as we would expect and is driven by the addition of the technology shocks. More interesting is the relative volatility of employment, which actually increases slightly (from 1.51 in the benchmark model to 1.56 in the model with real shocks). To understand this finding, note that here the log-deviations in output are approximately given by $\hat{y} \cong (1-\alpha)(\hat{A} + \hat{l})$, so that

$$\frac{\mathrm{var}\, \hat{l}}{\mathrm{var}\, \hat{y}} \cong \frac{1}{(1-\alpha)^2} - \frac{\mathrm{var}\, \hat{A}}{\mathrm{var}\, \hat{y}} - \frac{2\mathrm{cov}(\hat{A}, \hat{l})}{\mathrm{var}\, \hat{y}}.$$

From this expression, we see that introducing technology shocks can increase the variability of employment if technology shocks and employment are sufficiently negatively correlated. In the model, a positive technology shock leads on impact to a fall in employment since firms can meet the same demand with fewer workers. This feature of the model makes technology shocks and employment negatively correlated enough to raise the relative volatility of employment. Government consumption shocks, meanwhile, have a quantitatively insignificant role.

D. *Taylor rule*

Finally, we examine what happens when we change our modeling of monetary policy. There is a lively debate over the most appropriate way to model monetary policy. A recently popular way to do so has been with an interest rate rule. Here we discuss how our money growth rule can be interpreted as an interest rate rule, and we describe the properties of our model economy under several interest rate rules that stem from the work of Taylor (1993). Unfortunately, this type of change moves the persistence of real exchange rates in the wrong direction.

Logically, any interest rate rule can be interpreted as a money growth rule and vice versa. To see this, posit an interest rate rule and work out the equilibrium of the economy. This equilibrium has a corresponding money growth process associated with it. Clearly, if this money growth process is viewed as the policy, then the equilibrium for this economy with this money growth is the same as that for an economy with the interest rate rule. Of course, if the economy has multiple equilibria under the interest rule, then each equilibrium has a different money growth process that implements it. The converse also holds. (Of course, such rules can be represented either as a function of both past endogenous variables and exogenous shocks or as a function of solely the history of exogenous shocks.) Moreover, empirical evidence supports our choice for the money growth rule. In particular, Christiano, Eichenbaum, and Evans (1998) have shown with vector autoregression analysis that a money growth process of the kind considered here is a good approximation to a process that implements their estimated interest rate rule.

As a practical matter, however, some simple interest rate rule might be a better approximation to the policy reflected in the data than is our simple money growth rule. Thus, we consider the implications of replacing our simple rule for money growth rates with an interest rate rule similar to the rules studied by Taylor (1993) and Clarida, Gali, and Gertler (2000).

In particular, we assume that nominal interest rates r_t are set as a function of lagged nominal rates, expected inflation rates, and output according to

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(\alpha_\pi E_t \pi_{t+1} + \alpha_y [\log\, gdp_t]/4\right) + \varepsilon_{rt}, \tag{22}$$

where we have dropped the constant and converted units to quarterly rates. In (22), π_{t+1} is the inflation rate from t to $t+1$, gdp_t is real gross domestic product at t, and ε_{rt} is a normally distributed, mean-zero shock. We set $\rho_r = .79$, $\alpha_\pi = 2.15$, and $\alpha_y = .93/4$. (The numbers are from Clarida, Gali, and Gertler (2000, Table II).) We choose the volatility of the shocks to match the volatility of output and the correlation of the home shock ε_{rt} and the foreign shock to match the cross-correlation of output.

When we use this Taylor rule in our benchmark model, we are unable to generate reasonable business cycle behavior. Briefly, for low values of the adjustment cost parameter, the relative volatility of consumption is tiny. For high values of the adjustment cost parameter, the relative volatility of consumption increases, but the correlation between consumption and output is negative.

On closer investigation, we find that these features of the model are driven by the non-separability of consumption and money balances. Since we do not view this nonseparability as a crucial feature of our model, we try again: we investigate a version of our model with the Taylor rule and with preferences of the form

$$\frac{c^{1-\sigma}}{1-\sigma} + \omega\frac{(M/P)^{1-\sigma}}{1-\sigma} + \psi(1-l)^{(1-\gamma)}/(1-\gamma).$$

We set the parameters σ, ψ, and γ as before. (The parameter ω is not relevant since money demand is determined residually.) In Tables 5 and 6, we report the results for this exercise in the columns labeled "Taylor Rule." This model moves the volatilities of the price ratio and the exchange rates closer to those in the data. Unfortunately, however, the model's nominal and real exchange rates, with autocorrelations of .46 and .48, are much less persistent than those in either the data or the benchmark model.

We also investigated the properties of the economy that results from a Taylor rule estimated by Rotemberg and Woodford (1997). Their estimated Taylor rule uses three lags of nominal interest rates and inflation together with current output and two of its lags. When we use this rule, we obtain essentially the same results as we did with the rule estimated by Clarida, Gali, and Gertler (2000). For example, the autocorrelations of nominal and real exchange rates are .40 and .43.

Nominal exchange rates are less persistent in our Taylor rule model than in the benchmark model because the endogenous policy reaction tends to offset the exogenous shocks. For example, a negative shock to interest rates in (22) raises the quantity of money and leads to a rise in inflation in subsequent periods. This rise in inflation leads to an endogenous increase in interest rates that offsets the initial shock. As a result, interest rates are not very persistent and, hence, neither are movements in consumption or real exchange rates. We confirmed this intuition by analyzing the properties of the model for higher values of ρ_r. For example, when we raised ρ_r from .79 to .95, the autocorrelations of nominal and real exchange rates increased from .46 and .48 to .63 and .60, while for $\rho_r = .99$, these autocorrelations increased even further, to .64 and .63.

6. ATTEMPTS TO ELIMINATE THE TWO ANOMALIES

Now we make some changes in the benchmark model to try to eliminate, or at least shrink, the two anomalies we have discovered between the model and the data. Unfortunately, we don't manage to affect either anomaly much.

A. *The persistence anomaly*

Recall that our benchmark model generates somewhat less persistence in real exchange rates than is present in the data. We attempt to eliminate this *persistence anomaly* by pursuing an avenue for increasing persistence that seems promising: adding labor frictions by making wages sticky. However, this change leads to only a marginal improvement in the benchmark model's persistence performance.

Our logic for using sticky wages to increase persistence is as follows. In the benchmark model, wages immediately rise after a monetary shock. This rise in wages leads intermediate goods producers to increase their prices as soon as they can. Thus, the benchmark model generates little endogenous price-stickiness, that is, price-stickiness beyond that exogenously imposed. Preset nominal wages cannot rise after a monetary shock. If they do not, intermediate goods producers may choose not to raise prices much when they can. Hence, the model may lead to some endogenous price-stickiness and, consequently, more persistence in exchange rates.

We extend the benchmark model to include sticky wages by letting labor be differentiated and introducing monopolistically competitive unions that set wages in a staggered way for M periods.

The final goods producers in the model remain as before, while the problems of the intermediate goods producers and the consumers are altered. The only change in technology is that the labor input $l(i, s^t)$ of intermediate goods producer i is now a composite of a continuum of differentiated labor inputs $j \in [0, 1]$ and is produced according to

$$l(i, s^t) = \left[\int l(i, j, s^t)^{\vartheta} \, dj \right]^{1/\vartheta}, \tag{23}$$

where $l(i, j, s^t)$ denotes the amount of differentiated labor input j used by intermediate goods producer i in period t and ϑ is the parameter determining the substitutability across different types of labor.

The problem of the intermediate goods producers is the same as before except that now they have a subproblem of determining the cost-minimizing composition of the different types of labor. The term $w(s^t)l(i,s^t)$ in the intermediate goods producers' problem (7) is now replaced by

$$w(s^t)l(i,s^t) = \min_{\{l(i,j,s^t)\},j\in[0,1]} \int \frac{W(j,s^{t-1})}{P(s^t)} \, l(i,j,s^t)\, dj \tag{24}$$

subject to (23), where $W(j,s^{t-1})$ is the nominal wage for the jth type of labor in period t and where the dependence of this wage on s^{t-1} reflects our timing assumption on the setting of wages discussed below. The solution to this problem is the demand for labor of type j by intermediate goods producer i, namely,

$$l(i,j,s^t) = \left(\frac{\bar{W}(s^t)}{W(j,s^{t-1})}\right)^{\frac{1}{1-\vartheta}} l(i,s^t),$$

where $\bar{W}(s^t) = \left[\int W(j,s^{t-1})^{\frac{\vartheta}{\vartheta-1}}\, dj\right]^{\frac{\vartheta-1}{\vartheta}}$ is the nominal wage index. Substitution of the demand for labor $l(i,j,s^t)$ into (24) implies that the real wage index is given by $w(s^t) = \bar{W}(s^t)/P(s^t)$.

The consumer side of the labor market can be thought of as being organized into a continuum of unions indexed by j. Each union consists of all the consumers in the economy with labor of type j. Each union realizes that it faces a downward-sloping demand for its type of labor. The total demand for labor of type j is obtained by integrating across the demand of the intermediate goods producers and is given by

$$l^d(j,s^t) = \left(\frac{\bar{W}(s^t)}{W(j,s^{t-1})}\right)^{\frac{1}{1-\vartheta}} \int l(i,s^t)\, di. \tag{25}$$

We assume that a fraction $1/M$ of unions set their wages in a given period and hold wages fixed for M subsequent periods. The unions are indexed so that those with $j \in [0,1/M]$ set new wages in 0, M, $2M$, and so on, while those with $j \in [1/M, 2/M]$ set new wages in 1, $M+1$, $2M+1$, and so on, for the M cohorts of unions. In each period, these new wages are set before the realization of the current monetary shocks. Notice that the wage-setting arrangement is analogous to the price-setting arrangement for intermediate goods producers.

The problem of the jth union for, say, $j \in [0,1/M]$ is to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\, U\left(c(j,s^t), l^d(j,s^t), M(j,s^t)/P(s^t)\right) \tag{26}$$

subject to the total labor demand schedule (25), the budget constraints

$$\begin{aligned} P(s^t)c(j,s^t) + M(j,s^t) + \textstyle\sum_{s_{t+1}} Q(s^{t+1}|s^t)B(j,s^{t+1}) \\ \leq W(j,s^{t-1})l^d(j,s^t) + M(j,s^{t-1}) + B(j,s^t) + \Pi(s^t) + T(s^t), \end{aligned} \tag{27}$$

and the constraints that wages are set for M periods, $W(j,s^{t-1}) = W(j,s^{-1})$ for $t = 0,\ldots,M-1$, and $W(j,s^{t-1}) = W(j,s^{M-1})$ for $t = M, \ldots, 2M-1$, and so on. We choose the initial bond holdings B of the unions so that each union has the same present discounted value of income.

In this problem, the union chooses the wage and agrees to supply whatever labor is demanded at that wage. The first-order conditions are changed from those in the benchmark economy as follows. The condition for the consumer's labor choice (10) is replaced by the condition for nominal wages

$$W(j,s^{t-1}) = -\frac{\sum_{\tau=t}^{t+M-1}\sum_{s^\tau} Q(s^\tau)P(s^\tau)U_l(j,s^\tau)/U_c(j,s^\tau)l^d(j,s^\tau)}{\vartheta\sum_{\tau=t}^{t+M-1}\sum_{s^\tau} Q(s^\tau)l^d(j,s^\tau)}. \tag{28}$$

Notice that in a steady state, this condition reduces to $W/P = (1/\vartheta)(-U_l/U_c)$, so that real wages are set as a markup over the marginal rate of substitution between labor and consumption. The conditions (11) and (12) are now indexed by j. These conditions, together with our assumption on initial bond holdings, imply that $U_c(j,s^t)$ and $U_m(j,s^t)$ are equated across unions.

The new parameters in the model are the number of periods of wage-setting M and the markup parameter ϑ. Following Taylor's (1999) discussion of the evidence, we set $M = 4$. We set $\vartheta = .87$ so that the markup is about 15%. This markup is consistent with estimates of the markup of union wages over nonunion wages. (See Lewis (1986).)

The results of this adjusted model are in Tables 5 and 6, in the columns labeled "Sticky Wages." There we see that the sticky wage model improves on the benchmark model only slightly. The sticky wage model decreases the volatility of the price ratio a bit (from 3.00 to 2.11) and increases the volatility of real exchange rates enough to make it match the data (from 4.27 to 4.35). The sticky wage model also slightly increases the persistence of real exchange rates (from .62 to .69) and the cross-correlation of real and nominal exchange rates (from .76 to .88). The business cycle statistics remain basically unchanged, except for the cross-correlations of real exchange rates with GDP and net exports with GDP, which worsen slightly.

B. *The consumption–real exchange rate anomaly*

Now we turn to the *consumption–real exchange rate anomaly,* which, recall, is that the model generates a much higher correlation between these two variables than is seen in the data. To attempt to eliminate this anomaly, we try two ways of changing the model. First we restrict the set of assets that can be traded across countries, thereby making markets incomplete. This avenue weakens the link between real exchange rates and relative consumption. Then we allow for habit persistence in preferences. We find that neither avenue is successful in eliminating the consumption–real exchange rate anomaly.

With incomplete markets, the simple static relationship between the real exchange rate and the ratio of the marginal utilities given in (18) is replaced by a relationship that holds only in expected first differences. Furthermore, with incomplete markets, monetary shocks can lead to wealth redistributions across countries and can increase the persistence of real exchange rates.

We introduce market incompleteness into the benchmark model by replacing the complete set of contingent bonds traded across countries by a single uncontingent nominal bond. This bond is denominated in units of the home currency. The home consumer's budget constraint is now

$$\begin{aligned} &P(s^t)c(s^t) + M(s^t) + \bar{Q}(s^t)D(s^t) \\ &\qquad \leq P(s^t)w(s^t)l(s^t) + M(s^{t-1}) + D(s^{t-1}) + \Pi(s^t) + T(s^t), \end{aligned} \tag{29}$$

where D is the consumer's bond holdings. The real value of these bonds $D(s^t)/P(s^t)$ is bounded below. Here each unit of $D(s^t)$ is a claim on one unit of the home currency in all states s^{t+1} that can occur at $t+1$, and $\bar{Q}(s^t)$ is the corresponding price. The foreign consumer's budget constraint is modified similarly.

The first-order condition for bond-holding in the home country is now given by

$$\bar{Q}(s^t) = \sum_{s_{t+1}} \beta\pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)} \frac{P(s^t)}{P(s^{t+1})}, \tag{30}$$

while that in the foreign country is given by

$$\bar{Q}(s^t) = \sum_{s_{t+1}} \beta\pi(s^{t+1}|s^t) \frac{U_c^*(s^{t+1})}{U_c^*(s^t)} \frac{e(s^t)}{e(s^{t+1})} \frac{P^*(s^t)}{P^*(s^{t+1})}. \tag{31}$$

Equating (30) and (31) and log-linearizing the resulting equation gives

$$E_t\left[\hat{U}_{ct+1} - \hat{U}_{ct} + \hat{P}_t - \hat{P}_{t+1}\right] = E_t\left[\hat{U}^*_{ct+1} - \hat{U}^*_{ct} + \hat{e}_t + \hat{P}^*_t - \hat{e}_{t+1} - \hat{P}^*_{t+1}\right], \tag{32}$$

where carets denote log-deviations from a steady state with $D = 0$. Noting that $\hat{q}_t = \hat{e}_t + \hat{P}^*_t - \hat{P}_t$, we can rewrite (32) as

$$E_t\left[\hat{q}_{t+1} - \hat{q}_t\right] = E_t\left[(\hat{U}^*_{ct+1} - \hat{U}_{ct+1}) - (\hat{U}^*_{ct} - \hat{U}_{ct})\right]. \tag{33}$$

Thus, with incomplete markets, the relation between real exchange rates and marginal utilities only holds in expected first differences, rather than as it did in (14) with complete markets.

In Tables 5 and 6, we report statistics for an incomplete market economy which has the same parameters as does the benchmark economy, but has the asset structure just discussed. In the columns labeled "Incomplete Markets," the statistics in both tables are virtually identical to those for the benchmark economy with complete markets. Thus, while adding incomplete markets theoretically could help eliminate the consumption–real exchange rate anomaly, quantitatively it does not.

So we try something else. In our benchmark model, the tight link between real exchange rates and marginal utilities arises because of consumers' abilities to trade in asset markets. This observation suggests that the consumption–real exchange rate anomaly might be reduced by adding specifications of utility functions used to analyze other asset market anomalies. One such specification has external habit persistence, in that lagged aggregate consumption enters each household's period utility function.

In our context, we can add habit persistence by replacing c in (15) with $c_t - d\bar{c}_{t-1}$, where $\bar{c}_{t-1}$ is lagged aggregate consumption and d is the habit persistence parameter. With this formulation and the equilibrium conditions that $c_t = \bar{c}_t$ and $c_t^* = \bar{c}_t^*$, (18) becomes

$$\hat{q}_t = \frac{A}{1-d}[(\hat{c}_t - \hat{c}_t^*) - d(\hat{c}_{t-1} - \hat{c}_{t-1}^*)] + B(\hat{m} - \hat{m}^*), \tag{34}$$

where $A = -cU_{cc}/U_c$ and $B = -mU_{cm}/U_c$. In what follows, we set B to 0 since it is small.

We can use the data to see whether the habit persistence approach is promising. Using H-P–filtered data for the United States and Europe, we compute the correlation between real exchange rates and the right side of (34). We experimented with values for d between -1 and 1 and found that this correlation attains its maximum value of $-.19$ at d arbitrarily close to 1. If the theory is correct, then this correlation will be 1. We conclude that this popular version of the habit persistence approach is not particularly promising.

Since none of our models can produce the negative correlation between real exchange rates and relative consumption in U.S.-European data, we ask whether this negative correlation is pervasive across different sets of countries. In Table 7, we report the correlation between bilateral real exchange rates and bilateral relative consumption for five countries (France, Germany, Italy, the United Kingdom, and the United States). This correlation varies between $-.48$ and .24, which suggests that there is no tight link in the data. Clearly, we need models with asset market frictions to break the tight link between real exchange rates and marginal utilities and thus between real exchange rates and relative consumption.

7. CONCLUSION

The central puzzle in international business cycles is that fluctuations in real exchange rates are volatile and persistent. Here we have taken a step toward solving that puzzle. We have developed a general equilibrium, sticky price model which can generate real exchange rates that are appropriately volatile and quite persistent, though not quite persistent enough. We have found that for monetary shocks to generate these data, the model needs to have preferences separable in leisure, high risk aversion, and price-stickiness of at least one year. We have also found that if monetary shocks are correlated across countries, then the comovements in aggregates across countries in the model are broadly consistent with those in the data.

We have seen that without substantial price-stickiness, real exchange rates are not persistent. We have assumed that prices are exogenously fixed for one year. While this assumption generates movements in prices that are consistent with the evidence of Taylor (1999), simply assuming that firms cannot change their prices for a year is somewhat unappealing. A major challenge in this line of research is to find a mechanism that generates substantial amounts of endogenous price-stickiness from small frictions. By this, we mean a mechanism that leads firms to optimally choose not to change prices much even when they can freely do so.

The main failing of our model is the consumption–real exchange rate anomaly: the model predicts a high correlation between the real exchange rate and relative consumption across countries while none exists in the data. We have shown that complete asset markets have a tight link between the real exchange rate and relative consumption which generates the anomaly. In particular, such frictions as sticky prices, sticky wages, and trading frictions in goods markets play no role in breaking this link. We have also shown that the most widely used forms of asset market incompleteness and habit persistence do not eliminate—or even shrink—the anomaly.

Essentially all of the models in the international business cycle literature, real and nominal, have either complete markets or the type of incomplete markets considered here. Our analysis suggests that all of these models will display the consumption–real exchange rate anomaly. To attempt to eliminate it, future research should focus on incorporating richer forms of asset market frictions.

Acknowledgments. This paper is a revised version of our 1996 paper "Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business Cycles." A technical appendix, computer codes, and the data used in this paper are available at http://minneapolis fed.org/research/sr/sr277.html. Chari, Kehoe, and McGrattan thank the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Notes

[1]In particular, our constructed index is $e_t = \sum_{i \in \mathcal{I}} \omega_i e_{it}/e_{i0}$, where e_{it} is the exchange rate for country i in period t, e_{i0} is the exchange rate for country i in the first quarter of 1973, and the weight ω_i is the time series average of the ratio of the dollar value of exports plus imports between country i and the United States to the total dollar value of all exports plus imports between the European countries included in set $\mathcal{I}$ and the United States. The countries (and their trade weights) included in our data set are Austria (1.2), Denmark (1.8), Finland (1.2), France (13.8), Germany (25.7), Italy (11.6), the Netherlands (10.5), Norway (2.2), Spain (2.7), Switzerland (5.8), and the United Kingdom (23.5).

[2]Our real exchange rate measure is substantially more volatile than another measure of the real exchange rate between the United States and the rest of the world. The IMF, in its *International Financial Statistics,* reports the effective real exchange rate for the United States, based on weights derived from the multilateral exchange rate model (MERM). For the period 1980:1–2000:1, this exchange rate has a standard deviation of 4.63 and an autocorrelation of .82. The MERM measure is less volatile than the measure we use presumably because shocks affecting bilateral exchange rates are not perfectly correlated across countries, and the MERM measure averages across more countries than our measure does.

[3]In this discussion, we ignore a third source of real exchange rate fluctuations. When there are multiple traded goods and consumption baskets differ across countries, fluctuations in the relative prices of traded goods can induce fluctuations in real exchange rates. In our data, this source is minor because European and U.S. consumption baskets are similar. This source is present in our theoretical model and turns out to be relatively small there as well.

[4]To derive our benchmark model's implications for growth paths, we suppress uncertainty and add equal rates of productivity growth to both the market and nonmarket sectors. Suppose that in the market sector, the technology for each intermediate goods producer is given by $F(k_t, z_t l_t)$, where z_t grows at a constant rate z. And in the spirit of Becker (1993), suppose that in the nonmarket sector, technical progress raises the productivity of time allocated to nonmarket activities, so that an input of $1 - l_t$ units of time outside the market produces $z_t(1 - l_t)$ units of leisure services. With our benchmark preferences, if c_t and m_t grow at the same rate as z_t and if l_t is a constant, then equilibrium wages equal $-\frac{U_{lt}}{U_{ct}} = \kappa \frac{(1+z)^{(1-\gamma)t}}{(1+z)^{-\sigma t}}$, where κ is a constant. Along a balanced growth path, wages grow at the same rate as z_t, so in order to have a balanced growth path, the economy must have $\sigma = \gamma$.

[5]Our series for the foreign country are aggregates for France, Germany, Italy, and the United Kingdom obtained from the OECD. Our choices of countries and time period are

dictated by data availability. We convert these series into dollars using the OECD's 1990 purchasing power parity exchange rate and add the results to obtain our aggregates for Europe.

[6] If we keep the adjustment cost parameter unchanged, then as we increase σ, the relative volatility of consumption and output decreases somewhat. Hence, the volatility of the real exchange rate increases with σ, but at a somewhat slower rate. For example, with b held fixed, the volatilities of the real exchange rate at $\sigma = 1.01$ and 10 are 1.21 and 6.39, while with b adjusted, these volatilities are .87 and 8.75.

[7] Note that, across countries, there is greater heterogeneity in the correlations between real exchange rates and various aggregates—like output, net exports, and relative consumption—than for other statistics—like the volatility and persistence of real exchange rates or the cross-correlation of real and nominal exchange rates.

[8] One extension that might help sticky price models in this dimension is to have cyclical variations in the intensity that measured capital and labor are worked.

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