Misallocation or Risk-Adjusted Capital Allocation?

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Abstract

Standard, frictionless neoclassical theory of investment predicts that the expected corporate marginal product of capital (MPK) depends on firms' exposure to systematic risk and the price of that risk. This implies that the cross-sectional dispersion in MPK i) depends on cross-sectional variation in risk exposures, and ii) fluctuates with the price of risk, and thus is countercyclical. We empirically evaluate these predictions, and document strong support for them. In particular, a long-short portfolio of high minus low MPK stocks earns significant and countercyclical excess returns forecastable by standard return predictors. A calibrated investment model suggests that ex ante variation in risk exposure can rationalize permanent dispersion in MPK. These findings suggests that a substantial fraction of dispersion in MPK, often dubbed misallocation, is effectively risk adjusted capital allocation.

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1 Introduction

A large and growing body of work has documented the 'misallocation' of resources across firms, measured by dispersion in the marginal product of factors of production. This failure of marginal product equalization has been shown to have potentially sizable negative effects on aggregate outcomes, such as productivity and output. Recent studies have found that even after accounting for a host of leading candidates as sources of misallocation - for example, adjustment costs, financial frictions, or imperfect information - a large role is played by firm-level 'distortions,' specifically, of a class that are orthogonal to firm fundamentals and are permanent to the firm. Identifying exactly what underlying economic mechanism can lead to this type of distortion has proven puzzling.

In this paper, we propose, empirically test and quantitatively evaluate just such a mechanism. Our approach links capital misallocation to systematic investment risks. To the best of our knowledge, we are the first to make the connection between standard notions of the risk-return tradeoff faced by investors and the resulting dispersion in the marginal product of capital. Our point of departure is a standard model of firm investment in the face of both aggregate and idiosyncratic shocks. Firms discount future payoffs using the stochastic discount factor of a representative household, which is also a function of aggregate conditions. With little more structure than this, the framework gives rise to an asset pricing equation governing the firm's expected marginal product of capital (MPK): firms with higher exposure to the aggregate shock have a permanently higher expected MPK (realized MPKs may differ across firms for additional reasons, i.e., adjustment frictions and uncertainty over future shocks). In fact, the model implies a beta pricing equation of exactly the same form used to price the cross-section of stock market returns. That beta pricing equation simply states that a firm's expected MPK should be linked to the exposure of MPK to systematic risk, and the latter's price.

Although this result is quite general, we provide a number of illustrative examples. If we shut down aggregate shocks (or assume risk neutrality), expected MPKs are equated across firms. In this case, indeed, the standard notion from macroeconomics obtains that relates any dispersion of MPK to misallocation. In any other situation, our beta pricing equation implies that cross-sectional dispersion in risk exposure should be reflected in cross-sectional dispersion in MPK. If the utility function is CRRA, for example, expected MPKs are determined by the Consumption CAPM equation, i.e., by the covariance of each firm's MPK with aggregate consumption growth. If aggregate and firm-level conditions are driven by technology shocks, expected MPKs are determined by the covariance of the MPK with innovations in the aggregate process. In a world driven by multiple risk factors, as is typically considered in the current asset pricing literature, the average MPK is linked to exposure to these various factors, as well as their factor prices.

The bulk of our paper is devoted to demonstrating that the simple beta pricing equation inherent in the neoclassical model of firm investment, effectively has substantial empirical content. Our empirical analysis suggests that a substantial fraction of the disperson of MPK in US data can be rationalized by cross-sectional variation in risk exposure and movements in risk prices. More precisely, we state and empirically investigate four predictions of our general framework.

First of all, the beta pricing equation predicts that exposure to standard risk factors priced in asset markets is an important determinant of expected MPK. We provide empirical evidence supporting this prediction in two ways. First, we directly determine the exposure of firm and portfolio level MPK to various risk factors emerging in current asset pricing models, from consumption growth, the market portfolio, the Fama-French factors, and most recently, the Q-factors, and show that MPKs are indeed significantly related to these factors. Second, given a variety of empirical challenges to directly computing MPK betas at the firm level, such as seasonality, we exploit the tight relationship between MPK and investment returns on the one hand, and stock returns on the other hand, as pointed out by Cochrane (1991) and Restoy and Rockinger (1994), and use an asset pricing approach for testing. More precisely, we sort stocks into decile into portfolios based on their MPK and evaluate the portfolio returns. Remarkably, we find that the expected excess returns on these decile portfolios are increasing in MPK, so that a high minus low (HML) MPK portfolio earns an annual premium of more than six percent before, and more than 3 percent after industry adjustment. We show that the high MPK portfolios has higher exposure to standard risk factors, consistent with the prediction of the beta pricing equation, suggesting that a higher MPK is linked to higher systematic risk.

Consistent with standard asset pricing models, a beta pricing equation does not only entail a cross-sectional prediction regarding cross-sectional variation in expected MPK, but also a time-series prediction linked to movements in factor risk prices. In particular, it suggests that movements in factor risk prices are linked to fluctuations in the conditional expected MPK. Again, we test this prediction on both direct estimates of expected MPK as well as MPK sorted stock portfolios. Consistent with the emprical evidence from the return predictability literature in asset pricing, suggesting a countercyclical price of risk, we indeed find the excess returns on MPK sorted portfolios are predictable and countercyclical, as suggested both by common return predictors such as credit spreads, as well as lagged macroeconomic variables, such as consumption growth.

These tests suggest that risk factors are indeed a significant determinant of MPK, both in the cross-section and in the time series. Our next predictions and tests aim at further dissecting the role of risk factors in determining the cross-sectional dispersion in MPK, which has commonly been associated with 'misallocation'. In that regard, the beta pricing equation suggests that MPK dispersion should be positively related to beta dispersion. In particular, in the cross-section, industries with higher dispersion in betas should display higher dispersion in MPK. We test this prediction in a two stage procedure that first determines betas with respect to standard risk factors, and then uses the dispersion of the betas as an explanatory variable for industry level MPK dispersion. Consistent with the neoclassical investment model, we find that beta dispersion is a significant determinat of MPK dispersion, explaining a substantial fraction of its inherent variation.

The prediction regarding determinants of MPK dispersion has a natural time series analogon, in that movements in factor prices should be linked to fluctuations in MPK dispersion. In particular, given likely countercyclical risk prices, a frictionless neoclassical model predicts a countercyclical dispersion in MPK. We test this notion by evaluating the time series properties of the MPK-HML portfolio, and find that its positive expected excess returns are highly predictable, and in fact countercyclical, as indicated by standard return and macroeconomic predictors such as credit spreads, unemployment rates and lagged consumption growth. This suggests that not only do high MPK firms earn higher premia, but the spread in the cost of capital between high and low MPK firms is rising in downturns. In other words, high MPK firms, while potentially more productive, become riskier in recessions.

After establishing these empirical results, we interpret them and gauge their magnitudes through the lens of a quantitative model. To that end, we calibrate and structurally estimate a simple neoclassical, dynamic investment model with adjustment costs that we augment with an exogenously specified stochastic discount factor designed to match standard asset pricing moments, as has become standard in the cross-sectional asset pricing literature, as in Zhang (2005) and Gomes and Schmid (2010). Our point of departure from these model is that we allow for ex ante cross-sectional heterogeneity in exposure, that is, beta, with respect to a single source of risk. This extension allows us to gauge what fraction of dispersion in MPK can be rationalized by a realistic amount of permanent ex ante heterogeneity in betas, with or without additional frictions, such as convex adjustements costs. Our preliminary results suggest that in a simple dynamic model such ex ante heterogeneity can rationalize more than fifty percent of the empirical dispersion, the rest being accounted for by other frictions, such as adjustment costs or perhaps financial frictions.

Related Literature. Our paper relates to several branches of the literature. First is the large body of work investigating and quantifying the effects of resource misallocation across firms, seminal examples of which include Hsieh and Klenow (2009) and Restuccia and Rogerson (2008). A number of papers have explored the role of particular economic forces in leading to misallocation. For example, Asker et al. (2014) study the role of capital adjustment costs, Midrigan and Xu (2014), Moll (2014), Buera et al. (2011) and Gopinath et al. (2015) financial frictions, and David et al. (2016) information frictions. David and Venkateswaran (2016) provide a unified theoretical framework and empirical methodology to estimate the contribution of each of these forces to misallocation and find that each can explain only a limited portion of observed dispersion in the marginal product of capital. They conclude that firm-specific distortions account for the lion's share of misallocation and, specifically, point out the large role of a permanent component of firm-level idiosyncratic distortions. We build on this literature by exploring the implications of a different dimension of financial markets for marginal product dispersion, namely, the risk-return tradeoff faced by optimizing investors. Importantly, our theory generates what appears to be a permanent firm-specific 'wedge' exactly of the type found by David and Venkateswaran (2016), but which here is a function of each firm's exposure to aggregate risk. Additionally, our quantitative framework encompasses several of the channels highlighted in these papers, i.e., adjustment costs and information frictions. The addition of aggregate risk is a key innovation of our analysis - existing work has abstracted from this channel. We show that the link between aggregate risk and misallocation is quite tight in the presence of heterogeneous exposures to that risk.

A growing literature, starting with Eisfeldt and Rampini (2006), investigates the reasons underlying the observation that capital reallocation is procyclical. This indeed seems puzzling as given higher cross-sectional dispersion in MPK in downturns one should expect to see capital flowing to highly productive, high MPK firms in recessions. Our results bear on that observation by noting that given a countercyclical price of risk, and a countercyclical premium on the MPK-HML portfolio, from a risk perspective, capital reallocation to high MPK firms would require capital flow to the riskiest of firms.

Our work exploits the insight, due to Cochrane (1991) and Restoy and Rockinger (1994), that stock returns and investment returns are closely linked. Indeed, under the assumption of constant returns to scale, stock and investment returns effectively coincide. Crucially, for our purposes, investment returns are intimately linked with the marginal product of capital. Balvers et al. (2015) explore and confirm the close albeit more complicated relationship under deviations from constant returns to scale. In this context, our work is closely related to the growing literature that examines the cross-section of stock returns by viewing them from the perspective of investment returns, starting from Gomes et al. (2006); Liu et al. (2009), and recently forcefully summarized in Zhang (2017). This literature interprets common risk factors as the Fama-French factors through firms' investment policies, and most recently, shows that risk factors related to corporate investment patterns themselves capture risks priced in the cross-section of returns, culminating in the recent Q-factor model. Our objective is quite different and in some sense turns that logic on its head, in that we examine investment returns and the marginal product of capital as a manifestation of exposure to systematic risk, most readily measured through stock returns.

2 Motivation

We consider a discrete time, infinite-horizon economy. A continuum of firms produce output using capital and labor. Labor is chosen period-by-period in a spot market at a competitive wage. At the end of each period, firms choose investment in new capital, which becomes available for production in the following period, so that $K_{it+1} = I_{it} + (1-\delta) K_{it}$ where δ is the rate of depreciation.

Let $Y_{it} = Y\left(\tilde{Z}_{it}, K_{it}\right)$ denote the operating profits of the firm - revenues net of wages - where $\tilde{Z}_{it}$ is a firm-specific shock and K_{it} the firm's level of capital. The firm-specific fundamental $\tilde{Z}_{it}$ is composed of both an idiosyncratic and an aggregate component. The analysis accommodates a number of potential interpretations of $\tilde{Z}_{it}$, for example, firm-level productivity or demand shifters. Although no further assumptions are needed for now, we will generally work with the formulation $Y_{it} = \tilde{Z}_{it} K_{it}^{\theta}$, $\theta \leq 1$, which arises under both interpretations. The payout of the firm is equal to $D_{it} = Y_{it} - I_{it}$.

Denote by M_{t+1} the stochastic discount factor (SDF) of the household, which may be correlated with the aggregate component of firm fundamentals. We can write the firm's problem in recursive form as

$$V\left(\tilde{Z}_{it}, K_{it}\right) = \max_{K_{it+1}} \ F\left(\tilde{Z}_{it}, K_{it}\right) - K_{it+1} + (1-\delta) K_{it} + E_t\left[M_{t+1} V\left(\tilde{Z}_{it+1}, K_{it+1}\right)\right] \tag{1}$$

Standard techniques give the Euler equation

$$1 = E_t\left[M_{t+1}\left(MPK_{it+1} + 1 - \delta\right)\right] \quad \forall i, t$$

where $MPK_{it+1} = \frac{\partial Y_{it+1}}{\partial K_{it+1}}$ is the marginal product of capital of firm i at time $t+1$.

MPK dispersion. Assuming a single source of aggregate risk for the sake of illustration, from the Euler equation, it is straightforward to derive the following factor model for expected MPK:[1]

$$E_t\left[MPK_{it+1}\right] = \alpha_t + \beta_{it}\lambda_t \tag{2}$$

[1] The derivation is in Appendix B.

where α_t is the 'risk-free' MPK, which equals the riskless user cost of capital $r_{ft} + \delta$, $\beta_{it} \equiv -\frac{\text{cov}(M_{t+1}, MPK_{t+1})}{\text{var}(M_{t+1})}$ and $\lambda_t \equiv \frac{\text{var}(M_{t+1})}{E_t[M_{t+1}]}$ is the market price of risk. In the language of asset pricing,the Euler equation gives rise to a conditional one-factor model for conditional excepted MPK. Expression (2) shows that the expected MPK is not necessarily common across firms and is a function of the risk-free return, the firm's β on the SDF and the market price of risk. The cross-sectional variance of date-t conditional expected MPK is equal to

$$\sigma^2_{E_t[MPK_{it+1}]} = \sigma^2_{\beta_t} \lambda_t^2 \tag{3}$$

which shows that the extent to which risk considerations lead to dispersion in the MPK depends on the cross-sectional dispersion in β and the level of the price of risk. Taking unconditional expectations, the theory can clearly generate 'permanent' dispersion in MPK, which is equal to dispersion in required rates of return across firms:

$$E\left[MPK_i\right] = \alpha + \beta_i \lambda + Cov(\beta_{it}, \lambda_t) \tag{4}$$

where $\beta_i = E\left[\beta_{it}\right]$ and $\lambda = E\left[\lambda_t\right]$ denote the unconditional expectations of conditonal MPK factor betas and factor prices. A lower bound for the cross-sectional dispersion in MPK implied by this simple structure is therefore given by

$$\sigma^2_{E[MPK_i]} \simeq \sigma^2_{\beta} \lambda^2 \tag{5}$$

We note that this observation generalizes in a straightforward manner to environments more recently considered in the cross-sectional asset pricing literature emphasizing multiple factors driving aggregate risks. Most prominently, beyond excess returns on the market portfolio and innovations to aggregate consumption rowth considered in the classical CAPM and Breeden-Lucas Consumption CAPM, these risk factors have been linked to excess returns on size, as well as book-to-market sorted portfolios (Fama-French factors), or investment returns or profitability (Zhang's Q factors).

Quantitatively, the strength of the mechanism linking dispersion in MPK to exposure to aggregate risk can be gauged in back of the envelope calculations by inspection of relationship (5). Clearly, the predicted MPK dispersion is increasing in the dispersion in betas, but also in the market price of risk λ. A key observation underlying our analysis is that asset price data suggest that risk prices are rather high. A lower bound is given by the Sharpe ratio on the market portfolio, estimated to be around 0.4. However, even easily implementable trading strategies such as those based on value-growth portfolios, or momentum, suggest numbers closer to 0.8, while hedge fund strategies report Sharpe ratios in excess of one. Taken at face value, these numbers imply substantial MPK dispersion even in frictionless models after taking risk exposure in account.

Empirical Predictions. Even under the general structure we have outlined thus far, expressions (2), (3) and (5) have much empirical content. They all contain both a 1) cross-sectional and a 2) time seris prediction.Specifically:

1. Exposure to standard risk factors is a determinant expected MPK. Expression (2) shows that the same factors that determine the cross-section of stock returns - namely, exposure to the SDF - determine the cross-section of MPK.

2. Predictable variation in the price of risk λ_t leads to predictable variation in expected MPK. In particular, conditional, expected MPK should be countercyclical. This is the time-series equivalent of expression (2). Keeping the β's fixed, movements in λ_t should affect expected MPK.

3. MPK dispersion should be related to beta dispersion. Expression (5) shows that unconditional variation in the cross-section of MPK is proportional to the variation in β. Segments of the economy, e.g., industries with higher dispersion in β should display higher dispersion in MPK.

4. MPK dispersion should be positively correlated with the price of risk. Expression (5) links MPK dispersion to time variation in the price of risk. For a given degree of cross-sectional in β, when compensation for bearing risk increases, MPK dispersion should increase as well.

Illustrative examples. Section 3 investigates these predictions in detail. Before doing so, however, it is useful to consider a number of more concrete illustrative examples.

Example 1: no aggregate risk or risk neutrality. In the case of no aggregate risk, we have $\beta_{it} = 0 \; \forall \; i, t$, i.e., all shocks are idiosyncratic to the firm. Expression (3) shows that there will be no dispersion in expected MPK and for each firm, $E_t \left[MPK_{it+1} \right] = \frac{1}{\rho} - (1 - \delta) = r_{ft} + \delta$ where ρ denotes the household's subjective discount factor, which is simply the riskless user cost of capital. This is the standard expression from neoclassical growth theory determining the rate of return on capital and is reminiscent of the results from the stationary models widely used in the misallocation literature, for example, HK and RR, that in a frictionless environment, MPK should be equalized across firms.[2] It is straightforward to show this expression also determines the expected MPK in the presence of aggregate shocks but under risk neutral preferences, which implies $M_{t+1} = \rho \; \forall \; t$.

Example 2: CCAPM. In the case that utility is CRRA with coefficient of relative risk aversion γ, standard approximation techniques give the pricing equation from the consumption capital asset pricing model:

$$E_t \left[MPK_{it+1} \right] = \alpha_t + \underbrace{\frac{\text{cov}\left(\Delta c_{t+1}, MPK_{it+1}\right)}{\text{var}\left(\Delta c_{t+1}\right)}}_{\beta_{it}} \underbrace{\gamma \text{var}\left(\Delta c_{t+1}\right)}_{\lambda_t}$$

Expected MPK is determined by the covariance of each firm with consumption growth (i.e., its consumption β). The market price of risk is the product of the coefficient of relative risk aversion and the variance of consumption growth.

Example 3: Technology shocks. Thus far we have not taken a stand on the underlying shocks that drive the SDF and firm fundamentals. A natural case to consider is technology shocks. Assume that firm operating profits take the form $Y_{it} = \tilde{Z}_{it} K_{it}^{\theta}$ for $\theta \leq 1$. Firm-level productivity (in logs, denoted by lowercase) is defined as $\tilde{z}_{it} = z_{it} + \beta_i z_t$, where z_{it} is an idiosyncratic component, z_t an aggregate one, and β_i captures the exposure of firm i to the aggregate shock. The shocks follow AR(1) processes[3]

$$\begin{aligned} z_{t+1} &= \varphi z_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma_{\varepsilon}^2\right) \\ z_{it+1} &= \tilde{\varphi} z_{it} + \varepsilon_{it+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma_{\tilde{\varepsilon}}^2 - \beta_i^2 \sigma_{\varepsilon}^2\right) \end{aligned}$$

We directly specify the (log of the) SDF as $m_{t+1} = \log \rho + \gamma \left(z_t - z_{t+1}\right)$. We show in Appendix B that the realized MPK of firm i is equal to

$$mpk_{it+1} = \alpha_t + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \beta_i \gamma \sigma_{\varepsilon}^2$$

where α_t is a time-varying term, but is constant across firms, so does not lead to MPK dispersion. Dispersion in realized MPK can be due to uncertainty over the realization of idiosyncratic and aggregate shocks (the first two terms, respectively), as well as a risk premium that is increasing in the firm's exposure to the aggregate shock β_i. The expected MPK only depends on

[2]With the time-to build technology and idiosyncratic shocks, there may be dispersion in *realized* MPK, but not in *expected* terms, and so these do not lead to persistent deviations.

[3]The variance of the idiosyncratic shock ensures that all firms have the same expected value of $\tilde{Z}$.

the last term:

$$E_t\left[mpk_{it+1}\right] = \alpha_t + \beta_i \gamma \sigma_\varepsilon^2$$

so that dispersion in expected MPK depends only on dispersion in firm-level β, i.e., differences in exposure to the aggregate shock.

It is straightforward to extend this example to account for a time-varying price of risk. Movements in that price could come about by fluctuations in the amount of risk, that is σ_ε, which would be likely countercyclical, or else, by movements in agents' attitudes towards risk, such as γ. Indeed, countercyclical risk aversion features prominently as an explanation for return predictability in the habit paradigm, pioneered by Campbell and Cochrane (1999), while countercyclical volatility is proposed in the long-run risk framework of Bansal and Yaron (2004).

3 Empirical Results

In this section we investigate the empirical predictions outlined above. To do so, we assume $Y_{it} = \tilde{Z}_{it} K_{it}^{\theta}$ which implies the log MPK is $mpk_{it} = y_{it} - k_{it}$.

Data. We use data on a sample of nonfinancial firms from both the Center for Research in Security Prices (CRSP) and S&P's Compustat. From these sources, we obtain a panel of firms with measures firm marginal products of capital, equity returns, market capitalization, and other firm characteristics. Our dataset comprises of common equities listed on the NYSE, NASDAQ, or AMEX from 1962 to 2014, on which we have measures of the marginal product of capital. We supplement this panel with time series data on market factors and aggregate conditions related to the market price of risk. The market factors we consider are the Fama and French (1992) factors, Hou, Xue, and Zhang (2015) investment-capm factors, as well as the growth rate of the sum of aggregate non-durable and services consumption from the Bureau of Economic Analysis (BEA). We also use data on aggregate macroeconomic and financial market variables from the BEA and the Gilchrist and Zakrajsek (2012) (GZ) spread.[4]

To compute a measure the marginal product of capital of a firm, we require data on firm output and capital. Our primary measure of output is sales[5] Our primary measures of the capital stock is the (net of depreciation) value of plant, property, and equipment (PPENT). Our results yield qualitatively similar results when we use alternative measures of output and capital.[6]

[4]We obtain measures of the GZ spread from Simon Gilchrist's website.

[5]Sales is measured using Compustat item SALE (and SALEQ for quarterly data).

[6]We measure value added ias the sum of operating income before depreciation (OIBDP) and labor compensation (measured as XLR, unless it is missing, in which case we use XSGA). The alternative measures of firm capital stock we consider are the book value of firm assets (AT) and the gross value of plant, property, and

Empirical Strategy. We can now revisit the main predictions from Section 2.

1. Exposure to standard risk factors is a determinant of expected MPK.

The MPK and stock return are tightly related. For example, with $\rho = 1$, it is well known that they are identical state by state and period by period, i.e., $R^s_{it+1} = R^I_{it+1} = MPK + \delta$.[7] With decreasing returns to scale, the relationship is no longer exact, but we can prove (1) for sufficiently small decreasing returns, i.e., α sufficiently close to 1, $r_{it+1} \approx mpk_{it+1} + \delta$, i.e., the exact relationship holds to a first-order approximation. Moreover, in the more general case, we can show corr $\left(r^s_{it+1}, r^I_{it+1}\right) > 0$, i.e., the two are positively correlated.

To investigate this implication of our framework, we sort firms into 10 portfolios based on MPK, where portfolio 1 contains low MPK firms and portfolio 10 high MPK ones. To compute portfolio returns, we follow Fama and French (1992). We construct measures of firm marginal products of capital using data from COMPUSTAT. We then construct breakpoints for portfolios every year based on the percentiles of firm MPK. To avoid 'look-ahead bias', we use the MPK reported by firms in their fiscal-year-end filing in date t-1 with firm returns from July of year t to June of year t+1.

We report the results in Table 1. The table shows a strong relationship between MPK and stock returns - portfolios with higher MPK earn higher excess returns. This relationship persists even when we control for differences in MPK across industries. Table 2 reports this portfolio sort, where firm MPK is demeaned by industry-year.

Table 5 explores whether widely used asset pricing models capture the variation in excess returns across MPK-sorted portfolios, specifically, the Fama-French 3 factor model. The table shows that MPK portfolios in general load on all three factors. The final column of the table shows the difference between the high MPK and low MPK portfolio returns. This difference, while statistically significantly correlated with market factors, can only partially be explained by them. Table 6 reports these regressions on firm-industry demeaned MPK portfolios.

A direct implication of our theory states that firm-specific exposure to aggregates shocks, 'Betas', should predict cross-sectional variation in firm MPK (less the risk free rate). To test this, we first compute firm Betas from time series regressions of each firm's stock returns on return factors. We consider the capital asset pricing model, Fama and French (1992) three factors, Hou, Xue, and Zhang (2015) investment-capm factors, and consumption growth CAPM. We run this time series regression at the firm level using a rolling window of lagged observations.[8]

equipment (PPEGT).

[7] CITES

[8] For monthly returns, we use a two-year rolling window. For quarterly returns, we use a five-year rolling window. We use quarterly data on consumption growth, so we only run the consumption growth CAPM

We then compute firm average Betas and firm average log MPK. Table 7 displays the regression of firm average $log\,(MPK - rf)$ on firm average Betas (from time series regressions of returns). We see that firm betas are significant in explaining variation in MPK, and are usually positive, such as in the capital asset pricing model. These factors remain significant even when including firm average 'alphas', intercepts from the time series regression (Table 8), or after demeaning firm MPK by industry-year (Table 9).[9]

To address the possibility that market factors affect firm MPK and returns differentially, we repeat the exercise above instead computing firm 'MPK Betas', to compute the loadings of firm realized MPK on aggregate factors. We therefore run time-series regression of firm quarterly MPK on quarterly return factors, considering the same factors as before. Tables 10 and 11 display the regression of firm average $log\,(MPK - rf)$ on firm average MPK Betas, without and with industry-year demeaning. The result remains that firm betas are significant in explaining variation in MPK, and are usually positive, such as in the capital asset pricing model.

2. Predictable variation in the price of risk λ_t leads to predictable variation in expected MPK. The market price of risk, λ_t in equation (2), is positively related to the level of firm expected MPK in the following period. To test this, we run a time series regression of average mpk across firms on three lagged measures related to the market price of risk: the Gilchrist and Zakrajsek (2012) (GZ) spread, unemployment, and consumption growth. We consider both 1 quarter and 1 year lags. Table 12 reports the results of these regressions. In line with our theory, the GZ spread and unemployment (which are likely positively correlated with the market price of risk) predict higher future mpk, while consumption growth (likely negatively correlated with the market price of risk) predicts lower future mpk.

3. MPK dispersion should be related to beta dispersion.
Equation 5 implies that for a group of firms, dispersion in expected mpk for a group of firms should be positively related to the dispersion in betas among those firms. In particular, this suggests that dispersion of mpk within an industry, a common measure of misallocation, is affected by dispersion in firm betas. We investigate this prediction using variation in the dispersion of firm-level β across industries. For each industry, we compute the standard deviation of mpk and betas in each year, average across years, and regress the first on the second. Tables 13 and 14 show that indeed, industries with high dispersion in return betas exhibit more

regression for quarterly returns.

[9]The explanatory power as measured by R^2 is low in these regressions, which is not unexpected for cross-sectional regressions of MPK, which have significant challenges relating to measurement and measurement error. Our analyses looking of MPK dispersion within industries, where measurement error is less of a concern, have much greater explanatory power.

dispersion in *mpk*. These results are not only statistically significant, but an individual factor model can explain over 30% of the variation in MPK dispersion across industries. We obtain similar results and R^2 using the inter-quartile range of MPK instead of the standard deviation. Table 15 shows that we obtain the same result when regressing *mpk* dispersion on *mpk* Betas.

4. MPK dispersion should be positively correlated with the price of risk.

Lastly, we explore the relationship between *mpk* dispersion and the price of risk. Equation 2 suggests that the market price of risk is positively related to the dispersion of marginal products of capital across a group of firms. We examine this in our framework in two ways. First, we show that the measures of the market price of risk considered before (the GZ spread, unemployment rate, consumption growth) predict time series variation in measures of MPK dispersion. Second, we show that the future expected return on a long-short MPK portfolio are also predicted by these measures of the market price of risk.

We show that both the unconditional dispersion in *mpk*, and the dispersion of *mpk* within industries are positively correlated with the lagged price of risk. Table 16 displays a time series regression of the unconditional standard deviation of *mpk* on lagged (by one quarter or one year) measures of the GZ spread, unemployment rate, and consumption growth. All three measures of the market price of risk significantly predict *mpk* dispersion, and in the direction our theory would suggest: The GZ Spread and unemployment rate predict greater *mpk* dispersion, while consumption growth predicts lower *mpk* dispersion. Table 17 displays a time series regression of a measures of within-industry *mpk* dispersion on lagged (by one or two years) measures of the GZ spread, unemployment rate, and consumption growth. The dependent variable is computed as the average, across industries j at a given time t, standard deviation of *mpk* within industry j at time t. Again, all three measures predict *mpk* dispersion.

As a final test of this prediction, we construct a long-short MPK portfolio and investigate its relation with market price of risk. The portfolio is long the top decile of MPK firms and short the bottom decile, re-balancing every every June based on MPK from the previous year. Table 18 reports a regression of the cumulative one, two, four, and eight quarter returns on the long-short MPK portfolio on the GZ spread, unemployment rate, and consumption growth. The GZ spread and unemployment rate predict higher future returns on the MPK portfolio, while consumption growth predicts lower future returns.

4 Quantitative Analysis

To interpret our empirical findings, we now build and estimate a simple dynamic model that can reproduce some of our evidence. Our model is deliberately simple in order to isolate the impact of our basic mechanism, namely dispersion in exposure to systematic risk. Indeed, our model is a simple extension of a standard q-theory model of corporate investment which we augment with an exogenously specified stochastic discount factor that allows for a time-varying price of risk, and dispersion in risk exposure. Our setup also allows for convex adjustment costs, but we set those to zero in our baseline specification to focus squarely on our risk channel. Specifying the stochastic discount factor exogenously in partial equilibrium allows to sidetrack challenges with generating empirically relevant risk prices in general equilibrium, and focus on gauging the quantitative strength of our mechanism.

In the following, we briefly outline the model, and provide simulation evidence from a calibration. The estimation is in process.

4.1 Quantitative Model

The model consists of two building blocks: a stochastic discount factor, which we directly parameterize to be consistent with salient patterns in financial markets, and a cross-section of heterogenous firms i, which make optimal investment decisions in the presence of firm-level and aggregate risk, given the stochastic discount factor. We assume that firms differ in their exposure to aggregate risk.

Technology Production requires one input, capital, K_{it}, and is subject to both an aggregate shock X_t, and an idiosyncratic shock, Z_{it}. In the following, we assume that both these processes follow autoregressive processes in logs, so that

$$\begin{aligned}\log X_{t+1} &= \mu + \rho_X \log X_t + \sigma_X \epsilon_{t+1} \\ \log Z_{it+1} &= \rho_Z \log Z_{it} + \sigma_Z \eta_{it+1},\end{aligned}$$

where both are η and ϵ are standard normal, iid. While, for simplicity, we restrict attention to a single source of aggregate risk, X_t, we assume that firms differ in their exposure to it. More precisely, we assume that firms' production function is given by

$$Y_{it} = \tilde{X}_{jt} Z_{it} K_{it}^{\alpha}.$$

Here, $\alpha \leq 1$ captures the degree of returns to scale, which we allow to be decreasing for the sake of realism. More importantly, $\tilde{X}_{jt}$ is given by

$$\log \tilde{X}_{jt} = \beta_j \log X_t,$$

for some β_j. For the time being, we treat β_j as exogenous and parameterize it to capture permanent dispersion in exposure to aggregate risk. Note that this dispersion can capture both within and across industry variation. Heterogeneity thus stems both from idiosyncratic risks as well as from differential exposure to aggregate risk.

Firms are allowed to scale operations by choosing the level of productive capacity K_{it}. This can be accomplished through investment, I_{it}, which is linked to productive capacity by the standard capital accumulation equation

$$K_{it+1} = (1-\delta)K_{it} + I_{it}, \tag{6}$$

where $\delta > 0$ denotes the depreciation rate of capital. A common theme in the literature is to relate dispersion in MPK to adjustment costs to capital. For the sake of comparison, therefore, we allow to capture obstructions and frictions to capital accumulation in a standard way by assuming that firms face convex capital adjustment costs that accrue directly as a deduction from firms' payout. More specifically, we assume that adjustment costs are of the standard form

$$H(I_{it}, K_{it}) = \frac{\theta}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it},$$

where θ captures the magnitude of these costs. For the time being, we assume that they are the same across firms. Setting $\theta = 0$ reduces our model to the standard frictionless investment setup with ex ante heterogeneity in risk exposure considered previously. Varying θ and the cross-sectional dispersion in β_j allows us to gauge and tease out the relative quantitative contribution of these channels to dispersion in MPK. In our baseline results, we set $\theta = 0$ so that only source of variation in expected MPK across firms is differential exposure to the aggregate shock.

Stochastic Discount Factor In line with the vast literature on cross-sectional asset pricing in production economies, we parameterize directly the pricing kernel without explicitly modeling the consumer's problem. Following Zhang (2005), we choose the stochastic discount factor to be of the form

$$\log M_{t+1} \quad = \quad \log \beta + \gamma_t(\log X_t - \log X_{t+1}),$$

where

$$\gamma_t = \gamma_0 + \gamma_1(\log X_t - \mu)$$

and $\gamma_0 > 0$ and $\gamma_1 < 0$. This common specification allows to capture a high and time varying, and as a matter of fact, countercyclical ($\gamma_1 < 0$) price of risk in a simple manner. Additionally, directly calibrating γ_0 and γ_1 allows the model to be quantitatively consistent with key moments about asset returns, which are important for our analysis.

Optimization The optimization problem consists in choosing an investment policy to maximize firm value V_{ijt} in the presence of aggregate and idiosyncratic risk and capital adjustment costs. In the absence of financing frictions, firm value is simply the expected discounted sum of future distributions. Given our assumptions, distributions are given by

$$D_{ijt} = Y_{it} - I_{it} - H(I_{it}, K_{it})$$

Firms' problem can thus be summarized in the following standard Bellman equation

$$V_{it} \equiv V(\beta_j, K_{it}, X_{it}, Z_{it}) = \max_{I_{it}} D_{it} + E_t V(\beta_j, K_{it+1}, X_{it+1}, Z_{it+1}).$$

4.2 Calibration

Our calibration strategy is to pick parameters in order to generate simulated firm panels, comparable to those we consider in our empirical work, with realistic investment behavior and stock returns. On the basis of such panels, we can evaluate to what extent ex ante heterogeneity in risk exposure can contribute to dispersion in expected MPK, with and without additional, complementary frictions such as capital adjustment costs.

We calibrate the model at a quarterly frequency. We solve the model via value function iteration and assess its calibration by means of moments implied by a simulated panel of firms. Our parameterization of the stochastic discount factor is meant to capture a realistic risk free rate, as well as levels and dynamics of risk prices. Note that we restrict ourselves to a single source of aggregate risk in the model, so that it does not reflect the rich factor exposures that firms face in reality. Nevertheless, the model gives rise to a conditional one-factor model. The calibration of the aggregate shock process is standard in the macro literature, and follows Cooley and Prescott (1995). In particular, we set the quarterly conditional volatility, σ_X to 0.007 and the persistence, ρ_X to 0.95. Regarding the stochastic discout factor, we fix γ_0 at 20, to ensure that the model is consistent with high Sharpe ratios witnessed in financial markets. While admittedly high when interpreted as risk aversion, our model is framed in partial equilibrium, so that the parameter does not lend itself directly to that interpretation. Given our objective of examining the implications of high risk prices for corporate behavior, rather than explaining their level, we view this procedure as a reasonable first step. Relatedly, γ_1 is set to -1000, which implies a countercyclical price of risk. That choice implies that in simulated panels aggregate (log) price dividend ratios forecast future market excess returns with coefficient of the relevant negative sign, and realistic magnitudes. Finally, we set $\beta = 0.99$, at quarterly frequency. These choices imply an annualized real interest rate of 2.68% with an annualized volatility of 2.07%. Critically, the implied average Sharpe ratio is 0.38. The baseline calibration of the stochastic discount factor therefore matches well the Sharpe ratio on the market portfolio, but likely only gives a lower bound to Sharpe ratio attainable in financial markets by means of alternative invstment strategies. In particular, popular value-growth strategies attain Sharpe ratios closer to 0.8. We can trace out the implications of various indications of attainable Sharpe ratios in financial markets for MPK dispersion by suitable recalibrations of the stochastic discount factor.

We take most of the remaining parameters from the literature. Indeed, we set returns to scale to 0.7 and depreciation to 0.025, which are standard values in the literature. We set the volatility and persistence of firm-level shocks, σ_Z and ρ_Z to 0.05 and 0.9 respectively, in order to generate realistic cross-sectional dispersion in investment rates and profitability, similar to Gomes (2001).

Regarding, adjustment costs, our baseline case features zero adjustment costs in order to focus on the implications of ex ante heterogeneity in risk exposure. We provide sensitivity by considering a variety of choices of θ, so as to allow to gauge the additional impact of capital adjustment costs on MPK dispersion. In that respect, a benchmark value features low adjustments costs as estimated in David and Venkateswaran (2016), that is, setting $\theta = 0.2$. One application of our quantitative model is to evaluate the effects of varying θ on the cross-sectional dispersion of MPK.

Finally, and critically, we need to pin down the ex-ante dispersion in firms' exposure to systematic risk. We choose the dispersion so as to generate an ex-post dispersion of *stock return*, CAPM betas within an industry, as a benchmark. As reported in the empirical section, within industry that dispersion amounts to about 0.6. We choose the within industry dispersion as our target as other than risk expsoures our model does not feature any industry-specific elements. We choose CAPM betas as a becnhmark, as our model is a conditional one-factor model. We note, however, that the within-industry dispersion of portfolio loadings with respect to other factors, such as HML and SMB, are significantly higher. In that sense, our results are most likely to be interpreted as a lower bound.

4.3 Results

Our objective is to gauge the amount of MPK dispersion that a frictionless dynamic investment model with ex ante heterogeneity in risk exposure can generate, *once calibrated to salient asset market data*. Also, we explore the dynamics of the cross-sectional dispersion in MPK over the business cycle, when we allow for realistically countercyclical movements in risk prices. We do so by examining the statistical properties of simulated panels of firms, from our benchmark calibration as well as relevant variations.

Our first set of results are in table 19. A first account of the results is as follows. For various parameter choices for the basic risk price parameters γ_0 and γ_1, we compute the cross-sectional standard deviation of within industry MPK, our main measure of variation in capital allocation. Note that empirically relevant estimates of this number, obtained in David and Venkateswaran (2016), are in the order of magnitude of around 0.7. Our baseline case, with $\gamma_0 = 20$ and $\gamma_1 = -1000$, gives a model analogon of about 0.26. Tote that this number is obtained by setting $\theta = 0$. Naturally, adjustment costs would add to the endogenous disperison in MPK beyond that obtained from ex-ante heterogeneity in risk exposure, the main channel at work in our setup. Nevertheless, given the low adjustment costs estimated in David and Venkateswaran (2016), that endogenous component in MPK dispersion due to adjustment costs is likely small. Indeed, through the lens of our model, ex-ante heterogeneity in risk exposure accounts for a substantial fraction in the dispersion of MPK. Naturally, increasing the magnitude of adjustment costs would raise the dispersion in MPK, consistent with the intuition in the previous literature, but so does increasing the risk price parameters γ_0 and γ_1, giving theoretical support to the notion that risk exposure, and the pricing of risk are important determinants of MPK dispersion.

5 Conclusion

A long literature in macroeconomics and finance has examined the cross-sectional dispersion in marginal products of capital, from the vantage point of the neoclassical growth model that suggests that efficiency dictates that all capital should be put to its most productive use. That notion is often interpreted as implying that the marginal product of capital should be equalized across firms. Perhaps not surprisingly, however, in the data, there is a significant dispersion in MPK. The latter observation suggests that there should be large aggregate productivity gains by equalizing firms' MPK, so that the current dispersion effectively amounts to 'misallocation'.

In this paper, we revisit the notion of misallocation from the perspective of a risk-sensitive, or risk-adjusted, version of the neoclassical growth model. Indeed, we show that the standard first order condition for investment in that framework suggests that expected firm-level MPK should reflect exposure to factor risks, and their pricing. To the extent that firms are differentially exposed to factor risks, as the literature on cross-sectional asset pricing suggests, that implies that cross-sectional dispersion in MPK may not only reflect misallocation, but also risk-adjusted capital allocation.

We empirically evaluate this proposition, and find strong support for it. Indeed, from an asset pricing perspective, we show that a long short portfolio of high minus low MPK stocks earns a significant premium, so that high MPK firms are effectively riskier, and that this premium is predictable and countercyclical, so that their cost of capital rises disproportionately in bad times. A calibrated dynamic model suggests that, indeed, risk-adjusted capital allocation accounts for a substantial fraction of MPK dispersion.

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Appendix

A Tables

Table 1: Equal Weighted Portfolio Sorts, No industry Adjustment

	1	2	3	4	5	6	7	8	9	10	HML
Return	6.632*	10.48***	11.19***	12.68***	12.59***	13.25***	13.46***	13.01***	13.22***	13.58***	6.583**
	(1.87)	(2.83)	(2.99)	(3.44)	(3.45)	(3.36)	(3.30)	(3.12)	(3.03)	(3.00)	(2.46)
Observations	196	196	196	196	196	196	196	196	196	196	196

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Equal Weighted Portfolio Sorts, Industry Adjusted (sic4)

	1	2	3	4	5	6	7	8	9	10	HML
Return	10.11*	11.48***	10.88***	11.73***	11.15***	11.95***	11.39***	13.33***	13.08***	13.20***	3.286**
	(1.96)	(2.79)	(2.85)	(3.15)	(3.11)	(3.21)	(2.98)	(3.35)	(3.03)	(2.81)	(1.99)
Observations	196	196	196	196	196	196	196	196	196	196	196

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Equal Weighted Portfolio Contemporaneous Returns, No Industry Adjustment

	1	2	3	4	5	6	7	8	9	10	HML
Return	6.026*	9.288**	9.258**	10.26***	10.65***	12.21***	12.86***	14.57***	15.20***	17.69***	11.11***
	(1.68)	(2.44)	(2.42)	(2.72)	(2.86)	(3.13)	(3.11)	(3.36)	(3.39)	(3.74)	(4.05)
Observations	196	196	196	196	196	196	196	196	196	196	196

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Equal Weighted Portfolio Contemporaneous Returns, Industry Adjusted (sic4)

	1	2	3	4	5	6	7	8	9	10	HML
Return	8.909*	8.208*	9.408**	9.386**	10.06***	11.58***	11.05***	13.80***	16.03***	17.67***	8.870***
	(1.71)	(1.95)	(2.41)	(2.55)	(2.76)	(3.05)	(2.86)	(3.28)	(3.45)	(3.60)	(5.17)
Observations	196	196	196	196	196	196	196	196	196	196	196

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Regression of Equal Weighted MPK Porfolios Returns, No industry Adjustment

	1	2	3	4	5	6	7	8	9	10	HML
β_{MKT}	0.941***	1.029***	1.059***	1.020***	1.032***	1.046***	1.059***	1.036***	1.033***	1.016***	0.0749**
	(29.43)	(49.54)	(59.53)	(60.15)	(58.81)	(60.62)	(52.11)	(45.19)	(42.28)	(38.20)	(2.15)
β_{SMB}	0.533***	0.766***	0.851***	0.873***	0.851***	0.944***	1.000***	1.055***	1.153***	1.167***	0.634***
	(11.72)	(25.92)	(33.63)	(36.17)	(34.10)	(38.49)	(34.57)	(32.34)	(33.19)	(30.85)	(12.80)
β_{HML}	0.207***	0.227***	0.288***	0.252***	0.251***	0.176***	0.190***	0.131***	0.0955**	0.167***	-0.0400
	(4.17)	(7.04)	(10.43)	(9.59)	(9.24)	(6.60)	(6.04)	(3.70)	(2.52)	(4.04)	(-0.74)
Constant	-0.00186	0.0000807	0.0000224	0.00153**	0.00147*	0.00182**	0.00167*	0.00143	0.00134	0.00133	0.00319**
	(-1.35)	(0.09)	(0.03)	(2.08)	(1.93)	(2.44)	(1.90)	(1.45)	(1.27)	(1.16)	(2.12)
Observations	588	588	588	588	588	588	588	588	588	588	588
R^2	0.690	0.878	0.915	0.919	0.914	0.924	0.903	0.882	0.876	0.853	0.268

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Regression of Equal Weighted MPK Porfolios Returns, Industry Adjusted (sic4)

	1	2	3	4	5	6	7	8	9	10	HML
β_{MKT}	1.139***	1.064***	1.036***	1.006***	0.997***	1.008***	1.006***	1.006***	1.031***	1.058***	-0.0806***
	(29.03)	(42.46)	(50.08)	(49.94)	(46.43)	(43.40)	(42.23)	(40.14)	(35.94)	(34.51)	(-2.96)
β_{SMB}	1.234***	0.943***	0.804***	0.767***	0.794***	0.776***	0.845***	1.003***	1.110***	1.199***	-0.0346
	(22.11)	(26.46)	(27.33)	(26.78)	(25.98)	(23.50)	(24.95)	(28.12)	(27.19)	(27.48)	(-0.89)
β_{HML}	-0.132**	0.0257	0.0959***	0.108***	0.148***	0.0627*	0.0759**	0.0173	0.0263	0.0196	0.151***
	(-2.17)	(0.66)	(2.99)	(3.47)	(4.44)	(1.74)	(2.06)	(0.45)	(0.59)	(0.41)	(3.59)
Constant	-0.00132	0.000817	0.000707	0.00162*	0.00106	0.00203**	0.00125	0.00268**	0.00185	0.00123	0.00255**
	(-0.78)	(0.75)	(0.79)	(1.86)	(1.15)	(2.02)	(1.21)	(2.47)	(1.49)	(0.93)	(2.17)
Observations	588	588	588	588	588	588	588	588	588	588	588
R^2	0.775	0.860	0.886	0.884	0.869	0.854	0.853	0.856	0.835	0.829	0.057

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Cross-Sectional Regression of $log(MPK - rf)$, No industry Adjustment

	(1)	(2)	(3)	(4)
	$LEMPK$	$LEMPK$	$LEMPK$	$LEMPK$
β_{CAPM}	0.169***			
	(11.53)			
$\beta_{FF,MKT}$		0.0787***		
		(4.89)		
$\beta_{FF,HML}$		-0.0411***		
		(-3.66)		
$\beta_{FF,SMB}$		0.124***		
		(13.10)		
$\beta_{Q,MKT}$			0.00524	
			(0.38)	
$\beta_{Q,ME}$			0.111***	
			(11.77)	
$\beta_{Q,I/A}$			0.0121*	
			(1.81)	
$\beta_{Q,ROE}$			-0.0268***	
			(-3.43)	
$\beta_{CMKT,C}$				-0.000331
				(-0.47)
$\beta_{CMKT,MKT}$				0.188***
				(13.34)
Constant	0.0899***	0.0780***	0.0900***	0.0852***
	(6.81)	(5.89)	(6.81)	(6.47)
Observations	8454	8434	8430	8449
R^2	0.015	0.023	0.017	0.021

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: Cross-Sectional Regression of $log(MPK - rf)$, No industry Adjustment

	(1)	(2)	(3)	(4)
	$LEMPK$	$LEMPK$	$LEMPK$	$LEMPK$
β_{CAPM}	0.169***			
	(11.55)			
α_{CAPM}	0.561*			
	(1.80)			
$\beta_{FF,MKT}$		0.0795***		
		(4.89)		
$\beta_{FF,HML}$		-0.0413***		
		(-3.44)		
$\beta_{FF,SMB}$		0.123***		
		(13.02)		
α_{FF}		-0.0521		
		(-0.18)		
$\beta_{Q,MKT}$			0.00657	
			(0.47)	
$\beta_{Q,ME}$			0.111***	
			(11.76)	
$\beta_{Q,I/A}$			0.0170**	
			(2.17)	
$\beta_{Q,ROE}$			-0.0200**	
			(-2.05)	
α_Q			0.318	
			(1.15)	
$\beta_{CMKT,C}$				0.00441***
				(3.27)
$\beta_{CMKT,MKT}$				0.198***
				(13.91)
α_{CMKT}				0.987***
				(3.97)
Constant	0.0930***	0.0782***	0.0915***	0.0898***
	(7.01)	(5.88)	(6.89)	(6.80)
Observations	8442	8429	8425	8447
R^2	0.016	0.023	0.017	0.023

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 9: Cross-Sectional Regression of $log(MPK - rf)$, Industry Adjusted (sic4)

	(1)	(2)	(3)	(4)
	$LEMPK$	$LEMPK$	$LEMPK$	$LEMPK$
β_{CAPM}	0.000918			
	(0.08)			
α_{CAPM}	0.962***			
	(4.15)			
$\beta_{FF,MKT}$		-0.0236*		
		(-1.88)		
$\beta_{FF,HML}$		0.00778		
		(0.84)		
$\beta_{FF,SMB}$		0.0393***		
		(5.52)		
α_{FF}		0.372*		
		(1.77)		
$\beta_{Q,MKT}$			-0.0414***	
			(-3.94)	
$\beta_{Q,ME}$			0.0343***	
			(4.89)	
$\beta_{Q,I/A}$			0.0215***	
			(3.78)	
$\beta_{Q,ROE}$			0.0141*	
			(1.95)	
α_{Q}			0.495**	
			(2.44)	
$\beta_{CMKT,C}$				0.00419***
				(4.15)
$\beta_{CMKT,MKT}$				0.0205*
				(1.77)
α_{CMKT}				1.037***
				(5.63)
Constant	0.0165*	0.0101	0.0122	0.0150
	(1.80)	(1.09)	(1.33)	(1.64)
Observations	7326	7318	7310	7331
R^2	0.002	0.006	0.008	0.005

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Cross-Sectional Regression of MPK, No industry Adjustment

	(1)	(2)	(3)	(4)
	$LEMPK$	$LEMPK$	$LEMPK$	$LEMPK$
$\beta_{CAPM,MPK}$	0.0672***			
	(4.83)			
$\beta_{FF,MKT,MPK}$		0.0502***		
		(3.69)		
$\beta_{FF,HML,MPK}$		-0.0542***		
		(-6.42)		
$\beta_{FF,SMB,MPK}$		-0.0325***		
		(-4.99)		
$\beta_{Q,MKT,MPK}$			0.0397***	
			(3.31)	
$\beta_{Q,ME,MPK}$			-0.0582***	
			(-8.17)	
$\beta_{Q,I/A,MPK}$			0.00534	
			(1.09)	
$\beta_{Q,ROE,MPK}$			0.00635	
			(1.09)	
$\beta_{CMKT,C,MPK}$				0.000833**
				(2.29)
$\beta_{CMKT,MKT,MPK}$				0.0733***
				(5.00)
Constant	0.113***	0.112***	0.112***	0.113***
	(8.68)	(8.61)	(8.62)	(8.64)
Observations	8580	8561	8560	8572
R^2	0.003	0.014	0.017	0.004

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 11: Cross-Sectional Regression of MPK, Industry Adjusted (sic4)

	(1)	(2)	(3)	(4)
	$LEMPK$	$LEMPK$	$LEMPK$	$LEMPK$
$\beta_{CAPM,MPK}$	0.0308***			
	(3.04)			
$\beta_{FF,MKT,MPK}$		0.0355***		
		(3.53)		
$\beta_{FF,HML,MPK}$		-0.0433***		
		(-6.86)		
$\beta_{FF,SMB,MPK}$		-0.0121**		
		(-2.47)		
$\beta_{Q,MKT,MPK}$			0.0477***	
			(5.35)	
$\beta_{Q,ME,MPK}$			-0.0248***	
			(-4.65)	
$\beta_{Q,I/A,MPK}$			0.000740	
			(0.20)	
$\beta_{Q,ROE,MPK}$			-0.00373	
			(-0.85)	
$\beta_{CMKT,C,MPK}$				0.000191
				(0.72)
$\beta_{CMKT,MKT,MPK}$				0.0618***
				(5.75)
Constant	0.0127	0.0134	0.0120	0.0118
	(1.41)	(1.49)	(1.33)	(1.30)
Observations	7462	7445	7440	7458
R^2	0.001	0.011	0.014	0.005

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 12: Predictive Regression of Average MPK,

	Avg. LMPK	Avg. LMPK	Avg. LMPK	Avg. LMPK	Avg. LMPK	Avg. LMPK
L.GZ Spread	0.306***					
	(4.83)					
L4.GZ Spread		0.261***				
		(4.37)				
L.UNEMP			0.0964*			
			(1.97)			
L4.UNEMP				0.122**		
				(2.55)		
L.Cons. Growth					-57.08***	
					(-3.23)	
L4.Cons. Growth						-61.90***
						(-3.59)
Constant	-0.779***	-0.654***	-1.073***	-1.227***	-0.211*	-0.182
	(-6.00)	(-5.33)	(-3.36)	(-3.97)	(-1.91)	(-1.65)
Observations	164	161	182	182	182	182
R^2	0.126	0.107	0.021	0.035	0.055	0.067

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 13: Cross-Sectional Regression of MPK Dispersion, Industry Adjusted (sic4)

	(1)	(2)	(3)
	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$
$\sigma(\beta_{CAPM})$	0.327^{***}		
	(5.07)		
$\sigma(\alpha_{CAPM})$	14.70^{***}		
	(6.57)		
$\sigma(\beta_{FF,MKT})$		0.140^{**}	
		(2.15)	
$\sigma(\beta_{FF,HML})$		0.0722^{*}	
		(1.91)	
$\sigma(\beta_{FF,SMB})$		0.135^{***}	
		(3.36)	
$\sigma(\alpha_{FF})$		10.76^{***}	
		(5.03)	
$\sigma(\beta_{Q,MKT})$			0.132^{**}
			(2.47)
$\sigma(\beta_{Q,ME})$			0.185^{***}
			(4.50)
$\sigma(\beta_{Q,I/A})$			0.0658^{**}
			(2.54)
$\sigma(\beta_{Q,ROE})$			0.0584^{*}
			(1.81)
$\sigma(\alpha_{Q})$			1.982
			(0.96)
Constant	0.212^{***}	0.153^{***}	0.154^{***}
	(5.21)	(3.47)	(3.47)
Observations	493	493	493
R^2	0.300	0.310	0.311

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 14: Cross-Sectional Regression of MPK Dispersion, Industry Adjusted (sic4)

	(1)	(2)	(3)	(4)
	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$
$\sigma(\beta_{CAPM})$	0.614***			
	(12.40)			
$\sigma(\beta_{FF,MKT})$		0.251***		
		(4.00)		
$\sigma(\beta_{FF,HML})$		0.140***		
		(3.86)		
$\sigma(\beta_{FF,SMB})$		0.165***		
		(4.07)		
$\sigma(\beta_{Q,MKT})$			0.139***	
			(2.63)	
$\sigma(\beta_{Q,ME})$			0.189***	
			(4.64)	
$\sigma(\beta_{Q,I/A})$			0.0741***	
			(3.04)	
$\sigma(\beta_{Q,ROE})$			0.0714**	
			(2.44)	
ret				9.492***
				(11.86)
Constant	0.337***	0.207***	0.160***	0.396***
	(8.99)	(4.72)	(3.64)	(11.50)
Observations	493	493	493	493
R^2	0.239	0.274	0.310	0.223

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 15: Cross-Sectional Regression of MPK Dispersion, Industry Adjusted (sic4)

	(1)	(2)	(3)	(4)
	$\sigma(LEMPK)$	$\sigma(LEMPK)$	$\sigma(LEMPK)$	$\sigma(LEMPK)$
$\sigma(\beta_{CAPM,MPK})$	0.362^{***}			
	(11.43)			
$\sigma(\beta_{FF,MKT,MPK})$		0.141^{***}		
		(4.23)		
$\sigma(\beta_{FF,HML,MPK})$		0.0683^{***}		
		(2.68)		
$\sigma(\beta_{FF,SMB,MPK})$		0.0796^{***}		
		(4.80)		
$\sigma(\beta_{Q,MKT,MPK})$			0.111^{***}	
			(3.46)	
$\sigma(\beta_{Q,ME,MPK})$			0.0663^{***}	
			(3.43)	
$\sigma(\beta_{Q,I/A,MPK})$			0.0424^{***}	
			(3.41)	
$\sigma(\beta_{Q,ROE,MPK})$			0.0315^{*}	
			(1.82)	
$\sigma(\beta_{CMKT,C,MPK})$				0.00697^{***}
				(7.78)
$\sigma(\beta_{CMKT,MKT,MPK})$				0.218^{***}
				(6.27)
Constant	0.436^{***}	0.329^{***}	0.317^{***}	0.357^{***}
	(16.11)	(10.74)	(10.07)	(12.92)
Observations	493	493	493	493
R^2	0.210	0.273	0.278	0.291

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 16: Predictive Regression of unconditional MPK Dispersion,

	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$
L.GZ Spread	0.106***					
	(5.11)					
L4.GZ Spread		0.0987***				
		(5.28)				
L.UNEMP			0.0407**			
			(2.38)			
L4.UNEMP				0.0546***		
				(3.30)		
L.Cons. Growth					-19.65***	
					(-3.17)	
L4.Cons. Growth						-20.78***
						(-3.42)
Constant	0.903***	0.933***	0.748***	0.663***	1.092***	1.100***
	(21.33)	(24.33)	(6.72)	(6.19)	(28.18)	(28.36)
Observations	164	161	182	182	182	182
R^2	0.139	0.149	0.031	0.057	0.053	0.061

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 17: Predictive Regression of Average MPK Dispersion within Industry (sic4)

	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$	$\sigma(LMPK)$
L.GZ Spread	0.0492***					
	(2.77)					
L2.GZ Spread		0.0420**				
		(2.52)				
L.UNEMP			0.0324*			
			(1.68)			
L2.UNEMP				0.0382*		
				(2.00)		
L.Cons. Growth					-10.07	
					(-1.41)	
L2.Cons. Growth						-12.67*
						(-1.97)
Constant	0.629***	0.653***	0.446***	0.414***	0.694***	0.712***
	(15.98)	(17.91)	(3.62)	(3.44)	(14.63)	(15.28)
Observations	39	38	46	46	46	46
R^2	0.171	0.150	0.060	0.083	0.043	0.081

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 18: Predictive Regression of Equal Weighted MPK Portfolio Return, Industry Adjusted (sic4)

	1	2	4	8	1	2	4	8	1	2	4	8
GZ Spread	-0.000293	-0.000421	0.000279	0.000862***								
	(-0.35)	(-0.68)	(0.60)	(2.67)								
UNEMP					0.00112**	0.00116***	0.00111***	0.00104***				
					(2.57)	(3.64)	(4.73)	(6.18)				
Cons. Growth									-0.160	-0.223	-0.278*	-0.271**
									(-0.54)	(-1.05)	(-1.79)	(-2.44)
Constant	0.00329*	0.00355***	0.00238**	0.00129*	-0.00474*	-0.00502**	-0.00474***	-0.00431***	0.00285	0.00312**	0.00342***	0.00333***
	(1.92)	(2.81)	(2.51)	(1.96)	(-1.71)	(-2.47)	(-3.16)	(-4.06)	(1.49)	(2.24)	(3.36)	(4.53)
Observations	495	492	486	474	585	582	576	564	195	194	192	188
R^2	0.000	0.001	0.001	0.015	0.011	0.022	0.037	0.064	0.002	0.006	0.017	0.031

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 19: MPK and Risk

Model	(i)	(ii)	(iii)	(iv)	(v)	(vi)
A. Parameters						
γ_0	20	40	60	20	40	60
γ_1	-1000	-1000	-1000	0	0	0
B. Moments						
Average Sharpe Ratio	0.38	0.56	0.75	0.34	0.54	0.73
$\sigma_{E[MPK_i]}$	0.26	0.33	0.39	0.23	0.28	0.34
Corr($\sigma_{E[MPK_i]}, Y_t$)	-0.16	-0.22	-0.27	-0.06	0.03	0.02

This table reports average moments of simulated panels across various model specifications. We focus on varying the parameters underlying the stochastic discount factor, γ_0 and γ_1 to assess the effects of risk premia on the levels and dynamics of average MPK. To compute $\sigma_{E[MPK_i]}$, we compute average MPK per firm, and determine its cross-sectional standard deviation.

B Proofs

Proof of equation (2).

$$
\begin{aligned}
1 &= E_t\left[M_{t+1}\left(MPK_{it+1}+1-\delta\right)\right] \\
&= E_t\left[M_{t+1}\right]E_t\left[MPK_{it+1}+1-\delta\right]+\text{cov}\left(M_{t+1},MPK_{it+1}\right) \\
&= E_t\left[M_{t+1}\right]\left(MPK_{ft}+1-\delta\right) \\
\Rightarrow E_t\left[MPK_{t+1}\right] &= MPK_{ft}-\frac{\text{cov}\left(M_{t+1},MPK_{t+1}\right)}{E_t\left[M_{t+1}\right]} \\
&= \alpha_t+\beta_{it}\lambda_t
\end{aligned}
$$

where α_t, β_{it} and λ_t are as defined in the text, and MPK_{ft} is the MPK of the 'risk-free' firm defined by $\text{cov}\left(M_{t+1},MPK_{ft}\right)=0$. By a no-arbitrage condition, it must be the case that $\frac{1}{E_t[M]}=MPK_{ft}+1-\delta=R_{ft}$ where R_{ft} is the risk-free interest rate.

To put in log terms, approximate MPK around its unconditional mean MPK_f as $MPK_{it+1}\approx MPK_f\left(1+mpk_{it_1}-mpk_f\right)$ and rearrange to obtain

$$
\begin{aligned}
E\left[mpk_{it+1}\right] &\approx mpk_f-\frac{\text{cov}\left(M_{t+1},mpk_{t+1}\right)}{E\left[M_{t+1}\right]} \\
&= \log\left(r_f+\delta\right)-\frac{\text{cov}\left(M_{t+1},mpk_{t+1}\right)}{E\left[M_{t+1}\right]}
\end{aligned}
$$

where the last line follows from $MPK_f+1-\delta=R_f\Rightarrow MPK=R_f-1+\delta=r_f+\delta$.

MPK and stock returns. The ex-dividend stock prices is defined as $P_{it}=E_t\left[M_{t+1}V\left(A_{it},K_{it}\right)\right]$. We can show $P_{it}=K_{it+1}+N_{it}$, where $N_{it}=(1-\alpha)\,E_t\left[\sum M_{t+1}D_{it+1}\right]$.

No aggregate risk. With no aggregate risk,

$$
E_t\left[MPK_{it+1}\right]=\alpha_t=MPK_{ft}
$$

where MPK_{ft} satisfies

$$
\begin{aligned}
1 &= E_t\left[M_{t+1}\right]\left(MPK_{ft}+1-\delta\right) \\
&= \rho\left(MPK_{ft}+1-\delta\right)
\end{aligned}
$$

where the second line follows from $E_t\left[M_{t+1}\right]=\rho$ in the steady state. Combining and rearranging gives the expression in the text for $E_t\left[MPK_{it+1}\right]$.

CCAPM. A log-linear approximation to the SDF around its unconditional mean gives: $M_{t+1} \approx E[M_{t+1}](1 + m_{t+1} - E[m_{t+1}])$ and in the case of $CRRA$ utility, $m_{t+1} = -\gamma \Delta c_{t+1}$ where Δc_{t+1} is log consumption growth. Substituting for M_{t+1} in the left hand equation in (5), the terms involving $E[M_{t+1}]$ cancel, giving the CCAPM expression in the text.

Technology shocks. The Euler equation gives

$$\begin{aligned} 1 &= E_t\left[M_{t+1}\left(\alpha e^{z_{it+1}+\beta_i z_{t+1}} K_{it+1}^{\alpha-1} + 1 - \delta\right)\right] \\ &= (1-\delta) E_t[M_{t+1}] + \alpha K_{it+1}^{\alpha-1} E_t\left[e^{m_{t+1}+z_{it+1}+\beta_i z_{t+1}}\right] \end{aligned}$$

Substituting for m_{t+1} and rearranging,

$$\begin{aligned} E_t\left[e^{m_{t+1}+z_{it+1}+\beta_i z_{t+1}}\right] &= E_t\left[e^{\log\rho + z_{it+1} + (\beta_i - \gamma) z_{t+1} + \gamma z_t}\right] \\ &= E_t\left[e^{\log\rho + \tilde{\varphi} z_{it} + \varepsilon_{it+1} + (\beta_i-\gamma)(\varphi z_t + \varepsilon_{t+1}) + \gamma z_t}\right] \\ &= E_t\left[e^{\log\rho + \tilde{\varphi} z_{it} + \varepsilon_{it+1} + ((\beta_i-\gamma)\varphi + \gamma) z_t + (\beta_i-\gamma)\varepsilon_{t+1}}\right] \\ &= e^{\log\rho + \tilde{\varphi} z_{it} + ((\beta_i-\gamma)\varphi+\gamma) z_t + \frac{1}{2}(\beta_i-\gamma)^2\sigma_\varepsilon^2 + \frac{1}{2}\left(\sigma_{\tilde{\varepsilon}}^2 - \beta_i^2\sigma_\varepsilon^2\right)} \\ &= e^{\log\rho + \tilde{\varphi} z_{it} + ((\beta_i-\gamma)\varphi+\gamma) z_t - \beta_i\gamma\sigma_\varepsilon^2 + \frac{1}{2}\gamma^2\sigma_\varepsilon^2 + \frac{1}{2}\sigma_{\tilde{\varepsilon}}^2} \end{aligned}$$

so that

$$\theta K_{it+1}^{\theta-1} = \frac{1-(1-\delta)E_t[M_{t+1}]}{e^{\log\rho + \tilde{\varphi} z_{it} + ((\beta_i-\gamma)\varphi+\gamma) z_t - \beta_i\gamma\sigma_\varepsilon^2 + \frac{1}{2}\gamma^2\sigma_\varepsilon^2 + \frac{1}{2}\sigma_{\tilde{\varepsilon}}^2}}$$

and in logs,

$$k_{it+1} = \frac{1}{1-\theta}\left[\tilde{\alpha}_t + \tilde{\varphi} z_{it} + \beta_i \varphi z_t - \beta_i \gamma \sigma_\varepsilon^2\right]$$

where $\tilde{\alpha}_t = \log\theta - \log(1-(1-\delta)E_t[M_{t+1}]) + \log\rho + \gamma(1-\varphi) z_t + \frac{1}{2}\sigma_{\tilde{\varepsilon}}^2 + \frac{1}{2}\gamma^2\sigma_\varepsilon^2$ is a time-varying term that is constant across firms. Next,

$$\begin{aligned} mpk_{it+1} &= \log\theta + y_{it+1} - k_{it+1} \\ &= \log\theta + z_{it+1} + \beta z_{t+1} - (1-\theta) k_{it+1} \\ &= \log\theta + z_{it+1} + \beta_i z_{t+1} - \tilde{\alpha}_t - \tilde{\varphi} z_{it} - \beta_i \varphi z_t - \frac{1}{2}(\beta_i - \gamma)^2 \sigma_\varepsilon^2 \\ &= \alpha_t + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \beta_i \gamma \sigma_\varepsilon^2 \end{aligned}$$

where $\alpha_t = \log\theta - \tilde{\alpha}_t$ and the expected MPK is

$$E_t[mpk_{it+1}] = \alpha_t + \beta_i \gamma \sigma_\varepsilon^2$$