A Theory of Business Transfers

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Motivation

- Privately-owned firms
  - Account for 1/2 of US business net income
  - Relevant for growth, wealth, tax policy/compliance
- But pose challenge for theory and measurement
This Paper

- Proposes theory of firm dynamics and capital reallocation
- Characterizes properties of competitive equilibrium
- Uses administrative IRS data to discipline theory
- Studies transfers, wealth, and impact of capital gains tax
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- Proposes theory of firm dynamics and capital reallocation
- Characterizes properties of competitive equilibrium
  † Uses administrative IRS data to discipline theory
- Studies transfers, wealth, and impact of capital gains tax
  † Still very much in progress
Private Business Capital: What is Known?
Cash/securities ← Inventories ← Fixed assets ← Sec. 197 intangibles
Private Business Capital: What is Known?

- Transferred assets are primarily intangible
Private Business Capital: What is Known?

- Transferred assets are primarily intangible

  ⇒ evidence in IRS Forms 8594, 8883 data
  shows intangible, non-liquid share is ≈ 60%
• Transferred assets are primarily intangible
  ◦ Customer bases and client lists
  ◦ Non-compete covenants
  ◦ Licenses and permits
  ◦ Franchises, trademarks, tradenames
  ◦ Workforce in place
  ◦ IT and other know-how in place
  ◦ Goodwill and on-going concern value

⇒ Classified as *Section 197 intangibles* by IRS
Transferred assets are primarily:

- Intangible and neither pledgeable nor rentable
• Transferred assets are primarily
  ○ Intangible and neither pledgeable nor rentable
  ○ Sold as a group that makes up a business
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  ⇒ evidence in brokered sale data is \( \approx 290 \text{ days} \)
Private Business Capital: What is Known?

- Transferred assets are primarily
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  - Sold as a group that makes up a business
  - Exchanged after timely search and brokered deals

⇒ Existing models unsuitable for studying business transfers
Today’s Talk

- Study firm dynamics

- Characterize competitive equilibrium

- Estimate wealth and impact of capital gains tax
Today’s Talk

- Study firm dynamics with
  - Indivisible capital
  - Bilaterally traded
  - Requiring time to reallocate

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Today’s Talk

- Study firm dynamics with
  - Indivisible capital
  - Bilaterally traded
  - Requiring time to reallocate

- Characterize competitive equilibrium
  - Who trades with whom?
  - How are terms of trade determined?
  - What are the properties?

- Estimate wealth and impact of capital gains tax
Theory
Environment: A Helicopter View

- Infinite horizon with discrete time
- Preferences: (for today) owners are risk-neutral
- Technology:
  - Firms indexed by \( s = (z, \kappa) \)
  - Produce \( y(s) = z(s)\kappa(s)^\alpha = \max_n \hat{z}(s)\kappa(s)^\hat{\alpha} n^{\gamma} - wn \)
    - \( z \): non-transferable capital with \( z' | z \) exogenous
    - \( \kappa \): transferable capital
    - \( n \): all external rented factors
  - Investment: \( \theta = P\{\kappa(s') = \kappa(s) + 1\} \) at cost \( C(\theta) \)
- Birth/death: draw from \( G(s) \) at cost \( c_e \) and die at rate \( \delta \)
Timing of Decisions
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Birth/death &
Shocks
$\kappa$ to $\kappa + 1$ w.p. $\theta$
$z$ to $z'$
Timing of Decisions

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Shocks
$\kappa$ to $\kappa + 1$ w.p. $\theta$
$z$ to $z'$

trading
$\kappa^m, p^m$

- Terms of trade for pair $(s, \tilde{s})$
  - Allocations: $\kappa^m(s, \tilde{s})$ is post-trade capital for $s$
  - Prices: $p^m(s, \tilde{s})$ is payment by $s$ to $\tilde{s}$
Timing of Decisions

Birth/death & Shocks
\( \kappa \) to \( \kappa + 1 \) w.p. \( \theta \)
\( z \) to \( z' \)

Trading
\( \kappa^m, p^m \)

Production/investment

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Timing of Decisions

Pre-trade value: $W$

Post-trade value: $V$

Birth/death & Shocks
$\kappa$ to $\kappa + 1$ w.p. $\theta$
$z$ to $z'$

$\kappa$ to $\kappa + 1$ w.p. $\theta$
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  ○ Prices: $p^m(s, \tilde{s})$ is payment by $s$ to $\tilde{s}$
Dynamic Program of Incumbent Firms

- Given prices and allocations \( \{p^m(s, \tilde{s}), \kappa^m(s, \tilde{s})\}_{s, \tilde{s}} \)

- Compute values:

\[
V(s) = \max_{\theta \in [0,1]} z(s)\kappa(s)^\alpha - C(\theta) + (1 - \delta)\beta \mathbb{E} W(s')
\]

\[
W(s') = \max_{\lambda(\tilde{s}) \geq 0, \lambda_o \geq 0} \int \left[ V(z(s'), \kappa^m(s', \tilde{s})) - p^m(s', \tilde{s}) \right] \lambda(\tilde{s}) + V(s') \lambda_o
\]

where \( \{\lambda(\cdot), \lambda_o\} \) are probabilities over trading options
Aggregate Measures

• Measures:
  
  ○ $\phi(s)$: firms of type $s$
  
  ○ $\phi_e(s)$: entrants of type $s$
  
  ○ $\Lambda(s, \tilde{s}) = \lambda(\tilde{s} | s)\phi(s)$: matches between $s, \tilde{s}$
  
  ○ $\Lambda_o(s) = \lambda_o(s)\phi(s)$: unmatched firms of type $s$

• Law of motion for $\phi$:

$$\phi'(s) = \Gamma(\phi; \lambda, \lambda_o, \theta, \phi_e, k^m)$$
Recursive Equilibrium with Pairwise Stability

Objects: \[ \{ V, W, \kappa^m, p^m, \phi, \Lambda, \Lambda_0, \phi_e \} \]

such that

1. firms optimize and entrants make zero profits
2. bilateral trades are feasible and pairwise stable
3. measures are consistent with decisions and stationarity

Conditions 1) and 3) are standard. Next, consider 2)
Feasibility and Pairwise Stability

- Terms of trade satisfy
  
  - Feasibility:
    
    \[ \kappa_m(s, \tilde{s}) + \kappa_m(\tilde{s}, s) \leq \kappa(s) + \kappa(\tilde{s}) \]
    
    \[ p_m(s, \tilde{s}) + p_m(\tilde{s}, s) \geq 0 \]
    
    where \( \kappa_m(s, \tilde{s}) \in \{ \kappa(s) + \kappa(\tilde{s}), \kappa(s), 0 \} \) for buy, no trade, sell.

- Pairwise stability:
  
  \( \not\exists \) feasible trade for \((s, \tilde{s})\) increasing pair’s welfare
Back to RE Definition

Objects: \{ V, W, \kappa^m, p^m, \phi, \Lambda, \Lambda_o, \phi_e \}

such that

1. \( V, W \) solve firms problems and entrants make zero profits
2. \( \kappa^m, p^m \) are feasible and pairwise stable
3. \( \phi, \Lambda, \Lambda_o, \phi_e \) satisfy for all \( A \subseteq S, m \geq 0 \):

\[
\phi(A) = \int \Lambda(ds \in A, d\tilde{s} \in S) + \Lambda_o(ds \in A)
\]

\[
\phi(A) = \int \Lambda(d\tilde{s} \in S, d\tilde{s} \in A) + \Lambda_o(ds \in A)
\]

\[
\phi_e(A) = G(ds \in A)m
\]

\[
\phi'(A) = \Gamma(\phi; \lambda, \lambda_o, \theta, \phi_e, k^m)(A)
\]
Discussion

- Relative to models with
  - CES demand/ monopolistic competition
  - Frictional labor or asset markets

- Framework delivers (with few a priori restrictions)
  - Differentiated goods
  - Rich heterogeneity in market participants
  - Endogenously evolving matching sets
Characterizing Equilibria
Who Trades with Whom?

• Intuitive example:
  ◦ Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  ◦ Capital pre-trade: all have $\kappa = 1$

• Efficient reallocation:
  ◦ 10 low types sell to 10 of the high types
How are Terms of Trade Determined?

• Intuitive example:
  
  ◦ Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  
  ◦ Capital pre-trade: all have $\kappa = 1$

• Price leaves high types indifferent between:
  
  ◦ Trading, with $\kappa = 2$ post-trade
  
  ◦ Not trading, with $\kappa = 1$ post-trade
Equilibrium Policy Functions

- Intuitive example:
  - Productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - Capital pre-trade: all have $\kappa = 1$

- Capital allocations: $k^m(s_H, s_L) = 2$, $k^m(s_L, s_H) = 0$

- Prices: $p^m(s_H, s_L) = 1$, $p^m(s_L, s_H) = -1$

- Choice probabilities:

  $$
  \lambda(s_H|s_L) = 1, \quad \lambda(s_L|s_H) = 1/2, \quad \lambda_o(s_L) = 0, \quad \lambda_o(s_H) = 1/2
  $$
More Generally Given \((\phi, V)\)

- Who trades with whom?
  - Solve assignment problem maximizing total gains

- How are terms of trade determined?
  - Compute shadow prices from assignment problem

- Can solve dynamic program iteratively
  - Update: \((\phi, V) \rightarrow \text{equilibrium objects} \rightarrow (\phi, V)\)
Imagine splitting our businesses in two:

\[
\begin{align*}
&5 \left\{ 
\begin{array}{cc}
  z_L & z_L \\
  z_L & z_L \\
  z_L & z_L \\
  z_L & z_L \\
  z_L & z_L \\
\end{array}
\right\} 5 \\
&10 \left\{ 
\begin{array}{cc}
  z_H & z_H \\
  z_H & z_H \\
  z_H & z_H \\
  \vdots & \vdots \\
  z_H & z_H \\
\end{array}
\right\} 10
\end{align*}
\]
Monge-Kantorovich Assignment Problem

\[ Q(\phi, V) = \max_{\pi_s, \tilde{s} \geq 0, \pi_o, \tilde{\pi}_o \geq 0} \int X(s, \tilde{s}) \pi_s, \tilde{s}(ds, d\tilde{s}) + V(s)\pi_o(ds) + V(\tilde{s})\tilde{\pi}_o(d\tilde{s}) \]

s.t. \[ \int \pi_s, \tilde{s}(ds \in A, d\tilde{s} \in S) + \pi_o(ds \in A) = \phi(A)/2 \]
\[ \int \pi_s, \tilde{s}(ds \in S, d\tilde{s} \in A) + \tilde{\pi}_o(ds \in A) = \phi(A)/2 \]

where the gains to trade are

\[ X(s, \tilde{s}) = \max \left\{ V(z(s), \kappa(s) + \kappa(\tilde{s})), V(s) + V(\tilde{s}), V(z(\tilde{s}), \kappa(s) + \kappa(\tilde{s})) \right\} \]
\[ s \text{ buys } \quad \text{no trade} \quad \tilde{s} \text{ buys} \]
Role of MK Lagrange Multipliers

- Multipliers $\mu = \mu^a = \mu^b$ capture gains from trade

\[ \mu = \nabla_\phi Q \]

- Prices implement optimal gains from trade:

\[
\mu(s) = \begin{cases} 
V(z(s), k^m(s, \tilde{s})) - p^m(s, \tilde{s}) & \text{social} \\
\end{cases} = \text{private gains}
\]

- Updates of $\phi, V$ are easy to compute:

\[
V(s) = \max y(s) - C(\theta) + (1 - \delta) \beta E\mu(s')
\]

\[
\phi'(s) = \Gamma(\phi; \pi, \pi_o, \theta, \phi_e, k^m)
\]
Properties of Equilibrium

- Competitive allocations maximize

\[ \sum_t \beta^t \int \phi_t(s)[y(s) - C(\theta(s)) - m_t c_e] \]

- Competitive prices independent of \( z \)
Properties of Equilibrium

- Competitive allocations maximize

\[ \sum_t \beta^t \int \phi_t(s)[y(s) - C(\theta(s)) - m_t c_e] \]

- Competitive prices independent of \( z \), eg,

\[
\begin{align*}
    p^m(\tilde{s}, s) &= V(z(\tilde{s}), \kappa(s) + \kappa(\tilde{s})) - \mu(\tilde{s}) \\
    p^m(\tilde{s}, s') &= V(z(\tilde{s}), \kappa(s') + \kappa(\tilde{s})) - \mu(\tilde{s})
\end{align*}
\]

\( \Rightarrow \quad p^m(\tilde{s}, s') \) depends on \( \kappa \) but not \( z \)
Properties of Equilibrium

- Competitive allocations maximize

\[ \sum_t \beta^t \int \phi_t(s)[y(s) - C(\theta(s)) - m_t c_e] \]

- Competitive prices independent of \( z \)

\[ p^m(s, \tilde{s}) = \mathcal{P}(\kappa(s)) \]
Quantitative Results
## Model Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns to scale</td>
<td>$\alpha = 0.50$</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\beta = 0.95$</td>
</tr>
<tr>
<td>Investment cost, $C(\theta) = A\theta^\rho$</td>
<td>$A = 10, \rho = 2.0$</td>
</tr>
<tr>
<td>Productivity, $z'</td>
<td>z$ AR(1)</td>
</tr>
<tr>
<td>Entrant distribution, Zipf($z$)</td>
<td>tail = 1.20</td>
</tr>
<tr>
<td>Death rate</td>
<td>$\delta = 0.20$</td>
</tr>
</tbody>
</table>
Patterns of Trade

- Statistics to be matched to IRS data:
  - Roughly 4% of $\kappa$ units traded each period
  - Price is 4 to 7 times seller’s income
  - Buyer’s income is 2 to 4 times seller’s income

- Who trades with whom?
Patterns of Trade

![Graph showing the relationship between Buyer Productivity (z) and Seller Productivity (z).]
Patterns of Trade

![Graph showing the relationship between Buyer Productivity (z) and Seller Productivity (z).]
Capital Trades Upward in MPK Sense
Allocation of Capital

- Compare to “misallocation” literature benchmark
  - Divisible versus indivisible capital
  - Rental versus no rental markets
- Compute first-best:

\[
\kappa^{FB}(s) \in \arg\max \int z(s)[\kappa^{FB}(s)]^\alpha \phi(s)ds
\]

\[
\int \phi(s)\kappa^{FB}(s)ds = \int \phi(s)\kappa(s)ds
\]
Dispersion in MPKs without Frictions

Average Capital

Productivity (z)
Dispersion in Prices without Frictions

Per-unit Price $\left( \varphi(\kappa)/\kappa \right)$

Quantity of Capital ($\kappa$)
Dispersion in Prices without Frictions

Per-unit Price ($\phi(\kappa)/\kappa$)

Quantity of Capital ($\kappa$)

Low for low $\kappa$ because of time to build concerns

High for medium $\kappa$ because firms want to jump to optimal size

Somewhat lower at high $\kappa$ because of thin markets and decreasing returns to scale
Estimating Business Wealth

- Finance textbook: present value of owner dividends

- SCF survey: price if sold business today

- Both have clear model counterparts
Estimating Business Wealth

- Finance textbook: present value of owner dividends, $V(s)$
- SCF survey: price if sold business today, $P(\kappa(s))$
- Both have clear model counterparts
### Estimating Business Wealth

<table>
<thead>
<tr>
<th>Productivity Level ((z))</th>
<th>Transferable Share (\mathcal{P}(\kappa(s))/V(s))</th>
<th>Income Yield ([y(s) − C(\theta(s))]/V(s))</th>
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<tr>
<td>1.00</td>
<td>0.54</td>
<td>0.13</td>
</tr>
<tr>
<td>1.29</td>
<td>0.47</td>
<td>0.14</td>
</tr>
<tr>
<td>1.67</td>
<td>0.42</td>
<td>0.16</td>
</tr>
<tr>
<td>2.15</td>
<td>0.37</td>
<td>0.17</td>
</tr>
<tr>
<td>2.78</td>
<td>0.34</td>
<td>0.19</td>
</tr>
<tr>
<td>3.59</td>
<td>0.31</td>
<td>0.20</td>
</tr>
<tr>
<td>4.64</td>
<td>0.32</td>
<td>0.21</td>
</tr>
<tr>
<td>5.99</td>
<td>0.41</td>
<td>0.23</td>
</tr>
<tr>
<td>7.74</td>
<td>0.38</td>
<td>0.24</td>
</tr>
<tr>
<td>10.0</td>
<td>0.33</td>
<td>0.23</td>
</tr>
<tr>
<td>Avg</td>
<td>0.43</td>
<td>0.17</td>
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TAXING CAPITAL GAINS
Capital Gains Tax

- Introduce tax $\tau$ on gains
  - Seller receives $(1 - \tau)p^m(s, \tilde{s})$
  - Government receives $\tau p^m(s, \tilde{s})$

- Use tricks to handle nontransferable utility case
Effects of Tax

• Fewer trades (obvious)
  ○ Tax eliminates trades where gains are small

• Heterogeneity in tax incidence
  ○ Larger on buyer if transacted quantity small
  ○ Larger on seller if transacted quantity large
Eliminates Small-Gain Trades

- With tax, find larger distance between buyers/sellers
- For example, ratio of MPKs of buyer to seller:

<table>
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<tr>
<th>Moments</th>
<th>$\tau = 0%$</th>
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<tbody>
<tr>
<td>Mean</td>
<td></td>
<td></td>
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<td>Standard deviation</td>
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<tr>
<td>$5^{th}$ percentile</td>
<td></td>
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<td>$25^{th}$</td>
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<td></td>
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<tr>
<td>$50^{th}$</td>
<td></td>
<td></td>
</tr>
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<td>$75^{th}$</td>
<td></td>
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</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>8.2</td>
<td>10.7</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>5$^{th}$ percentile</td>
<td>5.9</td>
<td>8.0</td>
</tr>
<tr>
<td>25$^{th}$</td>
<td>7.0</td>
<td>9.5</td>
</tr>
<tr>
<td>50$^{th}$</td>
<td>8.0</td>
<td>10.4</td>
</tr>
<tr>
<td>75$^{th}$</td>
<td>9.3</td>
<td>12.0</td>
</tr>
<tr>
<td>95$^{th}$</td>
<td>12.0</td>
<td>13.4</td>
</tr>
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Next Steps

- Theory: add curvature and financing constraints
- Estimation: continue work with IRS data
- Applications: continue work studying capital taxation
Appendix: Galichon-Kominers-Weber
Galichon-Kominers-Weber Tricks

• Without capital gains tax
  ○ Labeling buyers/sellers a priori not necessary
  ○ Exploiting symmetry possible with MK

• With capital gains tax
  ○ Labeling buyers/sellers a priori is necessary
  ○ Exploiting MK requires complicated outer loop

• GKW’s trick is to introduce small “preference shocks”
  ○ All types are buyers and sellers
  ○ Numerical objects are equations not inequalities