Motivation

- Privately-owned firms
  - Account for 1/2 of US business net income
  - Relevant for growth, wealth, tax policy/compliance

- But pose challenge for theory and measurement
Meeting the Challenge

- Propose theory of firm dynamics and capital reallocation
  - Add transfers to model of firm dynamics
  - Add self-created intangibles as productive capital
- Use administrative IRS data to discipline theory
Today: 3 Main Take-aways

- IRS data make study of business transfers possible
- New theory is needed to analyze these data
- Theory provides insights for tax policy/administration
IRS DATA MAKE STUDY OF BUSINESS TRANSFERS POSSIBLE
Business Transfers are Taxable Events

- Seller and buyer both report sale
  - Seller has to pay capital gains
  - Buyer has to report depreciable assets

- Price allocated across asset types
  - Seller wants to allocate to long-term
  - Buyer wants to allocate to short-term

⇒ Conflicts of interest and thus consistent reporting
What Do Filings Reveal?

- Transferred assets are primarily intangible
  - Customer bases and client lists
  - Non-compete covenants
  - Licenses and permits
  - Franchises, trademarks, tradenames
  - Workforce in place
  - IT and other know-how in place
  - Goodwill and on-going concern value
  - Consulting contracts during transition

- Transferred assets are sold as a group
What Else Do We Use?

- From other tax filings before/after sale
  - Characteristics and business filings for buyers/sellers
  - Characteristics and individual filings for all owners

- From brokered sales
  - Time between listing and sale
New theory is needed to analyze these data
New Theory

- Model of firm dynamics with self-created intangibles
  - Indivisible and nonrentable capital
  - Bilaterally-traded assets making up business
  - Requiring time to find buyers/negotiate allocations

⇒ Adds intangible investment and transfers to Hopenhayn
Environment: A Helicopter View

- Infinite horizon with continuous time

- Business type indexed by $s = (z, \kappa)$
  - $z$: non-transferable capital/owner productivity
  - $\kappa$: transferable and accumulable capital

- Key decisions for owners
  - Production
  - Investment
  - Transfers
Production

- Technology:

\[ y(s) = \max_n y(s, n) \]
\[ \equiv \max_n \hat{z}(s) \kappa(s) \alpha n^\gamma - wn \]
\[ \equiv z(s) \kappa(s) \alpha \]

where

\( \hat{z} \): non-transferable capital/owner productivity
\( \kappa \): transferable and accumulable capital
\( n \): all external rented factors

- Idea: \( \hat{z} \) is owner-specific, \( \kappa \) is self-created intangibles
Firm Dynamics, $s \rightarrow s'$

- Entry $\rightarrow (z, \kappa)$
- Shocks to productivity $z \rightarrow z'$
- Investment $\kappa \rightarrow \kappa'$
- Capital transfer $\kappa \rightarrow \kappa'$
- Exit $(z, \kappa) \rightarrow$
Firm Dynamics: Some notation

• Entry and exit:

\[ G(s) = \text{initial distribution of type} \]
\[ c_e = \text{entry cost} \]
\[ \delta = \text{exit rate} \]

• Shocks to productivity:

\[ dz = \mu(z)dt + \sigma(z)d\mathcal{B} \]
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Note: just standard Hopenhayn so far
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Next: add self-created intangibles and transfers
Firm Dynamics: Build or Buy Capital?

- Given decreasing returns to scale

⇒ Owners build to optimal size through
  - Internal investment or
  - Business transfers
Firm Dynamics: Build or Buy Capital?

- Investment: $d\kappa = \theta - \delta \kappa$ with convex cost $C(\theta)$

- Transfers between $s$, $\tilde{s}$:
  
  - Bilateral meeting rate: $\eta$

  † Allocation: $\kappa^m(s, \tilde{s}) \in \{\kappa(s) + \kappa(\tilde{s}), 0\}$

  - Price: $p^m(s, \tilde{s})$

† More general specifications also explored
Adding it up: Owner’s Value

\[(r + \delta)V(s) = \max_n y(s, n) + \mu(z)\partial_z V(s) + \frac{1}{2}\sigma^2(z)\partial_{zz} V(s)\]

\[\text{production} \hspace{5cm} \text{shocks to productivity}\]

\[+ \max_{\theta} \partial_{\kappa} V(s)(\theta - \delta_k) - C(\theta) + \max_{\lambda} \eta W(s; \lambda)\]

\[\text{investment} \hspace{5cm} \text{transfer}\]

where expected gain from transfer is:

\[W(s; \lambda) = \sum_{\tilde{s}} \left\{ V([z, \kappa^m(s, \tilde{s})]) - V(s) - p^m(s, \tilde{s}) \right\} \lambda(s, \tilde{s})\]

Partner Distribution
Closing the Model

- Free entry condition

\[
\int V(s) dG(s) \leq c_e
\]

where measure of entrants is \( \phi_e(s) = mG(s) > 0 \)

- Evolution of types:

\[
\dot{\phi} = \Gamma(\theta, \lambda; \phi) + \phi_e
\]

induced by drivers of firm dynamics
Recursive Equilibrium

Objects: \( \{ V, \kappa^m, p^m, \theta, \lambda, \phi, \phi_e, w \} \)

\begin{align*}
\text{value function} & \quad \text{policy functions} & \quad \text{measures} & \quad \text{wage} \\
\end{align*}

that satisfy

1. business owners’ optimality
2. market clearing
3. consistency of measures

- Can solve dynamic program iteratively
  
  \( \circ \) Update: \((\phi, V) \rightarrow \text{static planner} \rightarrow (\phi, V)\)
Properties of Equilibrium

- Competitive allocations maximize
  \[
  \int e^{-rt} \sum_s [y(s) - C(\theta(s, t)) - m(t)c_e] \phi(s, t) dt
  \]
  \[\Rightarrow \text{achieves efficiency}\]

- Competitive prices independent of \( z \)
  \[p^m(s, \tilde{s}) = \mathcal{P}(\kappa(\tilde{s}))\]
  \[\Rightarrow \text{same good sold at same price}\]

- Bilateral trades are pairwise stable
  \[\not\exists \text{feasible trade for }(s, \tilde{s}) \text{ making pair strictly better off}\]
Model Predictions

- Who trades with whom?
- What are the terms of trade?
- What is the implied dispersion in MPKs?
- How do financing constraints affect predictions?
Model Predictions

• Who trades with whom?
• What are the terms of trade?
• What is the implied dispersion in MPKs?
• How do financing constraints affect predictions?

Let’s simulate the model and find out...
## Model Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns to scale</td>
<td>$\alpha = 0.5$</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$r = 0.06$</td>
</tr>
<tr>
<td>Investment cost†</td>
<td>$A = 20, \rho = 2.0$</td>
</tr>
<tr>
<td>Productivity</td>
<td>$\mu = 0, \sigma = 0.25$</td>
</tr>
<tr>
<td>Entrant distribution</td>
<td>mass at $z = z_0, \kappa = 1$</td>
</tr>
<tr>
<td>Death rate</td>
<td>$\delta = 0.10$</td>
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<td>Depreciation rate</td>
<td>$\delta_\kappa = 0.058$</td>
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<td>Bilateral meeting rate</td>
<td>$\eta = 0.40$</td>
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$\dagger$ $C'(\theta) = A\theta^\rho$
How are Key Parameters Identified?

• Key parameters
  ○ Meeting rate $\eta$
  ○ Investment costs $C'(\theta) = A\theta^\rho$
  ○ Returns to scale in $y = z\kappa^\alpha$

• Key moments from IRS (8594 and annual filings)
  ○ Frequency of business transfers
  ○ Growth in business net income
  ○ Quantile regressions of $y$ on $P$
- Size of square proportional to number of transactions
- Shows capital trading upward in MPK sense
- Suggests that unit prices would be higher at low $\kappa$
What are the Terms of Trade?

![Graph of Per-unit Price vs. Quantity of Capital (κ)](image)

Per-unit Price ($\varphi(\kappa) / \kappa$)

Quantity of Capital (κ)
What is the Implied Dispersion in MPKs?

- Compare to “misallocation” literature benchmark
  - Divisible versus indivisible capital
  - Rental versus no rental markets

- Compute first-best:

\[
\kappa^{FB}(s) \in \arg\max \int z(s)[\kappa^{FB}(s)]^{\alpha} \phi(s) ds \\
\int \phi(s) \kappa^{FB}(s) ds = \int \phi(s) \kappa(s) ds
\]
Dispersion in MPKs
Predictions with Financing Constraints

- Add constraint: $p^m \leq \text{year's income}$

- Main effects:
  - No sales with small buyers
  - Large drop in price for big-$\kappa$ sales
Predictions with Financing Constraints

Per-unit Price ($\varphi(\kappa)/\kappa$)

Quantity of Capital ($\kappa$)

- Red: No price cap
- Blue: With price cap
Theory provides insights for tax policy/administration
Taxing Self-Created Intangible Capital

• Most value in business is $\kappa$

• How is it taxed?
  • Income taxes on business owner
  • Capital gains taxes on realized gains
  • Biden proposal: taxes on unrealized gains

• What is the implied tax incidence?
Taxing Capital Gains

- Relevant input to analysis is business wealth

- Three different concepts:
  - Price if sold business today
  - Present value of owner dividends
  - Capitalized income

⇒ All have model counterparts
Taxing Capital Gains

- Relevant input to analysis is business wealth

- Three different concepts:
  - Price if sold business today, \( P(\kappa(s)) \)
  - Present value of owner dividends, \( V(s) \)
  - Capitalized income, \( \hat{V}(s) = \frac{y(s)}{\text{constant } R} \)

\[ \Rightarrow \text{All have model counterparts} \]
## Estimating Business Wealth

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<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>50</td>
<td>0.37</td>
<td>0.09</td>
</tr>
<tr>
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<td>0.50</td>
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- Two insights:
  - $P/V$ large: relevant for tax elasticities
  - $(y - C)/V$ dispersed: relevant for capitalizing income
Incidence When Taxing Realized Gains

- Introduce tax $\tau$ on realized gains
  - Seller receives $(1 - \tau)p^m(s, \tilde{s})$
  - Government receives $\tau p^m(s, \tilde{s})$

- Positive tax base due to $\kappa$ (not in Hopenhayn)
Effects of Tax

- Fewer trades (obvious)
  - Tax eliminates trades where gains are small

- Lower investment and entry (obvious)
  - Tax introduces lock-in effect

- Heterogeneity in tax incidence
  - Nonmonotonic in size of business sold
  - Larger on seller for small and large quantities
Heterogeneity in Tax Incidence

![Graph showing the effect of pre-tax and post-tax scenarios on the per-unit price (\(\frac{\varphi(\kappa)}{\kappa}\)) with respect to the quantity of capital (\(\kappa\)). The graph illustrates that the per-unit price decreases as the quantity of capital increases, with different slopes and levels for pre-tax and post-tax scenarios.](image-url)
Heterogeneity in Tax Incidence

The graph illustrates the per-unit price ($\varphi(\kappa)/\kappa$) vs. the quantity of capital ($\kappa$). Three lines represent different tax scenarios:

- **Green line:** Pre-tax case.
- **Red line:** No-tax case.
- **Blue line:** Post-tax case.

Key points:

- **Almost full incidence on seller:** The green line shows a significant drop, indicating a large portion of the tax burden is shifted to the seller.
- **Partial incidence on buyer:** The red line shows a moderate drop, indicating a partial shift to the buyer.

The x-axis represents the quantity of capital ($\kappa$), while the y-axis shows the per-unit price ($\varphi(\kappa)/\kappa$).
Recap

- IRS data make study of business transfers possible
- New theory is needed to analyze these data
- Theory provides insights for tax policy/administration