QUANTIFYING EFFICIENT TAX REFORM

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DECEMBER 2020
Question

- How large are welfare gains from efficient tax reform?
  - Baseline:
    - Positive economy matched to administrative data
  - Reform:
    - Pareto improvements on efficient frontier (full)
    - Optima given set of policy tools (restricted)
Idea in a Picture
Idea in a Picture

- Start with baseline OLG economy:
  - Incomplete markets
  - Heterogeneous households
    - Differing in education levels by individual
    - Facing productivity, marital, unemployment risks
    - Deciding on consumption, saving, hours
  - Technology parameters and tax policies

- Compute remaining lifetime utilities ($v_j$)
Idea in a Picture

- Start with baseline OLG economy:
  - Incomplete markets
  - Heterogeneous households
    - Differing in education levels by individual
    - Facing productivity, marital, unemployment risks
    - Deciding on consumption, saving, hours
  - Technology parameters and tax policies
- Compute remaining lifetime utilities ($v_j$)
- Let’s draw this for 2 households...
Idea in a Picture

Value for Household B, $v^B$

Value for Household A, $v^A$

Positive Economy
Idea in a Picture

- Typical starting point for most analyses
  - With constraints on policy instruments
  - Do counterfactuals or restricted optimal ("Ramsey")

- Let’s draw this in the picture
Idea in a Picture

Value for Household B, $v^B$

Value for Household A, $v^A$

Positive Economy
Idea in a Picture

- Not typical starting point for studies in Mirrlees tradition
  - With constraints on information sets
  - Characterize efficient allocations and policy “wedges”

- Let’s draw this in the picture
Idea in a Picture

Value for Household B, $v^B$

Value for Household A, $v^A$

Efficient Frontier
• This paper quantifies gains from:
  ○ Full Pareto-improving reform a la Mirrlees
  ○ Partial Pareto-improving reform a la Ramsey
  ○ Adding early-life transfer informed by Mirrlees

• Let’s draw this in the picture
Idea in a Picture

- Efficient Frontier
- Pareto Improving
- Positive Economy
Idea in a Picture

Value for Household B, $v^B$

Value for Household A, $v^A$

Efficient Frontier

Pareto Improving

Positive Economy
Idea in a Picture

Value for Household B, $v^B$

Value for Household A, $v^A$

Efficient Frontier

Pareto Improving

Restricted Optimal

Positive Economy
Our Approach

- Solve equilibrium for positive economy (●)
  - Inputs: fiscal policy and wage processes
  - Outputs: values under current policy

- Solve planner problem next (●)
  - Inputs: values under current policy
  - Outputs: labor and savings wedges and welfare gains

- Use results to inform current policy and reforms (●)
Main Findings (●→●)

- Maximum consumption equivalent gains (future cohorts):
  - 21% starting at age 25
  - Comparisons made to utilitarian planner

- Decompose by comparing allocations:
  - Consumption: level ↑ and variance ↓ for all groups
  - Leisure: level ↓ and variance ↑ for all groups

*Note:* Currently computing transitions
Main Findings (●→●)

- Informed by comparison of baseline (●) and full reform (●)
  - Most gains in lifting consumption levels for young
  ⇒ Exploring early-life transfers

*Note:* Computer is still hillclimbing
Contributions to Literature

• Theory and application of income tax design (●→)
  ⇒ Using administrative data from NL, go to (●)

• Pareto-improving reforms with fixed types
  Hosseini-Shourideh (2019)
  ⇒ Extend analysis to add dynamic risks

• Theory behind dynamic taxation and redistribution (●)
  ⇒ Link OLG (●) to planner (●) in full GE
Positive Economy
Positive Economy (●)

• Open OLG economy a la Bewley

• Household heterogeneity in:
  ○ Age
  ○ Education (observed, permanent)
  ○ Productivity (private, stochastic)
  ○ Marital risk
  ○ Divorce risk (in progress)
  ○ Unemployment risk (in progress)

• Transfers and taxes on consumption, labor income, assets
Positive Economy ($\bullet$)

- Household problem

$$v_j(a, \epsilon; \Omega) = \max_{c,n,a'} \left\{ U(c, \ell) + \beta E[v_{j+1}(a', \epsilon'; \Omega)|\epsilon] \right\}$$

s.t. $a' = (1 + r)a - T_a(ra) + w\epsilon n - T_n(j, w\epsilon n) - (1 + \tau_c)c$

where

- $j = \text{age}$
- $a = \text{financial assets}$
- $\epsilon = \text{productivity shock}$
- $\Omega = \text{factor prices and tax policies}$
- $c = \text{consumption}$
- $n = \text{labor supply} (n + \ell = 1)$
Positive Economy (●)

- Firms:
  - Technology: $F(K, N) = K^\alpha N^{1-\alpha}$
  - Prices: $r, w$ set internationally

- Government:
  - Taxes: consumption, incomes, assets
  - Borrows: at home and abroad
In Equilibrium

- Add it up:

\[ C_t + I_t + G_t + B_{t+1} = F(K_t, N_t) + RB_t \]

\[ \lim_{T \to \infty} \frac{1}{R^{T-1}}(B_T + K_T) \geq 0 \]

- Then use answers as inputs into planner’s problem
Data from Netherlands

- Merged administrative data, 2006-2014
  - Earnings from tax authority
  - Hours from employer provided data
  - Education from population survey

- National accounts

- Tax schedules

⇒ Big data advantage for estimating elasticities & shocks
Estimation of Wage Processes

- Construct hourly wages $W_{ijt}$ ($j=$age, $t=$time)

- Classify degrees:
  - High school or practical (Low)
  - University of applied sciences (Medium)
  - University (High)

- Construct residual wages $\omega_{ijt}$:
  - $\log W_{ijt} = A_t + X_{ijt} + \omega_{ijt}$
  - Estimate AR(1) process for idiosyncratic risk
Marriage and Household Structure

• In period 0, individuals are single
  ◦ Different by education (L,M,H)

• After that, individuals either
  ◦ Form a couple (LL,LM,LH,MM,MH,HH) or
  ◦ Remain single (included with LL,MM,HH)

*Note: Working on adding divorce risk*
Wage Profiles

Normalized wage profile vs Age for different categories:

- HH
- MH
- MM
- LH
- LM
- LL

Age ranges from 25 to 60 on the x-axis.
## Wage Process Estimates

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<td>High, High</td>
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</table>
Income and Asset Tax Schedules

![Income Tax Schedule](image1)

![Financial Asset Tax Schedule](image2)
Reform Problem
Reform Problem (•)

• Take inputs from positive economy:
  ◦ Parameters for preferences and technologies
  ◦ Wage profiles and shock processes
  ◦ Values under current policy \((v_A, v_B, \ldots)\)

• Compute maximum consumption equivalent gain
Notion of Efficiency

• Our focus is Pareto-improving reforms:
  ◦ There is no alternative allocation that is
    - Resource feasible
    - Incentive feasible
  ◦ Making all better off and some strictly better off

• Will report gain assuming same percentage for all
Pareto-improving Reforms

Value for Household B, $v^B$

Value for Household A, $v^A$

Efficient Frontier

Positive Economy

Pareto Improving
Pareto-improving Reforms

How to construct this?

Value for Household B, $v^B$

Value for Household A, $v^A$

Efficient Frontier

Pareto Improving

Positive Economy
Planner Problem in Words (Primal)

- Maximize weighted sum of lifetime utilities

- subject to
  - Incentive constraints for every household and history
  - Resource constraints
Planner Problem in Words (Primal)

• Maximize weighted sum of lifetime utilities

• subject to
  ○ Incentive constraints for every household and history
  ○ Resource constraints

• Computationally easier to solve dual problem
Planner Problem in Words (Dual)

- Maximize present value of aggregate resources

- subject to
  - Incentive constraints for every household and history
  - Value delivered exceeds that of positive economy
Planner Problem in Math (Dual)

\[
\max \sum_h \pi_0(h) \Pi_0(V^h, -, \epsilon)
\]

subject to

- Incentive constraints for all \( h \)
- \( V^h \geq \vartheta^h \) for all \( h \)
Planner Problem in Math (Dual)

\[
\max \sum_h \pi_0(h) \Pi_0(V^h, -, \epsilon)
\]

subject to

- Incentive constraints for all \( h \)

- \( V^h \geq \vartheta^h \) for all \( h \)

\[\Rightarrow\] Exploit separability to solve household by household
Planner Problem in Practice

- Exploit separability to solve household by household

- Include only local downward incentive constraints
  - Verify numerically that constraints are satisfied

- Solve recursively by introducing additional states
  - Promised value for truth telling
  - Threat value for local lie
Planner Problem in Practice

• Exploit separability to solve household by household

• Include only local downward incentive constraints
  ◦ Verify numerically that constraints are satisfied

• Solve recursively by introducing additional states
  ◦ Promised value for truth telling ($V$)
  ◦ Threat value for local lie ($\tilde{V}$)
An Aside

- Government:
  - Can *ex-post* infer type from choices
  - Can’t *ex-ante* observe type

- But, can design policy to *induce* truthful reporting of type
Planner Problem for a Household
Planner Problem for a Household

Max present value of resources
Planner Problem for a Household

\[ \Pi_j (V, \tilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j (\epsilon_i | \epsilon) \left[ w \epsilon_i n_j (\epsilon_i) - c_j (\epsilon_i) + \text{future value} \right] \]

As in positive economy,

- \( j = \text{age} \)
- \( \epsilon = \text{productivity shock} \)
- \( c = \text{consumption} \)
- \( n = \text{labor supply} \)
Planner Problem for a Household

\[ \Pi_j(V, \tilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[ w \epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) \right. \]

\[ \left. + \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_{i+1}), \epsilon_i) / R \right] \]

Additionally, planner chooses

- \( V_j = \) promise value
- \( \tilde{V}_j = \) threat value
Planner Problem for a Household

\[ \Pi_j(V, \tilde{V}, \varepsilon) \equiv \max \sum_{\varepsilon_i} \pi_j(\varepsilon_i | \varepsilon) [w\varepsilon_i n_j(\varepsilon_i) - c_j(\varepsilon_i) \]

\[ + \Pi_{j+1}(V_j(\varepsilon_i), \tilde{V}_j(\varepsilon_{i+1}), \varepsilon_i)/R] \]

s.t.  Local downward incentive constraints
Planner Problem for a Household

\[
\Pi_j(V, \tilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[ w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) + \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_{i+1}), \epsilon_i)/R \right]
\]

s.t. \quad U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)

\[\geq U(c_j(\epsilon_{i-1}), \ell_j^+(\epsilon_{i-1})) + \beta \tilde{V}_j(\epsilon_i), \quad i \geq 2\]

where \(\ell_j^+(\epsilon_{i-1}) = 1 - n_j(\epsilon_{i-1})\epsilon_{i-1}/\epsilon_i\)
Planner Problem for a Household

\[ \Pi_j(V, \tilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[ w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) \right] \\
+ \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_{i+1}), \epsilon_i)/R \]

s.t. \[ U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i) \]

\[ \geq U(c_j(\epsilon_{i-1}), \ell_j^+(\epsilon_{i-1})) + \beta \tilde{V}_j(\epsilon_i), \ i \geq 2 \]

Deliver at least the promised value
Planer Problem for a Household

\[ \Pi_j(V, \tilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) [w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) \]
\[ + \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_{i+1}), \epsilon_i)/R] \]

s.t. \[ U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i) \]
\[ \geq U(c_j(\epsilon_{i-1}), \ell_j(\epsilon_{i-1})) + \beta \tilde{V}_j(\epsilon_i), \ i \geq 2 \]
\[ V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) [U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)] \]
Planner Problem for a Household

\[ \Pi_j(V, \tilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[ w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) \right. \\
\quad \left. + \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_i+1), \epsilon_i)/R \right] \]

s.t. \[ U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i) \]

\[ \geq U(c_j(\epsilon_{i-1}), \ell_j^+(\epsilon_{i-1})) + \beta \tilde{V}_j(\epsilon_i), \ i \geq 2 \]

\[ V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[ U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i) \right] \]

Deliver no more than the threat value
Planner Problem for a Household

\[ \Pi_j(V, \tilde{V}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[ w\epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i) \right. \]

\[ \left. + \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_{i+1}), \epsilon_i) / R \right] \]

s.t. \[ U(c_j(\epsilon_i), l_j(\epsilon_i)) + \beta V_j(\epsilon_i) \]

\[ \geq U(c_j(\epsilon_{i-1}), l_j^+(\epsilon_{i-1})) + \beta \tilde{V}_j(\epsilon_i), \ i \geq 2 \]

\[ V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) \left[ U(c_j(\epsilon_i), l_j(\epsilon_i)) + \beta V_j(\epsilon_i) \right] \]

\[ \tilde{V} \geq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon^+) \left[ U(c_j(\epsilon_i), l_j(\epsilon_i)) + \beta V_j(\epsilon_i) \right] \]
Planner Problem for Future Generation ($j = 1$)

$$\Pi_j(V, -, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i | \epsilon) \left[ w_{\epsilon_i} n_j(\epsilon_i) - c_j(\epsilon_i) + \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_{i+1}), \epsilon_i) / R \right]$$

s.t.  $U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)$

$$\geq U(c_j(\epsilon_{i-1}), \ell_j^+(\epsilon_{i-1})) + \beta \tilde{V}_j(\epsilon_i), \ i \geq 2$$

$$V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i | \epsilon) \left[ U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i) \right]$$

No threat value
Planner Problem for Future Generation \((j = 1)\)

\[
\Pi_j(V, -, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) [w_\epsilon n_j(\epsilon_i) - c_j(\epsilon_i) \\
+ \Pi_{j+1}(V_j(\epsilon_i), \tilde{V}_j(\epsilon_{i+1}), \epsilon_i)/R]
\]

s.t. \[
U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i) \\
\geq U(c_j(\epsilon_{i-1}), \ell_j^+(\epsilon_{i-1})) + \beta \tilde{V}_j(\epsilon_i), \ i \geq 2
\]

\[
V \leq \sum_{\epsilon_i} \pi_j(\epsilon_i|\epsilon) [U(c_j(\epsilon_i), \ell_j(\epsilon_i)) + \beta V_j(\epsilon_i)]
\]

Replace arbitrary \(V\) with \(\vartheta(\epsilon_0) + \vartheta_\Delta\)
General Equilibrium

• Solve planner problem for positive economy values

• Evaluate resource constraints

\[ C_t + I_t + G_t + B_{t+1} = F(K_t, N_t) + RB_t \]

\[ \lim_{T \to \infty} \frac{1}{R^{T-1}} (B_T + K_T) \geq 0 \]

• Increase \( \vartheta_\Delta \) until resources exhausted
Pareto-improving Reforms

Value for Household A, $v^A$

Value for Household B, $v^B$

Efficient Frontier

Pareto Improving

Positive Economy
Pareto-improving Reforms

Efficient Frontier

Pareto Improving

Positive Economy

Value for Household A, $v^A$

Value for Household B, $v^B$
Putting this on the computer...
Next Quantitative Steps

1. Quantify efficient reform (●→●)

2. Use answer to inform restricted reform (●→●)
Other Key Parameters

- Number of productivity types
- Preferences
- Status quo policy

Baseline: 20 types, log preferences, NL wages & policy
Quantitative Deliverables

- Welfare gains
  - Total consumption equivalent ($\nu_\Delta$)
  - Decomposition

- Wedges
Wedges

- Labor wedge:
  \[
  \tau_n(\epsilon^j) = 1 - \frac{1}{w} \frac{U_\ell(c(\epsilon^j), \ell(\epsilon^j))}{U_c(c(\epsilon^j), \ell(\epsilon^j))}
  \]

- Savings wedge:
  \[
  \tau_a(\epsilon^j) = 1 - \frac{U_c(c(\epsilon^j), \ell(\epsilon^j))}{\beta RE[U_c(c(\epsilon^j+1), \ell(\epsilon^j+1))|\epsilon^j]}\]
Wedges

- Labor wedge:

\[ \tau_n(\epsilon^j) = 1 - \frac{1}{w} \frac{U_\ell(c(\epsilon^j), \ell(\epsilon^j))}{U_c(c(\epsilon^j), \ell(\epsilon^j))} \]

- Savings wedge:

\[ \tau_a(\epsilon^j) = 1 - \frac{U_c(c(\epsilon^j), \ell(\epsilon^j))}{\beta R E[U_c(c(\epsilon^j+1), \ell(\epsilon^j+1)|\epsilon_j]} \]

\[ \Rightarrow \] Hopefully informative for reforming current policy
Results
Labor Wedges

![Graph showing labor wedges across different age groups.](image)

- HH
- MH
- MM
- LH
- LM
- LL

Labor wedge (%) vs Age
Labor Wedges

Increasing with wage variance as planner trades off increasing distortions with more insurance

Increasing with age since the opportunity to promise future payoffs decreases
What We Learn

- Wedges are suggestive of
  - Informational frictions
  - Insurance needs

- But,
  - Wedges are not taxes
  - Averages mask significant variation
Labor Wedges for LL, HH

Note: 25th and 75th percentiles shown
Welfare, (●) vs (●)

- Consumption equivalent gain of 21% for future cohorts

- Large but maybe not surprising given:
  - Tax rates in NL over 40%
  - Tax wedges of planner in 4% to 20% range
Welfare, (●) vs (●)

- Consumption equivalent gain of 21% for future cohorts

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Welfare, (●) vs ( ○)

- Consumption equivalent gain of 21% for future cohorts

- Large but maybe not surprising given:
  - Tax rates in NL over 40%
  - Tax wedges of planner in 4% to 20% range

- What are the implied Pareto weights?
Recall: could also have solved:

- $\max \sum_i \pi_i \omega_i V^i$

- subject to incentive and incentive constraints

Note: $\omega_i > 1 \Rightarrow$ overweight $i$ relative to population share
Implied Pareto Weights

- Recall: could also have solved:
  - $\max \sum_i \pi_i \omega_i V^i$
  - subject to incentive and incentive constraints

- What are the implied $\omega_i$’s for L,M,H?
## Pareto Weights and Welfare Gains

| Education | Equal Gains | | Equal Weights | |  
|-----------|-------------| | | | |
|           | $\omega_i$ | $\Delta_i$ | $\omega_i$ | $\Delta_i$ | |
| Low       | 0.8         | 21         |            |  | |
| Medium    | 1.0         | 21         |            |  | |
| High      | 1.2         | 21         |            |  | |
### Pareto Weights and Welfare Gains

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<th>Equal Gains</th>
<th>Equal Weights†</th>
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<td>High</td>
<td>1.2</td>
<td>21</td>
</tr>
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† Utilitarian planner with $V^H \geq V^M \geq V^L$
Comparing Allocations, (●) vs (●)

- Consumption: level ↑ and variance ↓ for all groups
- Leisure: level ↓ and variance ↑ for all groups
- Intuition from simple static model:
  - No insurance: $c$ varies, $\ell$ constant
  - Full insurance: $c$ constant, $\ell$ varies
- What about magnitudes?
A Look Under the Hood: Group LL

Levels of Consumption

Variance of Log Consumption

Levels of Leisure

Variance of Log Leisure
A Look Under the Hood: Group LL
Informing Counterfactuals (●)

Value for Household B, \( v^B \) vs. Value for Household A, \( v^A \)

- Efficient Frontier
- Pareto Improving
- Restricted Optimal
- Positive Economy
Informing Counterfactuals (●)

- Results of planner problem suggest large gains to
  - Lower average marginal tax rates
  - Early life transfers
  - Income-tested transfers

Note: our results on restricted gains still tentative
• Points to certain:
  o Early life transfers
  o Income-tested transfers
Summary

• Ultimate deliverables of project:
  ◦ Estimates of gains for efficient reform
  ◦ Identification of sources of gains
  ◦ Ideas for new policy instruments
  ◦ Prototype for future analyses

• Stay tuned...
Appendix Slides
Comparison to Static Problem

![Graph showing wage income and labor wedge for Saez and Mankiw models.](image)

- **Labor wedge (%)**
  - X-axis: Wage income (thous.)
  - Y-axis: Labor wedge (%)

- **Saez**
- **Mankiw**
Comparison to Static Problem

![Graph showing labor wedge (%) against wage income (thous.).]
Comparison to Static Problem

![Graph showing labor wedge as a function of wage income. The graph has two axes: the x-axis represents wage income in thousands, ranging from 0 to 200, and the y-axis represents the labor wedge as a percentage, ranging from 0 to 80. There are four lines corresponding to different sources: Saez, Mankiw, HH, and LL. The graph illustrates the comparison between these sources over a range of wage incomes.]
Comparison to Static Problem

![Graph comparing labor wedge across different wage income levels. The graph shows three lines representing different models: Saez, Mankiw, and Aggregate. The x-axis is labeled Wage income (thous.), and the y-axis is labeled Labor wedge (%).]
Scaled Planner Problem ($\beta_j = 1 + \beta + \ldots + \beta^{J-j}$)

\[
\hat{\Pi}_j(\hat{V}, \hat{\tilde{V}}, \epsilon) \equiv \max \sum_{\epsilon_i} \pi_j(\epsilon_i | \epsilon) \left( \frac{1}{\beta_j} (w \epsilon_i n_j(\epsilon_i) - c_j(\epsilon_i)) + \frac{\beta_{j+1}}{\beta_j} \hat{\Pi}_{j+1}(\hat{V}_j(\epsilon_i), \hat{\tilde{V}}_j(\epsilon_i+1), \epsilon_i) / R \right)
\]

s.t. \[ U(c_j(\epsilon_i), l_j(\epsilon_i)) + \beta_{j+1} \hat{V}_j(\epsilon_i) \]

\[ \geq U(c_j(\epsilon_{i-1}), l_j^+ (\epsilon_{i-1})) + \beta_{j+1} \hat{V}_j(\epsilon_i), \ i \geq 2 \]

\[
\hat{V} \leq \sum_{\epsilon_i} \pi_j(\epsilon_i | \epsilon) \left[ \frac{1}{\beta_j} U(c_j(\epsilon_i), l_j(\epsilon_i)) + \beta \frac{\beta_{j+1}}{\beta_j} \hat{V}_j(\epsilon_i) \right]
\]

\[
\hat{\tilde{V}} \geq \sum_{\epsilon_i} \pi_j(\epsilon_i | \epsilon^+) \left[ \frac{1}{\beta_j} U(c_j(\epsilon_i), l_j(\epsilon_i)) + \beta \frac{\beta_{j+1}}{\beta_j} \hat{V}_j(\epsilon_i) \right]
\]
Modify State Space

- Map to multiplier grid

- Envelope conditions

\[-\nu_j = \hat{\Pi}_{j,1}(\hat{V}_-, \hat{V}_-, \epsilon_-)\]
\[\mu_j = \hat{\Pi}_{j,2}(\hat{V}_-, \hat{V}_-, \epsilon_-)\]

- \(\hat{V}_-\) and \(\hat{V}_-\) residually determined by FOC
Optimality Conditions and Unknowns \((5I - 2)\)

\[
\pi_j(\epsilon_i | \epsilon_-) = \left( \nu_j \pi_j(\epsilon_i | \epsilon_-) + q_j(\epsilon_i) - \mu_j \pi_j(\epsilon_i | \epsilon_+^+) \right) u_c(c_j(\epsilon_i)) - q_j(\epsilon_{i+1}) u_c(c_j(\epsilon_i))
\]

\[
w\pi_j(\epsilon_i | \epsilon_-) = \left( \nu_j \pi_j(\epsilon_i | \epsilon_-) + q_j(\epsilon_i) - \mu_j \pi_j(\epsilon_i | \epsilon_+^+) \right) \frac{v_l(l_j(\epsilon_i))}{\epsilon_i} - q_j(\epsilon_{i+1}) \frac{v_l(l_j^+(\epsilon_i))}{\epsilon_{i+1}}
\]

\[
\nu_{j+1}(\epsilon_i) = \beta R(\nu_j \pi_j(\epsilon_i | \epsilon_-) - \mu_j \pi_j(\epsilon_i | \epsilon_+^+) + q_j(\epsilon_i)) / \pi_j(\epsilon_i | \epsilon_-)
\]

\[
\mu_{j+1}(\epsilon_i) = \beta R q_j(\epsilon_{i+1}) / \pi_j(\epsilon_i | \epsilon_-)
\]

and the incentive constraints
Newton-Raphson Algorithm

- Guess consumption $\{c_i\}_1^{I-1}$

- Optimality condition $\{c_i\} \rightarrow c_N, \{q_i\}$

- Optimality condition $\{\hat{V}_j(\epsilon_i), \hat{\nu}_j(\epsilon_i)\} \rightarrow \nu_i, \mu_i$

- Optimality condition $y_I$ and incentive constraints $\rightarrow \{y_i\}$

- Residual equations are optimality conditions $\{y_i\}_1^{I-1}$

Observe guess in terms of consumption and parallelizable