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# Is the Stock Market Overvalued?*

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## 1. Introduction

The value of the stock market relative to income has nearly doubled since 1994. In the first quarter of 2000, the stock market value was 1.8 times U.S. gross national product (GNP).[1,2] This number is high by historical standards. The previous post World War II peak of the stock market value relative to GNP is 1, and this peak occurred in 1968. For the 1946-1999 period as a whole, the value of all U.S. corporations averaged only .67 GNPs, so the current value is two and a half times the average of this historical period.

Stocks in the Standard and Poor's (S&P) 500 index earned an 8.4 percent (compounded) annual return over the period 1946-1999. If stock returns are to remain as high as they have been historically, then the large increases in the stock market value must be accompanied by large increases in corporate earnings. But corporate earnings have not kept pace with stock prices. As a result, the ratio of U.S. corporate earnings to the value of U.S. corporations has fallen significantly from its 1946-1999 average of 9.2 percent down to its current level of 4 percent.

These observations raise the question addressed in this article, Is the stock market overvalued? Should Federal Reserve Chairman Alan Greenspan be concerned that the U.S. stock market is or is becoming overvalued? Should people readjust their retirement portfolios to include more debt and less equity?

In this article we use aggregate economic theory to answer these questions. One implication of theory is that the value of corporate equity should be equal to the value of productive assets in the corporate sector, including factories, machines, land, and inventories.[3] For the 1946-99 period, the value of corporate equity was .67 GNPs while the value of tangible productive assets in the United States was a little over 1 GNPs. Historically then, the value of corporate equity has been lower than theory predicts.[4]

In fact the undervaluation of the stock market during this period is more extreme than these numbers suggest. This is the case because tangible assets in the United States are only part of corporate productive assets. Many U.S. corporations have foreign subsidiaries with capital abroad. Corporations also have intangible productive assets that are the results of past investments, which are expensed. Examples of such investments include research and development expenditures, firm specific learning by doing, and resources allocated to improving organization procedures.

The principle finding of this study is that the stock market is not currently overvalued. We estimate the current value of corporate productive assets to be approximately 1.75 GNPs. The current stock market value is 1.8 GNPs. Thus, the stock market is valued just as theory says it should be. Chairman Greenspan should not be concerned with the current situation.

What about average investors who care about the returns that they realize on their portfolios? To address this, we need another relation from theory, namely that, on average, real stock returns should be equal to the ratio of corporate earnings to the value of corporate equities plus any premium for bearing nondiversifiable risk.[5] If the value of the stock market is equal to the value of productive assets, it follows trivially that the ratio of earnings to the value of stocks is equal to the ratio of earnings to the value of productive assets – which in turn is the real return on productive assets like factories and machines. For the 1946-1999 historical period, real stock return was 8.4 percent per year when compounded, 9.8 percent per year when arithmetically averaged. The average earnings-stock value ratio was close at 9.2 percent. In sharp contrast, the return on real assets was a little over 4 percent. Thus, stocks earned a 5.2 percent premium over real assets.

If our theory is right, someone investing in stocks today and holding for an extended period should expect a real annual return of a little over 4 percent, far less than the 8.4 percent return earned by stockholders for the 1946-1999 period. It is far less because a 8.4 percent compounded return delivers an 11.2-fold increase in the real value of the asset after 30 years while a 4 percent return delivers only a 3.2-fold increase. Thus, if we are right, then the golden age of investing in the stock market is no more.

Should investors switch from equity to debt? Historically, bondholders earned much lower returns than stockholders. Short-term bonds, which are virtually risk-free, earned 0.6 percent per year in real terms over the period 1946-1999. This return is far lower than the 4-plus return on productive assets. After 30 years, they deliver only a 1.2-fold increase in the real value of the asset. Using theory to determine the real return on risk-free debt, we find that this return should be almost 4 percent. At the beginning of year 2000, the real return on risk-free debt, in particular the real return on 5-year inflation-protected Treasury securities was roughly 4 percent. These observations lead us to the conclusion that the return on risk-free debt was too low during the post war period. Now the return is on risk-free debt is near what theory predicts.

But equity should be paid a premium given its risk characteristics. We should find that average returns on equity are higher than average returns on the inflation-protected securities. However, our prediction for the equity premium is small, less than .1 percent. In our opinion, investors using historical returns to make predictions for their retirement portfolios should rethink this procedure in light of the current financial data.

Our approach to valuing equity differs from the traditional finance approach of estimating dividends of traded corporations and then computing the present value of these dividends using estimates of market discount factors. The finance approach is the one used in Jagannathan and McGrattan (this issue). In this article we take a complimentary approach that permits us to make use of national income and product data. An innovative feature of this study is that we use these data to estimate the average return on all productive assets in the economy. Another feature of our study is that we make use not only of knowledge of people's willingness to substitute between consumption today and consumption in the future (that is, people's preferences), but also of knowledge of people's ability to transform current consumption into future consumption (that is, investment opportunities). Using established aggregate theory, that is our empirical knowledge about the ability and willingness of people to substitute along with market clearing, we find that the total value of equity should be about 1.8 GNPs, as it currently is. There would be cause for concern only if the value of the stock market were higher relative to income than its current value.

## 2. Theory and the Value of the U.S. Stock Market

We turn now to our main application: the U.S. stock market. Using standard growth theory, we estimate that the value of U.S. domestic corporations is roughly where it should be. We also estimate the return on risk-free debt to be slightly above four percent and the premium for holding corporate equities to be negligible. In this section, we describe the model and data that we use to reach these conclusions.

We refer readers who are unfamiliar with the historical facts to Appendix A. There we describe historical asset values and returns in detail and use evidence on asset holdings to motivate some key assumptions in our model economy. For readers unfamiliar with the basic concepts underlying standard asset-pricing we include a primer in Appendix B. There we emphasize the importance of both preferences and technologies for asset-pricing.

### A. The Model

The model that we use is a modified version of the real business cycle model of Kydland and Prescott (1982). Here, we include two sectors, a corporate sector and a noncorporate sector. Since our main focus is on the value of domestic corporations, we assume that output from our corporate sector is the gross domestic product of corporations located in the United States. The output of the noncorporate sector is the remaining product of U.S. GNP. Therefore, our noncorporate sector includes the household sector, the government sector, the noncorporate business sector, and the rest of the world.

***Willingness to Substitute***

Our model economy is inhabited by infinitely-lived households with preferences ordered by the expected value of

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t\, \ell_t^{\psi})^{1-\sigma}}{1-\sigma}\right] N_t \tag{1}$$

where $t$ indexes time, $c_t$ is per-capita consumption, and $\ell_t$ is the fraction of productive time allocated to non-market activities. The fraction of time allocated to market activities is denoted by $n = 1 - \ell$. The size of the household $N_t$ is assumed to grow at the rate of population growth, $\eta$. The parameter $\sigma \geq 0$ measures how risk-averse the household is. The larger its value, the more risk-averse are households. The parameter $0 < \beta < 1$ measures impatience with a smaller value implying greater impatience. The parameter $\psi$ measures the relative importance of leisure and consumption. The larger it is, the more important is leisure.

***Ability to Transform***

There are two intermediate good sectors – a corporate sector denoted by one and a noncorporate sector denoted by two. The noncorporate production technology is simple and given by

$$y_{2,t} \leq k_{2,t}^{\theta}(z_t n_{2,t})^{1-\theta}. \tag{2}$$

Here $y$ is sector output, $k$ is capital services, $n$ is labor services, $z$ is a stochastic technology parameter, and $\theta$ is the capital share parameter.

The corporate sector is the important sector and is more complicated. Corporations make large investments in intangible capital including firm-specific training, research and development, organization building, firm-specific learning by doing, and so on. These investments are large and the stock of intangible assets has important consequences for the pricing of corporate assets. Given this, we assume that production in the corporate sector requires both tangible assets which are measured, $k_{1m}$, and intangible assets which are unmeasured, $k_{1u}$. In addition to capital, labor services $n_1$ are required. The aggregate production function for the corporate sector is given by

$$y_{1,t} \leq k_{1m,t}^{\phi_{mt}} k_{1u,t}^{\phi_{ut}} (z_t n_{1,t})^{1-\phi_{mt}-\phi_{ut}} \tag{3}$$

where $\theta_{1m,t}$ and $\theta_{1u,t}$ are the capital shares for measured and unmeasured capital, respectively. We make this non-standard assumption in order to capture variations in profit shares over the business cycle. These variations affect the equity risk premium, and we want to quantify their impact.

The three per-capita capital stocks depreciate geometrically and evolve according to

$$k_{i,t+1} = [(1-\delta_i)k_{i,t} + x_{i,t}]/(1+\eta) \tag{4}$$

where $i = 1m$, $1u$, or 2, $\delta_i$ is the rate of depreciation for capital of type $i$, and $x_{i,t}$ is gross investment of type $i$ in period $t$. The right hand side of the capital accumulation equation is divided by the growth in population because $k_i$ and $x_i$ are in per-capita units.

There is a final good sector that combines the intermediate inputs from the corporate and noncorporate sectors to produce a composite output good that can be used for consumption and investment. This production function is

$$c_t + g_t + x_{1m,t} + x_{1u,t} + x_{2,t} \leq y_t \equiv A\left(\mu y_{1,t}^{\rho} + (1-\mu) y_{2,t}^{\rho}\right)^{\frac{1}{\rho}} \tag{5}$$

where $g$ is government consumption, $0 < \mu < 1$ is a parameter that determines the relative sizes of the two sectors, $\rho \leq 1$ is a parameter that governs the substitutibility of corporate and noncorporate goods, and $A > 0$ is a scale factor.

Government production is assumed to be included in noncorporate production. However, the government plays another key role in the economy: it taxes various activities to finance government purchases and make transfers. In particular, the government taxes consumption, labor income, property, and corporate and noncorporate profits.

***An Equilibrium***

There are two ways to decentralize our model economy that lead to the same equilibrium outcome. The first assumes that firms hire workers, make investment decisions, pay taxes directly to the government, and pay dividends to the households. Because of the investment decision, their problem, in this decentralization, is dynamic. A second decentralization assumes that firms rent capital and labor from households. Households make the investment decisions and they pay taxes to the government. In this scenario, the firms' problems are simple and static. The relevant equilibrium outcomes are the same in the two because the households effectively own the capital in both cases. In this section, we describe an equilibrium for the second economy. We find this economy simpler to work with because we can consolidate all of the interesting transactions for a period into the household's budget constraint.

The household budget constraint in period $t$ is given by

$$\begin{aligned}
&(1+\tau_{c,t})c_t + x_{1m,t} + x_{1u,t} + x_{2,t} \\
&\quad = r_{1m,t}k_{1m,t} + r_{1u,t}k_{u1,t} + r_{2,t}k_{2,t} + w_t n_t \\
&\qquad -\tau_{1k,t}k_{1m,t} - \tau_{2k,t}k_{2,t} - \tau_{n,t}w_t n_t \\
&\qquad -\tau_{1,t}\left[(r_{1m,t}-\delta_{1m,t})k_{1m,t} + r_{1u,t}k_{1u,t} - x_{1u,t} - \tau_{1k,t}k_{1m,t}\right] \\
&\qquad -\tau_{2,t}\left[(r_{2,t}-\delta_2)k_{2,t} - \tau_{2k,t}k_{2,t}\right] + \pi_t. \qquad (6)
\end{aligned}$$

Households rent tangible and intangible capital to corporations and receive rental rates $r_{1m}$ and $r_{1u}$, respectively. Additionally, they rent capital to noncorporate firms with the rental rate in this sector given by $r_2$. Wage income is $wn$ where $n = n_1 + n_2$ is total hours of work. Taxes are paid on consumption expenditures, property, wage income, and profits. The tax rate on consumption is given by $\tau_c$. Tax rates on property in the corporate and noncorporate sectors are given by $\tau_{1k}$ and $\tau_{2k}$, respectively. Wage income is taxed at a rate $\tau_n$. Corporate profits are taxed at a rate $\tau_1$. Notice that corporations can subtract depreciation and property taxes when computing their corporate profits tax. Notice also that unmeasured investment, like R&D, is untaxed. It too is subtracted from income when computing taxable income. Noncorporate profits are taxed at a rate $\tau_2$. Again, depreciation and property taxes are subtracted when computing taxable income. Finally, transfers from the government to households are denoted by $\pi$.

Households maximize expected utility (1) subject to the sequence of budget constraints (6) and the capital accumulation equations (4). They take initial capital stocks, tax rates, and current and future prices as given. Firms in all sectors behave competitively and solve simple, static optimization problems. The intermediate goods firms choose capital and labor to maximize profits subject to the constraint on their production, namely (3) or (2). Thus, wages and rental rates in these sectors are equal to their marginal value products. The final goods firms choose intermediate inputs to maximize $y - p_1 y_1 - p_2 y_2$ where $p_i$ is the price of intermediate goods of sector $i$. Maximization is done subject to (5). If households and firms choose allocations optimally, equilibrium prices are set so that markets for goods, labor, and capital all clear.

In this economy, the value of corporate equities is equal to the value of the end-of-period stock of capital used in the corporate sector. If we use the price of output as the unit of account, then the per-capita value is given by

$$v_t = k_{1m,t+1} + (1 - \tau_{1,t}) k_{1u,t+1}. \tag{7}$$

This follows from the fact that the cost on margin of a unit of measured capital is one and the cost on margin of a unit of unmeasured capital is one minus the corporate income tax rate. Expenditures on unmeasured investment are expensed and reduce taxable corporate income. (See the budget constraint (6).)

The return on corporate equities is given by

$$r^e_{t,t+1} = \frac{(1+\eta)v_{t+1} + d_{t+1}}{v_t} - 1 \tag{8}$$

where $\{d_t\}$ is the stream of payments to shareholders of the corporation (that is, households). Notice the term $1+\eta$ is included since the variables are in per-capita terms, and there is growth in the population. Payments to shareholders are given by

$$\begin{aligned} d_t &= p_{1,t}y_{1,t} - w_t n_{1,t} - \tau_{1k,t}k_{1m,t} \\ &\qquad -\tau_{1,t}\left[(r_{1m,t} - \delta_{1m} - \tau_{1k,t})k_{1m,t}) + r_{1u,t}k_{1u,t} - x_{1u,t}\right] - x_{1m,t} - x_{1u,t}, \end{aligned} \tag{9}$$

that is, what is left over after workers have been paid, taxes on property and profits have been paid, and new investments have been purchased.

The return on a one-period bond, which we refer to as the risk-free rate, is given by

$$r_{f,t} = \left[\beta E_t \frac{c_{t+1}^{-\sigma}\ell_{t+1}^{\psi(1-\sigma}}{c_t^{-\sigma}\ell_t^{\psi(1-\sigma}}\right]^{-1} - 1, \tag{10}$$

where $c^{-\sigma}\ell^{\psi(1-\sigma)}$ is the marginal utility of consumption. The value or price of the bond is simply the inverse of the gross return, $1/(1+r_{f,t})$.

We can use these formulas for the asset values and returns to assess whether or not our model is consistent with U.S. observations. We turn to that next.

**B. Findings**

In this section, we make theoretical predictions for the United States stock market. In doing so, we first abstract from uncertainty and price corporate equity and risk-free debt using a deterministic version of the model. Unlike for the stochastic version, calculations of these quantities are trivial. We then establish that, for all practical purposes, the results are the same in the deterministic and stochastic versions of the model when we introduce uncertainty consistent with the behavior of U.S. data.

### ***The deterministic version***

We work first with the steady state of a deterministic version of the model. We derive an estimate for the return to capital using data from the U.S. noncorporate sector. We then derive an estimate for the size of the intangible capital stock. We choose the level of intangible capital so that the returns to capital in the corporate and noncorporate sectors are equated. With the estimate for intangible capital and data on measured corporate capital and taxes paid in the corporate sector, we can estimate the value of the stock market.

Absent uncertainty, the return to corporate equities and the return to a bond that pays 1 for sure in the following period are both equal to $i$ defined to be

$$i = \frac{(1+\gamma)^{\sigma}}{\beta} - 1 \tag{11}$$

where $\gamma$ is the growth of the technology parameter, $z_t$. This follows directly from the first-order conditions of the household. In fact, if there is no uncertainty, then the returns to each type of capital are also given by $i$ and the following is true

$$i = (1-\tau_1)(r_{1m} - \delta_{1m} - \tau_{1k}) = (1-\tau_2)(r_2 - \delta_2 - \tau_{2k}) = r_{1u} - \delta_{1u}. \tag{12}$$

Assuming the United States economy is roughly in a steady state, we can estimate $i$ using the National Income and Product Accounts. In Table C1, we report average values for income, product, and capital of the United States. The first column are the accounting concepts used by NIPA. The second column is the average values over the period 1990-1999 relative to GNP. Notice that GNP, listed under 'Product', is normalized to 1. Notice also that the sum of value added for the corporate and noncorporate sectors is equal to GNP.

Corporate income is domestic income of corporations reported. (See Table 1.15 of the *Survey of Current Business.*) Noncorporate income is the difference between national income and corporate income. (See Table 1.9 of the SCB for national income.) Thus, noncorporate income includes income of households, the government, noncorporate business, and foreign subsidiaries. For compensation of the noncorporate sector, we include total compensation and 80 percent of proprietor's income. Profits of the noncorporate sector include profits of foreign subsidiaries, rental income, and 20 percent of proprietor's income.

We make several adjustments to noncorporate profits and noncorporate capital before constructing an estimate of the return to capital in that sector. Consider noncorporate profits. First, we add net interest (equal to .042 GNP) and subtract the part of this net interest (equal to .022 GNP) that is imputed financial services. Business purchases of financial services are intermediate goods and therefore not final product.[6] Most of the .022 GNP adjustment are for services implicitly purchased by homeowners with mortgages. Therefore, on the product side, we lower consumption services by an equal amount.

The second adjustment that we make to noncorporate profits is the addition of imputed capital services to government capital and to consumer durables. The U.S. system of accounts uses a zero percent interest rate when imputing services to government capital. We instead use the average return on capital in the noncorporate sector. In order that income equals product, we add imputed services to profits in the noncorporate sector and to government consumption. In the U.S. system, consumer durables are treated as consumption. We instead treat them as investment and impute services to these durables. These imputed capital services are added to profits in the noncorporate sector and to private consumption.

We make one adjustment to the capital stock of the noncorporate sector. Capital stocks reported by the Bureau of Economic Analysis include only capital located in the United States. However, some national income is produced by capital abroad, namely corporate profits of foreign subsidiaries. We estimate that the capital used abroad is equal to the profits divided by the return on capital.

We are now ready to estimate the return on capital in the noncorporate sector. It is equal to

$$i = \frac{\text{profits} + \text{net interest} - \text{intermediate services} + \text{imputed services}}{\text{capital} + \text{capital of foreign subsidiaries}} \tag{13}$$

$$= \frac{.044 + .042 - .022 + (.595 + .292)i}{2.162 + .012/i} \tag{14}$$

where .595 GNP is the net stock of government capital, .292 GNP is the net stock of consumer durables, and .012 GNP is the profits from foreign subsidiaries. All other numbers are taken from Table C1. The value of $i$ that satisfies (14) is 4.08%. Therefore, our estimate of the imputed services to capital is .036, and our estimate of the capital used by foreign subsidiaries is .294.

It is interesting to compare the return we calculate here with the risk-free rate on inflation-protected bonds. The average return on 5-year inflation-protected bonds was 3.99% in the first quarter of 2000. The average return on the 30-year inflation protected bond was 4.19%.

We turn next to our estimate of the value of domestic corporations. For this, we need the value of measured tangible capital and an estimate of the value of unmeasured intangible capital. In the deterministic version of our model, the after-tax returns must be equal for each of the three types of capital. This requirement ties down the size of unmeasured corporate capital.

In Table C1, measured capital as reported in the Survey of Current Business *Fixed Reproducible Tangible Wealth* is .822 GNP. This measure does not include inventories or land. Inventories are however available in NIPA so we add them. Land is not included in NIPA estimates but it is in the *Flow of Funds* for nonfinancial corporate business. The difference between real estate values reported in FOF and nonresidential structures reported in NIPA is .06 GNP. Thus, our estimate of measured capital, with land and inventories included, is 1.043 GNP.

The next step is obtaining an estimate for unmeasured capital in the corporate sector.

Assuming that returns of the noncorporate and corporate sector are equal, we have

$$\begin{aligned} i &= \frac{\text{after}-\text{tax profits from measured capital}}{\text{measured capital}} && (15)\\ &= (1-\text{corporate tax rate})[\text{NIPA profits} + \text{unmeasured investment} \\ &\qquad -\text{the return on unmeasured capital}]/\text{measured capital} && (16)\end{aligned}$$

Using values from Table C1, this implies

$$.0408 = (1 - .026/.073)(.073 + .03k_{1u} - .0408k_{1u})/1.043 \tag{17}$$

where .026 GNP is the tax paid on domestic corporate profits, .073 is NIPA profits, .03 is the growth rate of GNP, and .03 $k_{1u}$ is the value of unmeasured net investment in the steady state. The solution to this equation is $k_{1u} = .639$. Therefore, unmeasured investment is equal to .019 GNP.

Given our estimate for unmeasured capital, we can now compute the market value of domestic corporations using the formula (7). Assuming that the time period is not too long, the value is given by

$$v = k_{1m} + (1 - \tau_1)k_{1u} = 1.455 \tag{18}$$

where $\tau_1 = .356$ (or .026/.073).

To compare this estimate to market value of U.S. corporations, we need to add in foreign subsidiaries. Let $v_{US}$ be the market value of U.S. corporations. Then

$$v_{US} = v + .294 = 1.75. \tag{19}$$

According to the *Flow of Funds*, the market value of domestic corporations at the end of the first quarter of 2000 was 17,228 billion dollars. GNP in the first quarter of 2000 was 9,655 billion dollars. Therefore, the market value of domestic corporations is 1.78 GNP – very close to what theory predicts.

***The stochastic version***

We are now ready to work out the implications of a stochastic version of the model. With uncertainty, we expect that risky assets, like corporate equities, would be paid a risk premium. In this section, we quantify this premium.

To do this, we must first calibrate the model. We do this in three steps. First, we compute a steady state for the model that is consistent with the adjusted accounting measures in Table C1. Second, we choose parameters – or means of stochastic parameters – that are consistent with these steady state values. Third, we choose stochastic processes for shocks that lead to fluctuations in model variables that are comparable to their U.S. analogues.

To compute a steady state of the model we need to make some further adjustments to the NIPA data so that they are consistent with the model concepts. The adjustments that we have discussed so far are (*i*) the addition of unmeasured investment; (*ii*) the subtraction of intermediate financial services; (*iii*) the imputation of durable and government capital services); and (*iv*) adjustments to the capital stocks. The final adjustments are adjustments for sales and excise taxes, depreciation of consumer durables, and adjustments to investment and depreciation for foreign subsidiary capital.

The U.S. system of accounts includes sales taxes in its measure of private consumption. In our model we treat consumption $c$ as pre-tax and therefore subtract sales taxes from NIPA private consumption. Consumer durables are treated as private consumption in NIPA and as investment in our model. Therefore, we include depreciation of consumer durables in noncorporate depreciation. Finally, because profits of foreign subsidiaries are included in national income (and therefore in noncorporate profits), we add an estimate of the capital of these subsidiaries to noncorporate capital.

The adjusted values for income, product, and capital are treated as a steady state for the model. These values are reported in Table C2. In the rightmost column of Table C2, we show the relevant expression for the model. We also include values for hours, growth rates, and tax rates. In the U.S., hours per person are roughly 1/4 of discretionary time. The growth rates are averages for 1990-1999 for total factor productivity and population. With the exception of the labor tax rate, we used NIPA values reported in Table C1 to calculate tax rates. We have data for corporate profits before- and after-tax. We assume that the noncorporate tax rate is 0 because the main categories of noncorporate income are untaxed. Property taxes and consumption taxes are indirect business taxes. Consumption taxes are .047 GNP, and property taxes are the remaining .032 GNP.

The labor tax rate is more difficult since the U.S. income tax is progressive, while taxes in our model economy are proportional. Households in the federal 28 percent or higher tax bracket pay nearly all of the income tax. However, because of fringe benefits and before-tax contributions to retirement plans, their marginal tax rates effectively lower than 28 percent. Therefore, we chose the tax rate on labor income to be 25 percent. But our analysis is not sensitive to the exact rate used.

In Table C3, we back out depreciation rates, capital shares, and parameters for the final goods technology and the utility function. Most of these parameters can be tied down by steady state values. There are two exceptions: the elasticity of substitution of corporate and noncorporate goods $1/(1-\rho)$ and the curvature parameter on consumption $\sigma$, which measures the degree of risk aversion. For these we experiment with different values and compare second moments of the model and data.

Following Jermann (1998), we compute a linear approximation to the decision rules for capital. All other variables, including equity returns, can be determined in a nonlinear way. Once we compute the equilibrium we can simulate the economy and calculate model statistics and compare them to statistics for the U.S. data.

The final choices are the stochastic processes. We assume that the technology parameter $z_t$ is stochastic, with the process given by:

$$\log z_{t+1} = \log z_t + \log(1+\gamma) + \varepsilon_{zt+1}, \tag{20}$$

where $\varepsilon_{zt}$ is i.i.d. normal with mean 0. Notice that $z_t$ grows at rate $\gamma$ as do other nonstationary variables in this economy. We choose the variance of $\varepsilon$ so that the standard deviation of U.S. GNP and our model output are roughly the same once we log the series and Hodrick-Prescott filter it.

If we turn on no other shocks, we find ... This is line 2 of Table 1. Since we are interested in ...

Now consider adding shocks to corporate profit shares. Assume, for example, that $\phi_{mt}$ is an autoregressive process with,

$$\phi_{mt+1} = (1 - \rho_\phi)\bar{\phi}_m \rho_\phi \phi_{mt} + \varepsilon_{\phi t+1} \tag{21}$$

where $\bar{\phi}_m$ is the mean of the process and $\varepsilon_{\phi t}$ is i.i.d. normal with mean 0. We set $\bar{\phi}_m$ equal to .277 as shown in Table C3 so that the mean of $\phi_{mt}$ is equal to the mean corporate profit share in the data. We set $\rho_\phi$ and the variance of $\varepsilon_{\phi t}$ to replicate two properties of corporate profits over the post-war period, namely its autocorrelation and variance.

In line 3, we see that ...

Another variable that has an important effet on corporate earnings and consumption is the corporate tax rate. As in the case of the share parameter, we assume this tax rate is an autoregressive process with,

$$\tau_{1t+1} = (1 - \rho_1)\bar{\tau}_1 \rho_1 \tau_{1t} + \varepsilon_{\tau t+1} \tag{22}$$

where $\bar{\tau}_1$ is the mean of the process and $\varepsilon_{\tau t}$ is i.i.d. normal with mean 0. We choose

If we simulate the economy with both, little changes.

If we allow for variations in capital shares and fiscal variables consistent with variations in the data, we still find small risk premia – never larger than .1 percent. If we allow for larger values of $\sigma$ as suggested by the finance literature, we can increase the risk premia but we then have counterfactually large risk-free rates. If we additionally adjust $\beta$ to keep $i$ at 4.08 percent, then we do find that the equity premium is small.

Doing this, we find that our calculated return on corporate equities is only slightly higher than the return on a risk-free bond, roughly .02 percent.

Thus, the principal findings are as follows: The equity premium is small. The equity premium is less than 0.2 percent and our best estimate is 0.1 percent. The value of the stock market relative to GNP should be near 1.5 GNPs as it currently is and risk free real return should be near 4.5 percent as it currently is. These conclusions are robust to the value assumed for expected future growth. Unmeasured capital is needed for consistency of theory and observations.

Outline:

(1) $z$ on, low sigma, high sigma, low rho, high rho

(2) $z$ on + $\phi_m$, low sigma, high sigma, low rho, high rho

(3) $z$ on + $\tau_1$ or $\phi_m$ or both, low sigma, high sigma, low rho, high rho

(4) $z$ on + $\tau_n$

(5) everything ... get the same.

## 3. Conclusions

It was bothersome that economic theory failed to account for the returns on debt and equity. No longer is this a problem as now observations are in conformity with theory. Return on these financial assets are consistent with returns on non-financial assets. Currently there is not "irrational exuberance." Conceivably there was irrational pessimism. Trial and error is one way to develop valuation rules that have the property that they are rational if others use these rules. For high frequency daily trades, this rationality is quickly realized. This is not true for lower frequency movements in the stock and bond markets. The empirical approach takes a long time to achieve rationality. A reason why the theory used here is inconsistent with observed asset pricing in much of the post war period is that this theory abstracted from institutional constraints and transactions costs. People in fiducial roles were often precluded from investing in the stock market. Transaction costs were not small but now are, at least if people use the intermediate services of the low cost providers. Another way for theory to work is for people to know the predictions of theory. In the model worlds, if people base their actions on theory, their expectations will be rational. We think focusing on corporate after-tax earnings relative to the value of the stock market is what people should do. We fear that the stock market will get over valued and then suffer a crash and losers will demand that the taxpayers bail them out.

The popular press suggests that higher future growth justifies higher equity values. We find that this is not the case. There are two consequences of higher growth for the value of the stock market. First with more rapid growth, future corporate payouts will be larger. If market discount factors remained fixed, these higher payouts imply higher stock market values. But higher growth leads to greater discounting of future payouts, which reduces the current value of these future payouts. We find that these two consequences of more rapid growth for the value of corporate equities roughly offset. The expectation of more rapid economic growth does not justify higher equity values relative to gross national income.

A change that would justify higher corporate value relative to income is an increase in the corporate after-tax earning share of income. This we see as very unlikely given the historic stability of corporate after-tax earnings share of gross national income, once these earnings are corrected for the business cycle variation.

## Appendix A. The 1946-1999 Experience

In this section we provide a historical perspective on the U.S. stock market for those less familiar with the data. We also report some facts that guided the selection of the model that we use to determine whether the U.S. stock market is overvalued.

*Returns on equities have been high*

The past two hundred years has been a golden age for stocks. In Table A1 we report average real returns for stocks and short- and long-term bonds.[7] In the 1946-1999 period, the compounded real return on equity was 8.4 percent. This is a large number. One dollar invested in the stock market in the beginning of 1946 would buy 77.9 times as much at the end of 1999. Over longer periods of time, the historic average return on an investment in the stock market has not been this high, but has been high. If you had been so fortunate as to have inherited the portfolio of your great, great, great grandparent who invested a silver dollar in 1802 in the stock market and who was not subject to taxation, your inheritance would be worth 7.2 million dollars today. Some of this increase is just an increase in the general price level, but even after correcting for inflation, the buying power of this investment would have increased 660,000 times.

*Returns on debt have been low*

The average real return on debt – which is virtually free of default – was much lower than on equity. The growth factor for a portfolio of one-month Treasury bills in the 1946-1999 period is only 1.32, which is far lower that the growth factor of 77.9 for holding the market portfolio of corporate equity. The average return on long-term debt was higher in this 1946-1999 period than on short-term debt. The investment strategy of investing in long-term government debt would have yielded a real growth factor of 1.83, which is better than the 1.32 factor for short-term government debt, but pales in comparison with the 77.9 factor for the stock market.

*Average stock return is consistent with earnings-value ratio*

Accounting earnings are far from a perfect measure of true earnings. Some investment expenditures, such as for maintenance, training and R&D, are expensed rather than capitalized. This lowers accounting earnings in the accounting period they are incurred relative to true earnings and does just the reverse in future years. In a non- growing or slow-growing economy, these two consequences offset and accounting earnings are a good measure of true earnings. Given the U.S. economy grew at a relatively stable and low rate over the 1946-1999 period, national income and product account (NIPA) corporate earnings are a good, but not perfect, measure of U.S. corporate earnings. We estimate they on average underestimate earnings by only 0.4 percent.

An implications of theory is that over long period of times, the arithmetic average ratio of corporate earnings to the value of corporate equity should equal the arithmetic average return on corporate equity less any risk premium. The average ratio of corporate after tax earnings to corporate market value was 9.8 percent and the average return on traded corporate equity was 9.4 percent. Thus, the important finding: The arithmetic averages of the ratio of U.S. corporate earnings to the value of U.S. corporations and the return on the stock market were approximately equal in the 1946-1999 period This finding is consistent with theory. If the return on the stock market were significantly higher over this long period, it would suggest a price bubble, with value of an asset exceeding the present value of its payouts.

Over the 1946-1999 period U.S. corporate earnings relative to the corporate equity has fluctuated as pictured in figure 1. An important fact is that currently U.S. corporate earnings relative to corporate value is much lower than the historical average, being only 4 percent in 1999. This number is not low because of cyclical variation in earnings relative to gross national income. The economy boomed in 1999 and in boom year corporate earnings' share is above average.

Figure 2 plots corporate earnings as a fraction of gross national income and the value of U.S. corporate equity also relative to gross national income.[8] The figures shows that most of the non-cyclical variation in corporate earnings relative to equity value is due to variations in the value of corporate equity relative to GNP and not due to variations in corporate earnings relative to GNP.

*Large quantities of both debt and equity are held*

We now establish that people in the pre-retirement years now hold large quantities of both debt and equity. Table A2 reports the balance sheet of U.S. households in 1999 and the average balance sheet for the 1946-1999 period all relative to gross national income. The holdings of debt and equity are as big as annual gross national income, which is the value of all claims against the value of all final goods and services produced by U.S. citizens and permanent residents.

*People can and do shift between debt and equity*

People with defined contribution pension plans, that is IRA's, SEP's, Keough's, 401's and 403's, typically can shift between debt and equity with a phone call. In Table 3, we In 1999 assets in these plans were 0.24 GNPs. Of these assets, 0.13 GNPs are corporate equity and 0.11 GNPs are debt. Owners of defined contribution plans hold large quantities of debt and equity directly and many can select the proportion of their savings invested in debt and equity financial assets.

An equally important form of savings is state and local government defined benefit pension plans. Indeed the biggest holder of financial assets is the California State pension plan, which incidentally invested heavily in equity and, as a result, achieved a high return on its investments. State and local government plans with 0.18 GNPs of equity assets and 0.09 GNPs of debt hold large quantities of financial assets in 1999. These assets can be shifted between debt and equity holdings.

Survey data find that many people shift between debt and equity. Figure 3 captures this switching in a graphic manner. The figure is a scatter plot of the fraction of financial assets in equity in two different years for a sample of people. A circle depicts the positions of a person in the sample in 1984 and 1989. The circle for a person with the same equity share in the two years falls on the 45-degree line. The large number of circles that are far from the 45-degree line establishes that many people made large changes in the share of their portfolio in equity.

*Most variation in net worth is in equity holdings*

Figure 4 plots the time series of U.S. household net worth and its decomposition into debt assets, tangible assets, and corporate equity each divided by gross national income. Unlike debt and tangible assets, the value of corporate equity varies and varies a lot over time. Most variation in household net worth relative to income is due to variations in the value of corporate equity relative to income.

*Summary*

Historical data on asset returns and corporate earnings motivate our conclusion that the puzzles are in the past data not the present. The evidence on asset holdings motivated some of our modeling choices. The large amount of capital outside the corporate sector motivated us to include a corporate and noncorporate sector. The large holdings of both debt and equity and the large shifts between these categories led us to assume that people are not on corners – they can easily shift their portfolios.

## Appendix B. An Asset Pricing Primer

Here, we review simple concepts that underlie standard asset-pricing theory. A key idea is that consumption today and consumption at some future date are treated as different goods. Relative prices of these different goods are equal to both people's willingness to substitute between these goods and business' ability to transform these goods into each other. In this section we work through simple examples to illustrate this point.

We begin with a simple environment with neither capital accumulation nor uncertainty. There is firm-specific uncertainty that averages out over the economy and consequently introduces no aggregate uncertainty. With this economy, the value of firm equity equals the present value of expected firm payouts and all assets have the same expected return. Next we add economy-wide uncertainty that gives rise to uncertainty in consumption. Now expected returns differ across assets. An asset that makes relatively large payments when consumption is high will have a higher expected return than one that has relatively large payouts when consumption is low.

Finally we add capital accumulation opportunities by adding a storage technology than can transform the date $t$ consumption good into the date $t+1$ consumption good on a one-to-one basis. This technology specifies the ability of people to transform goods at some date into goods at some other date. The addition of this storage technology has major consequence for the value of firm equity and for average returns. An implication of this is that when deriving the implication of theory for the pricing of assets and determining the behavior of their returns, the ability of people to substitute must be explicitly modeled as well as the willingness of people to substitute.

We emphasize that we are applying the discipline of applied general equilibrium, where people's willingness to substitute and ability to transform date-event-contingent consumption goods into each other are equated to relative prices. In the next section we use this theory, national income and product data, and measurement based on a wealth of micro-economic studies concerning people's willingness and ability to substitute. These are the data of general equilibrium models. In particular, we use the standard growth model, which is used not only to model economic growth but also to evaluate tax policies and to assess the contribution of various shocks to business cycle fluctuations. The growth model is established aggregate economic theory and is fast becoming the textbook model in intermediate and advanced undergraduate courses concerned with the aggregate behavior of the economy. However, before applying this theory we first deal with simpler environment to make clear the underlying principles. Then, we use the growth model to derive the implications of theory for the value of equity and the average returns on risk-free debt and corporate equity.

*Willingness to Substitute*

Following established theory, the willingness to substitute is described in the following way. There are a large number of households that maximize expected discounted utility,

$$u(c1) + \beta u(c_2) + \beta^2 u(c_3) + \ldots + \beta^t u(c_t) + \ldots \tag{23}$$

The parameter $\beta$ is positive and less than one. Parameter ? describes how impatient households are. If $\beta$ is small, people are impatient with a strong preference for consumption now versus consumption in the future. These households live forever, which implicitly assume the utility of parents depends upon the utility of their children. This is the case for some people and not for others. However, worlds with both people who care about their children's utility and people who do not care have essentially the same implications for asset prices and returns.[9] Thus we use this simple abstraction to build quantitative economic intuition as to what the return on equity and debt should be. The function $u(c)$ is increasing, but at an ever-decreasing rate ? that is marginal utilities are decreasing in the level of consumption.

In empirical work, constant relative risk aversion is typically assumed. This means that if some household will accept a gamble, then that household will accept that gamble if both its wealth and the gamble amounts are scaled by any positive factor. For purposes of this section we use $u(c) = \log(c)$, which empirically is not a bad representation of people's aggregate willingness to substitute. With this utility function, an individual is indifferent between a gamble that provides fifty-fifty chances of a \$10,000 per year consumption or a \$20,000 per year consumption and a certainty consumption of \$14,142 as

$$0.5 \log(10,000) + 0.5 \log(20,000) = \log(14,142). \tag{24}$$

Given the logarithmic utility function displays constant relative risk aversion, the above equality holds if the three consumption levels are scaled by any factor.

*Example 1: No aggregate uncertainty and no ability to transform goods*

We assume that there is one firm for every 10 households. Each firm produces 100 units of output with probability one-half and 0 with probability one-half. These outcomes are randomly distributed both across firms and across time. Given that there is a large number of firms, output per firm in every period is 50 and output per person is 5. With the assumed utility function, the wealth distribution does not matter for pricing of assets, so for simplicity we will assume that everyone owns an equal share of every firm.

Equilibrium consumptions of every household is 5 units of consumption every period. Consumptions at different dates are different commodities and have different prices. The equilibrium prices of the dated consumption goods $c_t$ is

$$p_t = p_0 \beta^t. \tag{25}$$

Because of household impatience, consumption in the future has a lower price than consumption today.

These prices can be used to value a firm. Given there is no aggregate uncertainty, the ex-dividend value of a firm for this economy is the present value of its expected payouts. Note, firm-specific randomness does not matter, just the expected distributions matters. If some firm has a distribution of 1000 with probability .05 and a distribution of 0 with probability 0.95, it would have the same value as a firm with a certainty distribution of 50. The reason that this is the case is that households can diversify firm- risk away by holding a small share of a large number of firms. Thus, the value of a firm is

$$v_0 = 50p_1 + 50p_2 + 50p_3 + \ldots = 50\beta/(1-\beta). \tag{26}$$

If $\beta$ is 0.95, a firm's value is 950. The return on equity is expected dividend per firm, 50, divided by value a firm's value. Consequently, the real return on equity is 5.56 percent.

The one-period real interest rate in this economy is $r_t = (p_t/p_{t+1} - 1$ or 5.56 percent. By investing one unit of the consumption good in a security that pays real interest rate $r_t$ at date $t$, a household receives units of the consumption good at date. In this world with no aggregate uncertainty, returns on debt and equity are equal.

*Example 2: Aggregate uncertainty and no ability to transform goods*

The amount of aggregate uncertainty was selected so that the premium for holding equity is about five percent. In order to introduce aggregate uncertainty, we assume that the probability of good times is one-half as is the probability of bad times. The probability of good times and bad times is independent over time. It is as if a fair coin is tossed each period and if it comes up heads, there are good times and if it comes up tails, there are bad times. In good times the probability of a firm producing 100 units of the consumption good is two-thirds and the probability of 0 output is one-third. In bad times, these probabilities are reversed. In good times output per household is 6.67 and in bad times output per household is 3.33. Given the equal likelihood of good and bad times, expected output per household at future dates is 5.0 as in the previous example.

For this example, a richer class of commodities is needed. Consumption at date $t$ if times are good has a different price than consumption at t if times are bad. The reason is that in bad times, consumption is lower and people value an additional unit of consumption more.

Now we have to index consumption by date and nature of the times. Consumption at date $t$ if times are good is $c_{gt}$ and if times are bad is $c_{bt}$. Given the prices, the value of the firm at date $t$ conditional upon the state $s = b$ or $g$ is

$$v_{st} = \sum_{\tau=t+1}^{\infty} \left[p_{gt}\text{Expected payout given } g + p_{bt}\text{Expected payout given } b\right]/p_{st}. \tag{27}$$

The ex-dividend value of a firm at date 0 if the state is $s$ is

$$v_{s0} = \left[p_{g1}66.7 + p_{b1}33.3 + p_{g2}66.7 + p_{b2}33.3 + \ldots\right]/p_{s0}. \tag{28}$$

A question is what is the appropriate set of equilibrium prices. The price of consumption in bad times will be higher than in good times. With the assumed utility function, the prices are

$$p_{bt} = \beta^t, \qquad p_{gt} = \beta^t/2. \tag{29}$$

These price relations are obtained by equating marginal rates of substitution to the corresponding goods price ratio.

The ex dividend values of a firm in terms of that date consumption good are $v_b = 633$ and $v_g = 1267$. The effect of aggregate uncertainty is to raise the value of the firm in good times and lower it in bad times. The average return on equity is 11.67 percent, which is more than double the return when there was no aggregate uncertainty. We turn now to the return on debt. The price of a real bill if the state is $s$ is

$$q_s = \beta(p_b + p_g)/(2p_s). \tag{30}$$

Thus, the risk free interest rates are

$$r_s = 1/q_s - 1. \tag{31}$$

Using these equations, the risk free interest rates are $r_{bt} = r_b = 40.35$ percent and $r_{gt} = r_g = -29.82$ percent. The average risk free interest rate is $-0.76$, which is far less than the average return on equity. In this economy, the average economy premium, that is the difference between the average return on debt and equity is over 12 percent. Absent aggregate uncertainty, the equity premium is zero.

*Exampe 3: Aggregate uncertainty and the ability to transform good intertemporally*

This example is identical to example 2 in every way except that goods can be stored. Via storage, one unit of the date $t$ good can be transformed into one unit of the date $t + 1$ good. Negative storage is not feasible. We find that the ability to intertemporally transform good dramatically reduces the premium for holding equity.

For this economy, equilibrium values of assets and consumptions depend not only on whether times are good or bad, but also on the stock of stored goods. With this complication, computing the average returns on debt and equity requires the use of a computer. Intuitively in this world, people save in good times and draw on savings in bad times in order to smooth consumption over time. A consequence of this is that returns on both debt and equity are lower. In fact, the average returns over long periods of time are 3.62 percent for debt and 5.28 percent for equity. The average return on debt for this economy with a storage technology is higher than for the economy without the storage technology and the average return on equity lower. This example establishes that any theory of equity and debt return must model people's ability to transform consumption over time as well as people's willingness to substitute consumption over time.

The finance approach could be applied to this economy. This first step in determining the value of the stock market is to determine an appropriate list of commodities; the second step is to find payments of each of these commodities by firms; and the third is to find the prices of the commodities.

The needed list of commodities is as follows. The first three commodities, namely contracts to deliver sate 0 consumption goods, date 1 consumption good if times are good, and date 1 consumption good if times are bad are the same as went there is no storage technology. However, at date 2 there are four, not two event-contingent commodities. This is because people on marginal value a unit of consumption at date 2 differently if times were bad at date 1 then if they were good at date 1. This is true because equilibrium consumption levels are different. A consequence of this fact is that the date 2 commodities must be jointly indexed by the nature of times at date 1 and the nature of times at date 2. In general, date $t$ commodities must be index by the nature of times for dates 1 through date $t$. Consequently there are $2t$ date commodities. With this expanded commodity space, the present value calculation work just as they did for the simpler environment considered previously.

For this set of commodities the problem is to find the date, event-contingent consumptions and prices for which all markets clear. The simplest way to find these quantities is to exploit the invisible-hand result that the competitive equilibrium consumptions maximize welfare. We use standard computational methods to find consumption as a function of inventories x and the current state s, which is either g or b. This function is denoted ct+1 = c(x, s). Next periods stock of inventories is

$$x_{t+1} = h(x_{t+1}, s_t) = x_t - c(x_t, s_t) + s_t. \tag{32}$$

The interest rate if the current state of the economy is $(x, s)$ is given by

$$r(x, s) = \beta c(x, s)/c(h(x, s)) - 1. \tag{33}$$

Standard computational methods can be used to find the value of the stock market as a function of the state or position of the economy, $v(x, s)$. The function v satisfies the functional equation

$$v(x, s) = \quad c(x, s)(0.5\beta(v(x', b) + c(x', b))/c(x', b)) + \tag{34}$$

$$c(x, s)(0.5\beta(v(x', g) + c(x', g))/c(x', g)) \tag{35}$$

We find that the introduction of a storage technology reduces the average return on equity from 11.67 to 5.28 percent, while its introduction increases the return on debt from

$-0.76$ percent to 3.62 percent. This establishes that the nature of the technology, that is ability to transform goods, matters for valuing assets and determining their returns

## Appendix C. NIPA Data and the Model

In this appendix, we describe in detail the adjustments that we make to the data reported in the *National Income and Product Accounts* before comparing these data to our model.

In Table C1, we report average values for income, product, and capital of the United States. The first column are the accounting concepts used by NIPA. The second column is the average values over the period 1990-1999 relative to GNP. Notice that GNP, listed under 'Product', is normalized to 1. Notice also that the sum of value added for the corporate and noncorporate sectors is equal to GNP.

Corporate income is domestic income of corporations with operations in the United States. (See Table 1.15 of the *Survey of Current Business.*) Noncorporate income is the difference between national income and corporate income. (For national income, see Table 1.14 of the SCB.) Thus, noncorporate income includes income of households, the government, noncorporate business, and foreign subsidiaries. For compensation in the noncorporate sector, we include total compensation and 80 percent of proprietor's income. Profits of the noncorporate sector include profits of foreign subsidiaries, rental income, and 20 percent of proprietor's income.

Total product is the sum of private consumption, public consumption, and investment in the three types of capital, namely measured corporate, unmeasured corporate, and noncorporate. (See Table 1.1 of SCB.) We include net exports in noncorporate investment since production in the rest of the world is included in our notion of noncorporate production.

In the middle column of Table C1, we provide descriptions of the adjustments that we make to NIPA accounts to make them consistent with the theory. We now describe each adjustment in detail.

The U.S. system of accounts includes sales taxes in its measure of private consumption. On the income side, these taxes are included in indirect business taxes. In our model we treat consumption $c$ as pre-tax and therefore subtract sales taxes from both sides of the accounts. We estimate that .047 GNP out of .079 GNP is sales or excise taxes which we model as a tax on private consumption. The remainder is attributed to property taxes.

Unmeasured intangible investment is estimated to be .019 GNP. (See the main text for details of our estimation.) This is investment that is expensed so it does not show up in GNP. We include it in our notion of GNP since it raises both after-tax corporate profits and investment.

We make an adjustment to net interest – both in the corporate and noncorporate sectors. We subtract that part of financial services purchased by businesses that are intermediate goods. The U.S. system of accounts treats net interest of financial intermediaries as purchases of services by the lender, typically the household. The United Nations system of accounts treats it as purchases of services by the borrower. Thus, in the U.N. system, no entry for imputed interest is made and imputed interest and consumption services are correspondingly lower. Here, we compute lenders' (borrowers') purchases of financial services as the product of the short-term interest rate and the amount lent (borrowed).

Most of the .022 GNP adjustment are for services implicitly purchased by homeowners with mortgages. Therefore, on the product side, we lower consumption services by an equal amount.

The remainder of net interest is included in profits.

Consumer durables are treated as private consumption in NIPA and as investment in our model. Therefore, we include depreciation of consumer durables. On the income side this depreciation appears in noncorporate capital consumption. On the product side this is consumption services. This is the procedure used for housing services which are included in NIPA.

Because profits of foreign subsidiaries are included in national income (and therefore in noncorporate profits), we added an estimate of the capital of these subsidiaries to noncorporate capital. To make the depreciation and investment of the noncorporate sector comparable to the capital stock, we include depreciation and net investment for the foreign subsidiaries. We assume that depreciation rates and growth rates are the same at home and abroad to make these estimates.

A second adjustment that we make to noncorporate profits is the addition of imputed capital services to government capital and to consumer durables. The U.S. system of accounts uses a zero percent interest rate when imputing services to government capital. We instead use the average return on capital in the noncorporate sector. In order that income equals product, we add imputed services to profits in the noncorporate sector and to government consumption. In the U.S. system, consumer durables are treated as consumption. We instead treat them as investment and impute services to these durables. These imputed capital services are added to profits in the noncorporate sector and to private consumption.

We make one adjustment to the capital stock of the noncorporate sector. Capital stocks reported by the Bureau of Economic Analysis include only capital located in the United States. However, some national income is produced by capital abroad, namely corporate profits of foreign subsidiaries. We estimate that the capital used abroad is equal to the profits divided by the return on capital.

In Table C1, measured capital as reported in the Survey of Current Business *Fixed Reproducible Tangible Wealth* is .822 GNP. This measure does not include inventories or land. Inventories are however available in NIPA so we add them. Land is not included in NIPA estimates but it is in the *Flow of Funds* for nonfinancial corporate business. The difference between real estate values reported in FOF and nonresidential structures reported in NIPA is .06 GNP. Thus, our estimate of measured capital, with land and inventories included, is 1.043 GNP.

The adjusted values for income, product, and capital are treated as a steady state for the model. In Table C2, notice that the values for income, product, and capital are the adjusted values of Table C1. In the rightmost column, we show the relevant expression for the model. We also include values for hours, growth rates, and tax rates. In the U.S., hours per person are roughly 1/4 of discretionary time. The growth rates are averages for 1990-1999 for total factor productivity and population. With the exception of the labor tax rate, we used NIPA values reported in Table C1 to calculate tax rates. We have data for corporate profits before- and after-tax. We assume that the noncorporate tax rate is 0 because the main categories of noncorporate income are untaxed. Property taxes and consumption taxes are indirect business taxes. Consumption taxes are .047 GNP, and property taxes are the remaining .032 GNP.

The labor tax rate is more difficult since the U.S. income tax is progressive, while taxes in our model economy are proportional. Households in the federal 28 percent or higher tax bracket pay nearly all of the income tax. However, because of fringe benefits and before-tax contributions to retirement plans, their marginal tax rates effectively lower than 28 percent. Therefore, we chose the tax rate on labor income to be 25 percent. But our analysis is not sensitive to the exact rate used.

In Table C3, we back out depreciation rates, capital shares, and parameters for the final goods technology and the utility function. Most of these parameters can be tied down by steady state values. There are two exceptions: the elasticity of substitution of corporate and noncorporate goods $1/(1-\rho)$ and the curvature parameter on consumption $\sigma$, which measures the degree of risk aversion. For these we experiment with different values and compare second moments of the model and data.

## Notes

[1]We report values of assets relative to gross national product or equivalently gross national income (GNI), which differs from national income in that it includes all claims against product. The two important claims omitted from national product are indirect business taxes and depreciation.

[2]The two main data sources used in this paper are the Board of Governors of the Federal Reserve System *Flow of Funds Accounts of the United States* and the U.S. Department of Commerce *National Income and Product Accounts*. For details of how these data were used to calculate the statistics reported in this article, see our data appendix.

[3]Actually the market value of equity plus the market value of liabilities debt should equal the market value of debt assets plus the value of tangible assets. As net indebtedness of corporations is near zero, to simplify the discussion we ignore corporate debt holdings and liabilities.

[4]The theory we refer to abstracts from transaction costs and assumes a large number of households, or their agents, are knowledgeable about financial markets.

[5]Technically, this is true if there is no unmeasured capital. Some adjustments have to be made to standard NIPA concepts if there is unmeasured intangible capital.

[6]The U.S. system of accounts treats net interest of financial intermediaries as purchases of services by the lender, typically the household. The United Nations system of accounts treats it as purchases of services by the borrower. Thus, in the U.N. system, no entry for imputed interest is made and imputed interest and consumption services are correspondingly lower. Here, we compute lenders' (borrowers') purchases of financial services as the product of the short-term interest rate and the amount lent (borrowed).

[7]The data for the long period is from Seigal (1998). In the earlier period, when government debt of the appropriate maturity was not available, he used the returns on the highest quality corporate debt.

[8]We consolidate private defined benefit pension plans and life insurance companies with the corporate sector. The motivation is that even though corporations are required to hold reserves to cover liabilities, the residual claimants are typically a corporation as they must make additional payments if there is a deficiency or can make smaller payments than the additional liabilities incurred if there is a surplus. Given this consolidation, to avoid double counting we subtract the value of U.S. corporate equity held by private defined pension plans and life insurance companies from the market value of U.S. corporate equity that is reported in the Federal Reserve System Flow of Funds Accounts.

[9] Which environment one uses sometimes matter for the average returns. Baby boomers' saving for retirement may lower expected returns on all financial securities, but they have little effect on differences in average returns on debt and equity. See Constantinities, Donaldson and Mehra (1900).

## References


Board of Governors of the Federal Reserve System, (1946-2000), *Flow of Funds Accounts of the United States.*

Cochrane, J. H. (1991), "Production-Based Asset Pricing and the Link between Stock Returns and Economic Fluctuations," *Journal of Finance* 46: 209-237.

Constantinides, G. M., J. B. Donaldson, and R. Mehra (2000), "Junior Can't Borrow: A new Perspective on the Equity Premium Puzzle," University of Chicago working paper, NBER working paper # 6617.

Hall, R. E., (2000), "e-Capital," Hoover Institution working paper.

Ibbotson Associates, (2000), *2000 Yearbook: Market Results for 1926-1999.*

Jagannathan, R., E. R. McGrattan, and A. Scherbina, (2000), "The Equity Premium: A Vanishing Puzzle," *Federal Reserve Bank of Minneapolis Quarterly Review*, this issue.

Jagannathan, R. and N. R. Kocherlakota (1996), "Why Should Older People Invest Less in Stocks than Young People," *Federal Reserve Bank of Minneapolis Quarterly Review*, 20 : 11-23.

Jermann, U., (1998), "Asset Pricing in Production Economies," *Journal of Monetary Economics*, 41: 257-276.

Kocherlakota, N. R., (1996), "The Equity Premium is still a Puzzle," *Journal of Economic Literature*, 34: 42-71.

Mehra, R., "On the Volatility of Stock Prices: An Exercise in Quantitative Theory," *International Journal of Systems Science*, 29: 1203-1211.

Miller, M. H., (1977), "Death and Taxes," *Journal of Finance*, 32: 261-275.

Siegel, J. J., (1998), *Stock Returns for the Long Run: The Definitive Guide to Financial Market Returns and long-term Investment Strategies*, (New York: McGraw-Hill).

U.S. Department of Commerce, (1946-1999), *National Income and Product Accounts.*