1. Theory and the Value of the U.S. Stock Market

We turn now to our main application: the U.S. stock market. Using standard growth theory, we estimate that the value of U.S. domestic corporations is roughly where it should be. We also estimate the return on risk-free debt to be slightly above four percent and the premium for holding corporate equities to be negligible. In this section, we describe the model and data that we use to reach these conclusions.

A. The Model

The model that we use is a modified version of the real business cycle model of Kydland and Prescott (1982). Here, we include two sectors, a corporate sector and a noncorporate sector. Since our main focus is on the value of domestic corporations, we assume that output from our corporate sector is the gross domestic product of corporations located in the United States. The output of the noncorporate sector is the remaining product of U.S. GNP. Therefore, our noncorporate sector includes the household sector, the government sector, the noncorporate business sector, and the rest of the world.

Willingness to Substitute

Our model economy is inhabited by infinitely-lived households with preferences ordered by the expected value of

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t\, \ell_t^{\psi})^{1-\sigma}}{1-\sigma}\right] N_t \tag{1}$$

where t indexes time, c_t is per-capita consumption, ℓ_t is the fraction of productive time allocated to non-market activities. The fraction of time allocated to market activities is denoted by $n = 1 - \ell$. The size of the household N_t is assumed to grow at the rate of population growth, η. The parameter $\sigma \geq 0$ measures how risk-averse the household is. The larger its value, the more risk-averse are households. As before, the parameter $0 < \beta < 1$ measures impatience with a smaller value implying greater impatience. The parameter ψ

measures the relative importance of leisure and consumption. The larger it is, the more important is leisure.

Ability to Transform

There are two intermediate good sectors – a corporate sector denoted by one and a noncorporate sector denoted by two. The noncorporate production technology is simple and given by

$$y_{2,t} \leq k_{2,t}^{\theta}(z_t n_{2,t})^{1-\theta}. \tag{2}$$

Here y is sector output, k is capital services, n is labor services, z is a stochastic technology parameter, and θ is the capital share parameter.

The corporate sector is the important sector is a somewhat more complicated. Corporations make large investments in intangible capital including firm-specific training, research and development, organization building, firm-specific learning by doing, and so on. These investments are large and the stock of intangible assets has important consequences for the pricing of corporate assets. Given this, we assume that production in the corporate sector requires both tangible assets which are measured, k_{1m}, and intangible assets which are unmeasured, k_{1u}. In addition to capital, labor services n_1 are required. The aggregate production function for the corporate sector is given by

$$y_{1,t} \leq k_{1m,t}^{\phi_{mt}} k_{1u,t}^{\phi_{ut}} (z_t n_{1,t})^{1-\phi_{mt}-\phi_{ut}} \tag{3}$$

where $\theta_{1m,t}$ and $\theta_{1u,t}$ are the capital shares for measured and unmeasured capital, respectively. We make this non-standard assumption in order to capture variations in profit shares over the business cycle. These variations affect the equity risk premium, and we want to quantify their impact.

The three per-capita capital stocks depreciate geometrically and evolve according to

$$k_{i,t+1} = [(1-\delta_i)k_{i,t} + x_{i,t}]/(1+\eta) \tag{4}$$

where $i = 1m$, $1u$, or 2, δ_i is the rate of depreciation for capital of type i, and $x_{i,t}$ is gross investment of type i in period t. The right hand side of the capital accumulation equation is divided by the growth in population because k_i and x_i are in per-capita units.

There is a final good sector that combines the intermediate inputs from the corporate and noncorporate sectors to produce a composite output good that can be used for consumption and investment. This production function is

$$c_t + g_t + x_{1m,t} + x_{1u,t} + x_{2,t} \leq y_t \equiv A\left(\mu y_{1,t}^{\rho} + (1-\mu) y_{2,t}^{\rho}\right)^{\frac{1}{\rho}} \tag{5}$$

where g is government consumption, $0 < \mu < 1$ is a parameter that determines the relative sizes of the two sectors, $\rho \leq 1$ is a parameter that governs the substitutibility of corporate and noncorporate goods, and $A > 0$ is a scale factor.

Government production is assumed to be included in noncorporate production. However, the government plays another key role in the economy: it taxes various activities to finance government consumption. In particular, the government taxes consumption, labor income, property, and corporate and noncorporate profits.

An Equilibrium

There are two ways to decentralize our model economy that lead to the same equilibrium outcome. The first assumes that firms hire workers, make investment decisions, pay taxes directly to the government, and pay dividends to the households. Because of the investment decision, their problem, in this decentralization, is dynamic. A second decentralization assumes that firms rent capital and labor from households. Households make the investment decisions and they pay taxes to the government. In this scenario, the firms' problems are simple and static. The relevant equilibrium outcomes are the same in the two because the households effectively own the capital in both cases. In this section, we describe an equilibrium for the second economy. We find this economy simpler to work with because we can consolidate all of the interesting transactions for a period into the household's budget constraint.

The household budget constraint in period t is given by

$$
\begin{aligned}
&(1+\tau_{c,t})c_t + x_{1m,t} + x_{1u,t} + x_{2,t} \\
&\quad = r_{1m,t}k_{1m,t} + r_{1u,t}k_{u1,t} + r_{2,t}k_{2,t} + w_t n_t \\
&\qquad -\tau_{1k,t}k_{1m,t} - \tau_{2k,t}k_{2,t} - \tau_{n,t}w_t n_t \\
&\qquad -\tau_{1,t}\left[(r_{1m,t}-\delta_{1m,t})k_{1m,t} + r_{1u,t}k_{1u,t} - x_{1u,t} - \tau_{1k,t}k_{1m,t}\right] \\
&\qquad -\tau_{2,t}\left[(r_{2,t}-\delta_2)k_{2,t} - \tau_{2k,t}k_{2,t}\right] + \pi_t. \qquad (6)
\end{aligned}
$$

Households rent tangible and intangible capital to corporations and receive rental rates r_{1m} and r_{1u}, respectively. Additionally, they rent capital to noncorporate firms with the rental rate in this sector given by r_2. Wage income is wn where $n = n_1 + n_2$ is total hours of work. Taxes are paid on consumption expenditures, property, wage income, and profits. The tax rate on consumption is given by τ_c. Tax rates on property in the corporate and noncorporate sectors are given by τ_{1k} and τ_{2k}, respectively. Wage income is taxed at a rate τ_n. Corporate profits are taxed at a rate τ_1. Notice that corporations can subtract depreciation and property taxes when computing their corporate profits tax. Notice also that unmeasured investment, like R&D, is untaxed. It too is subtracted from income when computing taxable income. Noncorporate profits are taxed at a rate τ_2. Again, depreciation and property taxes are subtracted when computing taxable income. Finally, transfers from the government to households are denoted by π.

Households maximize expected utility (1) subject to the sequence of budget constraints (6) and the capital accumulation equations (4). They take initial capital stocks, tax rates, and current and future prices as given. Firms in all sectors behave competitively and solve simple, static optimization problems. The intermediate goods firms choose capital and labor to maximize profits subject to the constraint on their production, namely (3) or (2). Thus, wages and rental rates in these sectors are equal to their marginal value products. The final goods firms choose intermediate inputs to maximize $y - p_1 y_1 - p_2 y_2$ where p_i is the price of intermediate goods of sector i. Maximization is done subject to (5). If households and firms choose allocations optimally, equilibrium prices are set so that markets for goods, labor, and capital all clear.

In this economy, the value of corporate equities is equal to the value of the end-of-period stock of capital used in the corporate sector. If we use the price of output as the unit of account, then the per-capita value is given by

$$v_t = k_{1m,t+1} + (1 - \tau_{1,t}) k_{1u,t+1}. \tag{7}$$

This follows from the fact that the cost on margin of a unit of measured capital is one and the cost on margin of a unit of unmeasured capital is one minus the corporate income tax rate. Expenditures on unmeasured investment are expensed and reduce taxable corporate income. (See the budget constraint (6).)

The return on corporate equities is given by

$$r^e_{t,t+1} = \frac{(1+\eta) v_{t+1} + d_{t+1}}{v_t} - 1 \tag{8}$$

where $\{d_t\}$ is the stream of payments to shareholders of the corporation (that is, households). Notice the term $1 + \eta$ is included since the variables are in per-capita terms, and there is growth in the population. Payments to shareholders are given by

$$\begin{aligned} d_t \;&=\; p_{1,t} y_{1,t} - w_t n_{1,t} - \tau_{1k,t} k_{1m,t} \\ &\quad - \tau_{1,t} \left[(r_{1m,t} - \delta_{1m} - \tau_{1k,t}) k_{1m,t}) + r_{1u,t} k_{1u,t} - x_{1u,t} \right] - x_{1m,t} - x_{1u,t}, \end{aligned} \tag{9}$$

that is, what is left over after workers have been paid, taxes on property and profits have been paid, and new investments have been purchased.

The return on a one-period bond, which we refer to as the risk-free rate, is given by

$$r_{f,t} = \left[\beta E_t \frac{c_{t+1}^{-\sigma} \ell_{t+1}^{\psi(1-\sigma}}{c_t^{-\sigma} \ell_t^{\psi(1-\sigma}} \right]^{-1} - 1, \tag{10}$$

where $c^{-\sigma}\ell^{\psi(1-\sigma)}$ is the marginal utility of consumption. The value or price of the bond is simply the inverse of the gross return, $1/(1 + r_{f,t})$.

We can use these formulas for the asset values and returns to assess whether or not our model is consistent with U.S. observations. We turn to that next.

B. Findings

In this section, we make theoretical predictions for the United States stock market. In doing so, we first abstract from uncertainty and price corporate equity and risk-free debt using a deterministic version of the model. Unlike for the stochastic version, calculations of these quantities are trivial. We then show that, for all practical purposes, the results are the same in the deterministic and stochastic versions of the model when we introduce uncertainty consistent with the behavior of U.S. data.

The deterministic version

We work first with the steady state of a deterministic version of the model. We derive an estimate for the return to capital using data from the U.S. noncorporate sector. We then derive an estimate for the size of the intangible capital stock. We choose the level of intangible capital so that the returns to capital in the corporate and noncorporate sectors are equated. With the estimate for intangible capital and data on measured corporate capital and taxes paid in the corporate sector, we can estimate the value of the stock market.

Absent uncertainty, the return to corporate equities and the return to a bond that pays 1 for sure in the following period are both equal to i defined to be

$$i = \frac{(1+\gamma)^\sigma}{\beta} - 1 \tag{11}$$

where γ is the growth of the technology parameter, z_t. This follows directly from the first-order conditions of the household. In fact, if there is no uncertainty, then the returns to each type of capital are also given by i and the following is true

$$i = (1 - \tau_1)(r_{1m} - \delta_{1m} - \tau_{1k}) = (1 - \tau_2)(r_2 - \delta_2 - \tau_{2k}) = r_{1u} - \delta_{1u}. \tag{12}$$

Assuming the United States economy is roughly in a steady state, we can estimate i using the National Income and Product Accounts. In Table X, we report average values for income, product, and capital of the United States. The first column are the accounting concepts used by NIPA. The second column is the average values over the period 1990-1999 relative to GNP. Notice that GNP, listed under 'Product', is normalized to 1. Notice also that the sum of value added for the corporate and noncorporate sectors is equal to GNP.

Corporate income is domestic income of corporations reported. (See Table 1.15 of the *Survey of Current Business*.) We take noncorporate income to be the difference between national income and corporate income. (See Table 1.9 of the SCB for national income.) Thus, noncorporate income includes income of households, the government, noncorporate business, and foreign subsidiaries. For compensation of the noncorporate sector, we include total compensation and 80 percent of proprietor's income. Profits of the noncorporate sector include profits of foreign subsidiaries, rental income, and 20 percent of proprietor's income.

We make several adjustments to noncorporate profits and noncorporate capital before constructing an estimate of the return to capital in that sector. Consider noncorporate profits. First, we add net interest (equal to .042 GNP) and subtract the part of this net interest (equal to .022 GNP) that is imputed financial services. Business purchases of financial services are intermediate goods and therefore not final product.[1] Most of the .022 GNP adjustment are for services implicitly purchased by homeowners with mortgages. Therefore, on the product side, we lower consumption services by an equal amount.

The second adjustment that we make to noncorporate profits is the addition of imputed capital services to government capital and to consumer durables. The U.S. system of accounts uses a zero percent interest rate when imputing services to government capital. We instead use the average return on capital in the noncorporate sector. In order that income equals product, we add imputed services to profits in the noncorporate sector and to government consumption. In the U.S. system, consumer durables are treated as consumption. We instead treat them as investment and impute services to these durables. These imputed capital services are added to profits in the noncorporate sector and to private consumption.

We make one adjustment to the capital stock of the noncorporate sector. Capital stocks reported by the Bureau of Economic Analysis include only capital located in the United States. However, some national income is produced by capital abroad, namely corporate profits of foreign subsidiaries. We estimate that the capital used abroad is equal to the profits divided by the return on capital.

We are now ready to estimate the return on capital in the noncorporate sector. It is equal to

$$i = \frac{\text{profits} \ + \ \text{net interest} \ - \ \text{intermediate services} \ + \ \text{imputed services}}{\text{capital} \ + \ \text{capital of foreign subsidiaries}} \tag{13}$$

$$= \frac{.044 + .042 - .022 + (.595 + .292)i}{2.162 + .012/i} \tag{14}$$

where .595 GNP is the net stock of government capital, .292 GNP is the net stock of consumer durables, and .012 GNP is the profits from foreign subsidiaries. All other numbers are taken from Table X. The value of i that satisfies (14) is 4.08%. Therefore, our estimate of the imputed services to capital is .036, and our estimate of the capital used by foreign subsidiaries is .294.

It is interesting to compare the return we calculate here with the risk-free rate on inflation-protected bonds. The average return on 5-year inflation-protected bonds was 3.99% in the first quarter of 2000. The average return on the 30-year inflation protected bond was

4.19%.

We turn next to our estimate of the value of domestic corporations. For this, we need the value of measured tangible capital and an estimate of the value of unmeasured intangible capital. In the deterministic version of our model, the after-tax returns must be equal for each of the three types of capital. This requirement ties down the size of unmeasured corporate capital.

In Table X, measured capital as reported in the Survey of Current Business *Fixed Reproducible Tangible Wealth* is .822 GNP. This measure does not include inventories or land. Inventories are however available in NIPA so we add them. Land is not included in NIPA estimates but it is in the *Flow of Funds* for nonfinancial corporate business. The difference between real estate values reported in FOF and nonresidential structures reported in NIPA is .06 GNP. Thus, our estimate of measured capital, with land and inventories included, is 1.043 GNP.

The next step is obtaining an estimate for unmeasured capital in the corporate sector. Assuming that returns of the noncorporate and corporate sector are equal, we have

$$\begin{aligned} i &= \frac{\text{after}-\text{tax profits from measured capital}}{\text{measured capital}} && (15) \\ &= (1-\text{corporate tax rate})[\text{NIPA profits} + \text{unmeasured investment} \\ &\qquad -\text{the return on unmeasured capital}]/\text{measured capital} && (16) \end{aligned}$$

Using values from Table X, this implies

$$.0408 = (1 - .026/.073)(.073 + .03k_{1u} - .0408k_{1u})/1.043 \qquad (17)$$

where .026 GNP is the tax paid on domestic corporate profits, .073 is NIPA profits, .03 is the growth rate of GNP, and .03 k_{1u} is the value of unmeasured net investment in the steady state. The solution to this equation is $k_{1u} = .639$. Therefore, unmeasured investment is equal to .019 GNP.

Given our estimate for unmeasured capital, we can now compute the market value of domestic corporations using the formula (7). Assuming that the time period is not too long, the value is given by

$$v = k_{1m} + (1 - \tau_1)k_{1u} = 1.455 \tag{18}$$

where $\tau_1 = .356$ (or .026/.073).

To compare this estimate to market value of U.S. corporations, we need to add in foreign subsidiaries. Let v_{US} be the market value of U.S. corporations. Then

$$v_{US} = v + .294 = 1.75. \tag{19}$$

According to the *Flow of Funds*, the market value of domestic corporations at the end of the first quarter of 2000 was 17,228 billion dollars. GNP in the first quarter of 2000 was 9,655 billion dollars. Therefore, the market value of domestic corporations is 1.78 GNP – very close to what theory predicts.

The stochastic version

We are now ready to work out the implications of a stochastic version of the model. With uncertainty, we expect that risky assets, like corporate equities, would be paid a risk premium. In this section, we quantify this premium.

To do this, we must first calibrate the model. We do this in three steps. First, we compute a steady state for the model that is consistent with the adjusted accounting measures in Table X. Second, we choose parameters – or means of stochastic parameters – that are consistent with these steady state values. Third, we choose stochastic processes for shocks that lead to fluctuations in model variables that are comparable to their U.S. analogues.

To compute a steady state of the model we need to make some further adjustments to the NIPA data so that they are consistent with the model concepts. The adjustments that we have discussed so far are (*i*) the addition of unmeasured investment; (*ii*) the subtraction of intermediate financial services; (*iii*) the imputation of durable and government capital services); and (*iv*) adjustments to the capital stocks. The final adjustments are adjustments for sales and excise taxes, depreciation of consumer durables, and adjustments to investment and depreciation for foreign subsidiary capital.

The U.S. system of accounts includes sales taxes in its measure of private consumption. On the income side, these taxes are included in indirect business taxes. In our model we treat consumption c as pre-tax and therefore subtract sales taxes from both sides of the accounts. Consumer durables are treated as private consumption in NIPA and as investment in our model. Therefore, we include depreciation of consumer durables. On the income side this depreciation appears in noncorporate capital consumption. On the product side this is consumption services. This is the procedure used for housing services which are included in NIPA. Finally, because profits of foreign subsidiaries are included in national income (and therefore in noncorporate profits), we added an estimate of the capital of these subsidiaries to noncorporate capital. To make the depreciation and investment of the noncorporate sector comparable to the capital stock, we include depreciation and net investment for the foreign subsidiaries. We assume that depreciation rates and growth rates are the same at home and abroad to make these estimates.

The adjusted values for income, product, and capital are treated as a steady state for the model. In Table Y, notice that the values for income, product, and capital are the adjusted values of Table X. In the rightmost column, we show the relevant expression for the model. We also include values for hours, growth rates, and tax rates. In the U.S., hours per person are roughly 1/4 of discretionary time. The growth rates are averages for 1990-1999 for total factor productivity and population. With the exception of the labor tax rate, we used NIPA values reported in Table X to calculate tax rates. We have data for corporate profits before- and after-tax. We assume that the noncorporate tax rate is 0 because the main categories of noncorporate income are untaxed. Property taxes and consumption taxes are indirect business taxes. Consumption taxes are .047 GNP, and property taxes are the remaining .032 GNP.

The labor tax rate is more difficult since the U.S. income tax is progressive, while taxes in our model economy are proportional. Households in the federal 28 percent or higher tax bracket pay nearly all of the income tax. However, because of fringe benefits and before-tax contributions to retirement plans, their marginal tax rates effectively lower than 28 percent. Therefore, we chose the tax rate on labor income to be 25 percent. But our analysis is not sensitive to the exact rate used.

In Table Z, we back out depreciation rates, capital shares, and parameters for the final goods technology and the utility function. Most of these parameters can be tied down by steady state values. There are two exceptions: the elasticity of substitution of corporate and noncorporate goods and the risk aversion parameter. For these we experiment with different values and compare second moments of the model and data.

The final choices are the stochastic processes. We start simply assuming that only the technology parameter z_t is stochastic, with the process given by:

$$\log z_{t+1} = \log z_t + \log(1+\gamma) + \varepsilon_{t+1}, \tag{20}$$

where ε_t is i.i.d. normal with mean 0. For now, assume capital shares and fiscal variables are constant.

Following Jermann (1998), we compute a linear approximation to the decision rules for capital. All other variables, including equity returns, can be determined in a nonlinear way. Once we compute the equilibrium we can simulate the economy and calculate model statistics comparable to the data. Doing this, we find that our calculated return on corporate equities is only slightly higher than the return on a risk-free bond, roughly .02 percent.

If we allow for variations in capital shares and fiscal variables consistent with variations in the data, we still find small risk premia – never larger than .1 percent. If we allow for larger values of σ as suggested by the finance literature, we can increase the risk premia but we then have counterfactually large risk-free rates. If we additionally adjust β to keep i at 4.08 percent, then we do find that the equity premium is small.