

A Theory of Business Transfers

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Privately-owned firms

- Account for 1/2 of US business net income
- Dominate discussions on growth, wealth inequality, tax policy
- But pose challenge for
 - theory: technology of capital accumulation and transfer
 - measurement: no reliable data on private wealth
- This paper:
 - propose a theory of firm dynamics and capital allocation that is appropriate for such firms
 - use IRS data to bring discipline to the theory
 - Study business taxation

What do we know about Private Business Capital?

Form **8594** **Asset Acquisition Statement** Under Section 1060 OMB No. 1545-0074
(Rev. November 2021) Department of the Treasury Internal Revenue Service **Attachment Sequence No. 169**
▶ Attach to your income tax return. ▶ Go to www.irs.gov/Form8594 for instructions and the latest information.

Name as shown on return Identifying number as shown on return

Check the box that identifies you:
 Purchaser Seller

Part I General Information

1 Name of other party to the transaction Other party's identifying number
Address (number, street, and room or suite no.)
City or town, state, and ZIP code

2 Date of sale **3** Total sales price (consideration)

Part II Original Statement of Assets Transferred

4 Assets	Aggregate fair market value (actual amount for Class I)	Allocation of sales price
Class I	\$	\$
Class II	\$	\$
Class III	\$	\$
Class IV	\$	\$
Class V	\$	\$
Class VI and VII	\$	\$
Total	\$	\$

5 Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document signed by both parties? Yes No
If "Yes," are the aggregate fair market values (FMV) listed for each of asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document? Yes No

6 In the purchase of the group of assets (or stock), did the purchaser also purchase a license or a covenant not to compete, or enter into a lease agreement, employment contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)? Yes No
If "Yes," attach a statement that specifies **(a)** the type of agreement and **(b)** the maximum amount of consideration (not including interest) paid or to be paid under the agreement. See instructions.

← Cash/securities
← Inventories
← Fixed assets
← Sec. 197 intangibles

What do we know about Private Business Capital?

- Transferred assets are primarily **intangible** (from form 8594 \approx 70%)
 - Customer bases and client lists, non-compete covenants
 - Licenses and permits, trademarks, tradenames
 - Workforce in place
 - Goodwill and on-going concern value
- Assets are **sold as a group**
- Sale **requires time** to find buyers/negotiate (from brokered data \approx 290 days)

⇒ Add intangible investment and transfers to Hopenhayn-style model

- Firm Dynamics
 - Hopenhayn (1992), Hsieh and Klenow (2009, 2014), Sterk et al. (2021)
- Capital Reallocation
 - Holmes and Schmitz (1990), Ottonello (2014), Guntin and Kochen (2020), Gaillard and Kankanamge (2020), David (2021)
- Entrepreneurship and Private Wealth
 - Cagetti and De Nardi (2006), Saez and Zucman (2016), Smith et al. (2019)
- Capital Gain Taxes and Wealth Taxes
 - Chari et al. (2003), Scheuer and Slemrod (2020), Guvenen et al. (2021), Agersnap and Zidar (2021)

- Infinite horizon, continuous time
- Demographics:
 - total population N : workers and business owners
 - newborns enter the economy, choose occupation, exit at rate δ
- Preferences: risk-neutral
- Workers supply labor inelastically

- Production technology:

$$y(s, n) = z(s)k(s)^\alpha n^\gamma$$

where n is a rentable input (labor)

- Productivity, z

- non-transferable
- evolves according to $dz = \mu z dt + \sigma z dB$

- Business capital, k

- transferable
- built through investment: $dk = \theta - \delta_k$, convex cost $C(\theta)$

- Entry technology: entry cost $n_0 w$, draw $s \sim G(s)$, where $s = (z, k)$

- Capital:
 - Firms access market at rate η
 - Bilaterally traded:
 - type $s = (z, k)$ can trade with **any** type $\tilde{s} = (\tilde{z}, \tilde{k})$
 - Allocation between s and \tilde{s} :
 - $k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\} \Rightarrow$ indivisibility
(extension w/ costly divisibility)
 - Price paid by s to \tilde{s} :
 - $p^m(s, \tilde{s})$, negative if selling
(extension w/ financing constraints: $p^m(s, \tilde{s}) \leq \xi y(s, n)$)
- Labor:
 - competitive spot markets

- The owner's value solves the following HJB

$$(r + \delta)V(s) = \underbrace{\max_n y(s, n) - wn}_{\text{production}} + \underbrace{\max_{\theta} \partial_k V(s)(\theta - \delta_k) - C(\theta)}_{\text{investment}} \\ + \underbrace{\mu z \partial_z V(s) + \frac{1}{2} \sigma^2 z^2 \partial_{zz} V(s)}_{\text{evolution of productivity}} + \underbrace{\max_{\lambda} \eta W(s; \lambda)}_{\text{trade}}$$

where

$$W(s; \lambda) = \int [V(z, k^m(s, \tilde{s})) - V(z, k) - p^m(s, \tilde{s})] \lambda(s, \tilde{s}) d\tilde{s}$$

and

$$\int \lambda(s, \tilde{s}) d\tilde{s} + \lambda(s, 0) = 1$$

- Occupational Choice ("free-entry")

$$\int V(s)dG(s) - n_0 w \leq \frac{w}{r + \delta}, \quad \phi_e \geq 0, \quad \text{w/ c.s.}$$

- Distribution over the state space ϕ evolves according to the Kolmogorov Forward (KF) equation

$$\dot{\phi} = \Gamma(\theta, \lambda; \phi) + \phi_e$$

- Evolution of ϕ induced by

▶ investment ▶ trade ▶ entry/exit ▶ individual productivity process

Definition of Recursive Equilibrium

A (stationary) equilibrium is a set of value functions $V(s)$, policy functions for investment $\theta(s)$ and trade $\lambda(s, \tilde{s})$, terms of trade $(k^m(s, \tilde{s}), p^m(s, \tilde{s}))$, wage w , and distribution over the state space $\phi(s)$ that satisfy

- business owners' optimality
- no-arbitrage in occupational choice
- market clearing
- consistency of measures

⇒ Trade of multiple differentiated goods

- Standard approach:
 - CES demand/monopolistic competition
 - frictional market with fixed point on matching set
- Our model:
 - rich heterogeneity in market participants
 - friction less matching with a competitive forces

- Who trades with whom?
 - Solve assignment problem maximizing total gains
- How are terms of trade determined?
 - Compute shadow prices from assignment problem
- Flexible block structure: $(\phi, V) \rightarrow (\lambda, p^m, k^m) \rightarrow (\phi', V')$
 - Easy to extend to non-transferable utility environment

details

- **Competitive prices** are independent of seller's z

$$p^m(s, \tilde{s}) = \mathcal{P}(\kappa(\tilde{s}))$$

Intuition: competitive nature of the equilibrium, same good sold at same price

- **Pairwise stability:** $\tilde{\mathcal{H}}(s, \tilde{s})$ and feasible trade that makes the pair (strictly) better off
- **Competitive allocation** solves the planner's problem

$$\int \exp(-\rho t) \int [y(s) - C(\theta(s, t)) - m(t)c_e] \phi(s, t) dt ds$$

given $\phi(s, 0) = \phi^{ss}(s)$

- Calibration using data on
 - firm dynamics
 - business transfers
- Model deliverables
 - dispersion in mpk
 - business price and value
- Tax Policy Analysis

Parameter	Value
Discount rate	$r = 0.06$
Share of rentable input	$\gamma = 0.70$
Entry distribution, G	mass point at $z = z_0, k = 1$
Death rate, depreciation rate	$\delta = 0.1, \delta_k = 0.058$
Investment cost, $C(\theta) = A\theta^\rho$	$A = 13, \rho = 2$
Trading rate	$\eta = 1$
Returns to scale	$\alpha = 0.09$
Productivity process	$\mu_z = 0, \sigma_z = 0.075$

- Life-cycle firm dynamics \Rightarrow productivity process, rentable input share, exit rate
- Transaction data \Rightarrow production, investment, meeting technology

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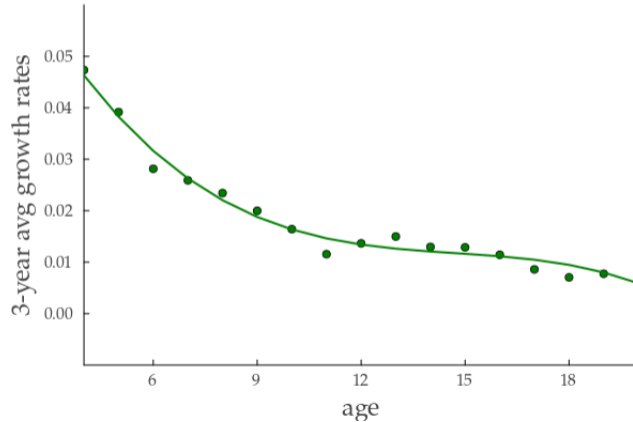
Key parameters

- meeting rate η
- investment cost $C(\theta) = A\theta^p$
- output elasticity wrt k , $y(z, k, n) = zk^\alpha n^\gamma$
- volatility of $\log(z)$, σ_z

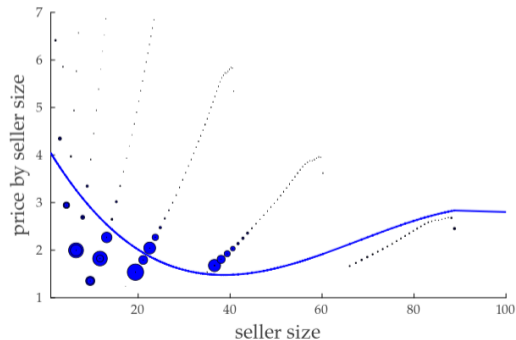
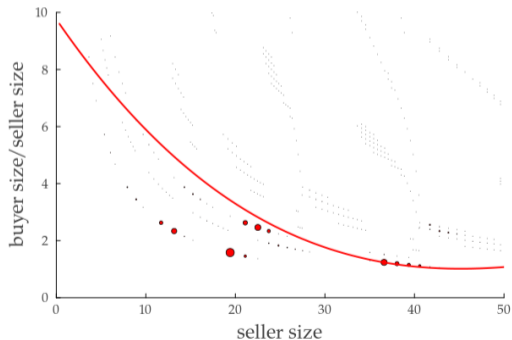
Key moments from data

- brokered sales: time to sell
- IRS filings
 - relative size of buyer/seller
 - sale price/wage bill
 - level and volatility of growth rates

- Declining growth rates over the life cycle (from 5% to $< 1\%$)

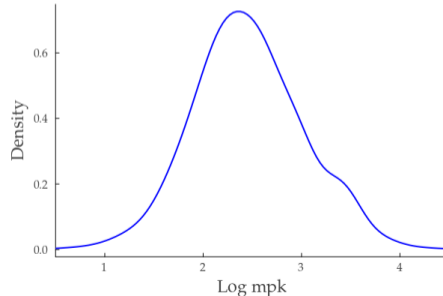


- Buyer's size does not scale up with seller's
- Lower price per unit for large sellers (less competition)



Dispersion in MPK

- Idiosyncratic change in productivity \rightarrow input reallocation toward higher MPK
- Dispersion in marginal product of capital induced by
 - decentralized trading
 - indivisibility of asset sold
- Standard deviation of log-mpk: 55%



- Finance textbook: Present value of owner's dividend
 - Model counterpart: $V(s)$

- SCF respondent: Answer to the survey question—"What could you sell it for?"
 - Model counterpart: $\mathcal{P}(k)$

Model Predictions for Business Wealth

- Heterogeneity in transferable share and returns
- Inputs to analysis of capital and wealth taxation

Distribution Pctile	Transferable Share	Income Yield
	$\frac{\mathcal{P}(k)}{V}$	$\frac{y - wn - C(\theta)}{V}$
5	0.00	-0.06
25	0.14	0.09
50	0.21	0.10
75	0.29	0.11
95	0.43	0.13
99	0.57	0.14

- Recent debate on business taxation
- What to tax
 - flows: business income
 - stocks: business capital or wealth (Guvenen et al. 2022)
 - transfers: capital gains (Sarin et al 2022, Agersnap and Zidar 2021)
- Our model can speak to all three forms of taxation

[details](#)

Comparison of

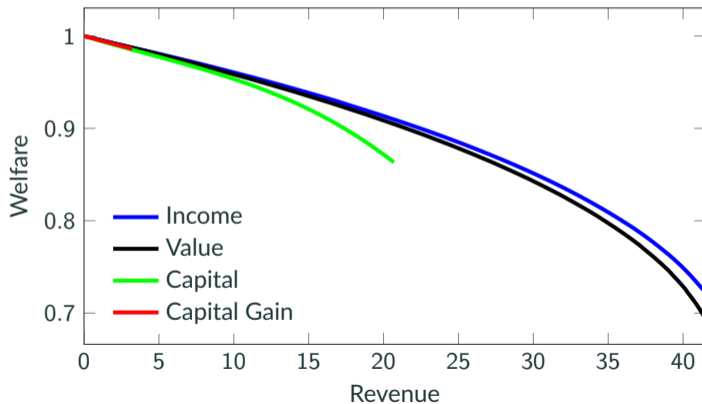
- **capital gains:** $\tau_c \mathcal{P}(k)$ [capital transaction]
- **business income:** $\tau_b(y - wn)$
- **business capital:** $\tau_k \mathcal{P}(k)$ [capital ownership]
- **wealth:** $\tau_v V$

Welfare measure: steady-state value at birth conditional on raising revenue R

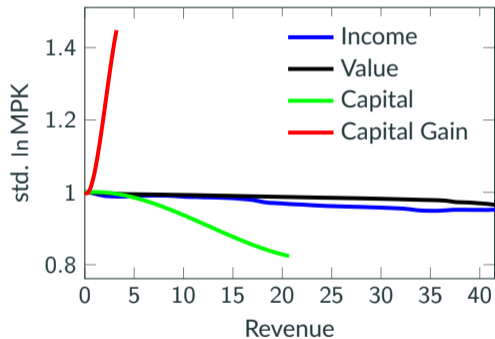
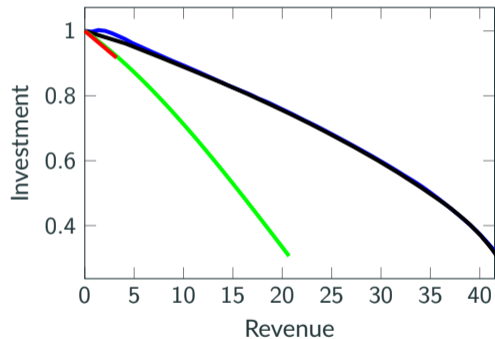
- by indifference at entry, all agents' ex-ante value is proportional to w

Main Results: Welfare

- For most levels of R , exclusively use τ_b



Main Results: Investment and MPK dispersion



Compared with tax on income,

- tax on capital gains
 - distorts capital reallocation across firms
 - decreases investment to sell
- tax on business capital plot
 - higher incidence on low and medium z firms which are more elastic
- tax on wealth \approx tax on income + tax on option value of selling capital

Practical implementation: k and V are not observed

- Add other salient features
 - undiversifiable risk
 - other motives (retirements, etc)
 - financing constraints
- Fuller study of tax policy

- Buyers and sellers both report sale
 - seller has to pay capital gains
 - buyer has to report depreciable assets

- Price allocated across asset types
 - seller wants to allocate to long-term
 - buyer wants to allocate to short-term

⇒ Conflict of interest and thus consistent reporting

Define gains from trade between s, \tilde{s} :

$$X(s, \tilde{s}) = \max_{k^m \in \{k(s) + k(\tilde{s}), 0\}} \{V(z(s), k^m) + V(z(\tilde{s}), k(s) + k(\tilde{s}) - k^m)\} - (V(s) + V(\tilde{s}))$$

$$Q(\phi, V) = \max_{\pi \geq 0} \sum_{s, \tilde{s}} X(s, \tilde{s}) \pi(s, \tilde{s})$$

$$\text{s.t. } \sum_{\tilde{s}} \pi(s, \tilde{s}) + \pi(s, 0) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^a(s)]$$

$$\sum_{\tilde{s}} \pi(\tilde{s}, s) + \pi(0, s) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^b(s)]$$

Lemma

- $W(s) = \frac{\partial Q}{\partial \phi(s)} = \frac{\mu^a(s) + \mu^b(s)}{2} \equiv \mu(s)$
- $\lambda(s, \tilde{s}) = \frac{2\pi(s, \tilde{s})}{\phi(s)}$
- $k^m(s, \tilde{s}) = \arg \max X(s, \tilde{s}) \quad p^m(s, \tilde{s}) = V(z, k^m(s, \tilde{s})) - V(z, k) - W(s)$

- Multipliers $\mu = \mu^a = \mu^b$ capture gains from trade

$$\mu = \nabla_{\phi} Q$$

- Prices implement gains from trade

$$p^m(s, \tilde{s}) = V(z(s), k^m(s, \tilde{s})) - \mu(s)$$

- Post-trade values are intuitively connected

$$V(s) = \max y(s) - C(\theta) + (1 - \delta)\beta \mathbb{E}\mu(s')$$

- From the minimax thm, the solution of the primal problem is equal to the solution of the dual
- The multipliers in the primal are equal to the choice variable in the dual, and vice versa

$$Q(\phi) = \min_{\mu^a \geq 0, \mu^b \geq 0} \sum_s \left(\mu^a(s) + \mu^b(s) \right) \frac{\phi(s)}{2}$$
$$s.t. \quad \mu^a(s) + \mu^b(\tilde{s}) \geq X(s, \tilde{s}) \quad \forall s, \tilde{s} \quad [\pi(s, \tilde{s})]$$

- After-trade values for buyers (v_b) and sellers (v_s)
 - $v_b(s, \hat{k}; p)$: value from buying \hat{k}
 - $v_s(s, 0; p)$: value from selling $k(s)$
- Matching probability

$$\lambda(s, \hat{k}; p) = \exp\left(\frac{v_b(s, \hat{k}; p) - W(s)}{\sigma}\right)$$

$$\lambda(s, 0; p) = \exp\left(\frac{v_s(s, 0; p) - W(s)}{\sigma}\right)$$

where $W(s) = \mathbb{E} \max\{v_b(s, \hat{k}; p), v_s(s, 0; p)\}$

- Find $\{p(s)\}$ such that $\forall \hat{k}$

$$\underbrace{\int \lambda(s, \hat{k}; p)}_{\text{demand}} = \underbrace{\int \lambda(s, 0; p) \mathbb{I}\{k(s) = \hat{k}\}}_{\text{supply}}$$

- Under capital gain tax τ ,

$$\begin{aligned}v_b(s; \hat{k}) &= V(z, k(s) + \hat{k}) - p(\hat{k}) \\v_s(s) &= V(\tilde{s}, 0) + (1 - \tau)p(k(s))\end{aligned}$$

- Under cap on paid price equal to $\xi y(s, n)$

$$\begin{aligned}v_b(s; \hat{k}) &= \begin{cases} V(z, k(s) + \hat{k}) - p(\hat{k}) & \text{if } p(\hat{k}) \leq \xi y(s, n) \\ -\infty & \text{o/w} \end{cases} \\v_s(s) &= V(\tilde{s}, 0) + p(k(s))\end{aligned}$$

Feasibility and Pair-wise stability

Terms of trade $\{p^m, k^m\}$ satisfy

- feasibility

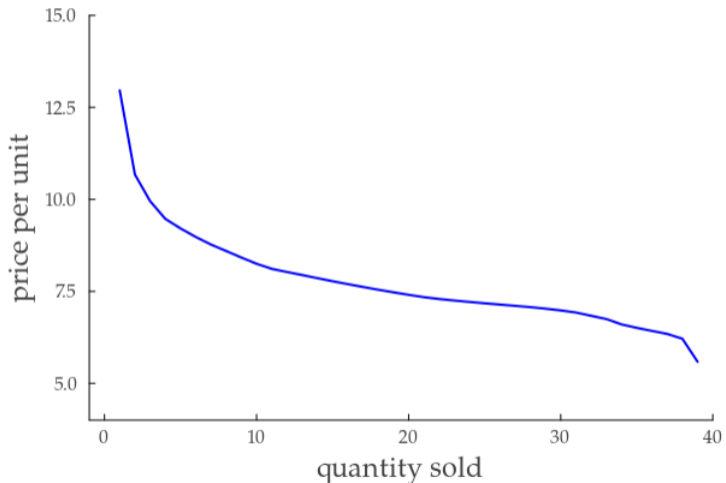
$$k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\}$$

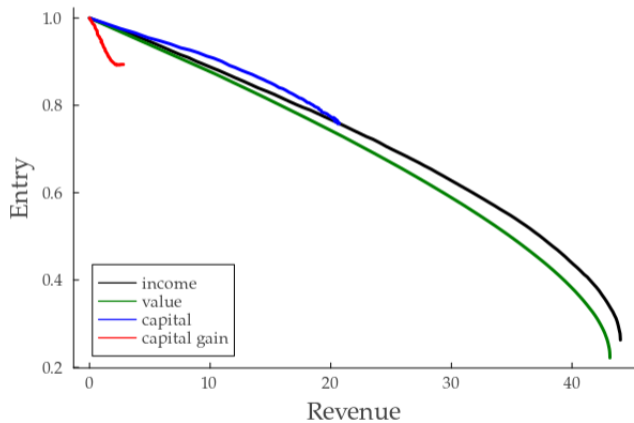
$$k^m(s, \tilde{s}) + k^m(\tilde{s}, s) \leq k(s) + k(\tilde{s})$$

$$p(s, \tilde{s}) + p(\tilde{s}, s) \geq 0$$

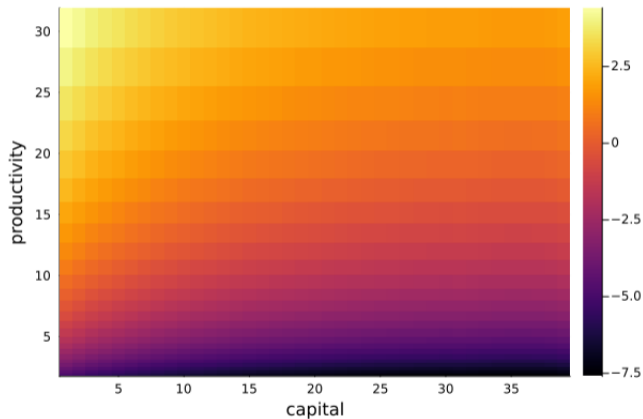
- pair-wise stability: $\nexists(s, \tilde{s})$ and feasible trade that makes the pair (strictly) better off

Price per Unit of Capital





Who pays more from taxing business income instead of business capital?



$$\text{log-ratio of taxes paid: } \log\left(\frac{\tau_b(y-wn)}{\tau_k P(k)}\right)$$