A Theory of Business Transfers

Anmol Bhandari  
Minnesota

Paolo Martellini  
Wisconsin

Ellen McGrattan  
Minnesota
Motivation

Privately-owned firms

- Account for 1/2 of US business net income
- Dominate discussions on growth, wealth inequality, tax policy
- But pose challenge for
  - theory: technology of capital accumulation and transfer
  - measurement: no reliable data on private wealth
- This paper:
  - propose a theory of firm dynamics and capital allocation that is appropriate for such firms
  - use IRS data to bring discipline to the theory
  - Study business taxation
### What do we know about Private Business Capital?

**Asset Acquisition Statement**

**Under Section 2660**

- Attach to your income tax return.
- Go to www.irs.gov/Form5696 for instructions and the latest information.

#### Part I: General Information

<table>
<thead>
<tr>
<th>Name on return</th>
<th>Identifying number as shown on return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchaser</td>
<td></td>
</tr>
<tr>
<td>Seller</td>
<td></td>
</tr>
</tbody>
</table>

#### Part II: General Statement of Assets Transferred

<table>
<thead>
<tr>
<th>Assets</th>
<th>Aggregate fair market value actual</th>
<th>Allocation of sales price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class I</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>Class II</td>
<td>$</td>
<td>$</td>
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<tr>
<td>Class III</td>
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<td>$</td>
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<tr>
<td>Class IV</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>Class V</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>Class VI and VII</td>
<td>$</td>
<td>$</td>
</tr>
</tbody>
</table>

**Total** $  

6. Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document signed by both parties?  
   - Yes  
   - No

If "Yes," the aggregate fair market values (FMV) listed for each asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document?  
   - Yes  
   - No

6. In the purchase of the group of assets (or stock), did the purchaser also purchase a license or a covenant not to compete, or enter into a lease agreement, employment contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)?  
   - Yes  
   - No

If "Yes," attach a statement that specifies (a) the type of agreement and (b) the maximum amount of consideration (not including interest) paid or to be paid under the agreement. See instructions.
What do we know about Private Business Capital?

- Transferred assets are primarily **intangible** (from form 8594 $\approx 70\%$
  
  - Customer bases and client lists, non-compete covenants
  
  - Licenses and permits, trademarks, tradenames
  
  - Workforce in place
  
  - Goodwill and on-going concern value

- Assets are **sold as a group**

- Sale **requires time** to find buyers/negotiate (from brokered data $\approx 290$ days)

$\Rightarrow$ Add intangible investment and transfers to Hopenhayn-style model
Related Literature

- Firm Dynamics

- Capital Reallocation

- Entrepreneurship and Private Wealth
  - Cagetti and De Nardi (2006), Saez and Zucman (2016), Smith et al. (2019)

- Capital Gain Taxes and Wealth Taxes
  - Chari et al. (2003), Scheuer and Slemrod (2020), Guvenen et al. (2021), Agersnap and Zidar (2021)
Environment

- Infinite horizon, continuous time
- Demographics:
  - total population $N$: workers and business owners
  - newborns enter the economy, choose occupation, exit at rate $\delta$
- Preferences: risk-neutral
- Workers supply labor inelastically
Environment

- Production technology:
  \[ y(s, n) = z(s) k(s)^\alpha n^\gamma \]
  where \( n \) is a rentable input (labor)

- Productivity, \( z \)
  - non-transferable
  - evolves according to \( dz = \mu z dt + \sigma z dB \)

- Business capital, \( k \)
  - transferable
  - built through investment: \( dk = \theta - \delta_k \), convex cost \( C(\theta) \)

- Entry technology: entry cost \( n_0 w \), draw \( s \sim G(s) \), where \( s = (z, k) \)
Markets

- Capital:
  - Firms access market at rate $\eta$
  - Bilaterally traded:
    - type $s = (z, k)$ can trade with any type $\tilde{s} = (\tilde{z}, \tilde{k})$
  - Allocation between $s$ and $\tilde{s}$:
    - $k^m(s, \tilde{s}) \in \{ k(s) + k(\tilde{s}), 0 \} \Rightarrow$ indivisibility
    (extension w/ costly divisibility)
  - Price paid by $s$ to $\tilde{s}$:
    - $p^m(s, \tilde{s})$, negative if selling
    (extension w/ financing constraints: $p^m(s, \tilde{s}) \leq \xi y(s, n)$)

- Labor:
  - competitive spot markets
The owner’s value solves the following HJB

\[(r + \delta) V(s) = \max_n y(s, n) - wn + \max_\theta \partial_k V(s)(\theta - \delta_k) - C(\theta)\]

production  
investment

\[+ \mu z \partial_z V(s) + \frac{1}{2} \sigma^2 z^2 \partial_{zz} V(s) + \max_\lambda \eta W(s; \lambda)\]

evolution of productivity  
trade

where

\[W(s; \lambda) = \int [V(z, k^m(s, \tilde{s})) - V(z, k) - p^m(s, \tilde{s})] \lambda(s, \tilde{s}) d\tilde{s}\]

and

\[\int \lambda(s, \tilde{s}) d\tilde{s} + \lambda(s, 0) = 1\]
• Occupational Choice ("free-entry")

\[
\int V(s) \, dG(s) - n_0 w \leq \frac{w}{r + \delta}, \quad \phi_e \geq 0, \quad \text{w/ c.s.}
\]

• Distribution over the state space \( \phi \) evolves according to the Kolmogorov Forward (KF) equation

\[
\dot{\phi} = \Gamma(\theta, \lambda; \phi) + \phi_e
\]

• Evolution of \( \phi \) induced by

- investment
- trade
- entry/exit
- individual productivity process
A (stationary) equilibrium is a set of value functions $V(s)$, policy functions for investment $\theta(s)$ and trade $\lambda(s, \tilde{s})$, terms of trade $(k^m(s, \tilde{s}), p^m(s, \tilde{s}))$, wage $w$, and distribution over the state space $\phi(s)$ that satisfy

- business owners’ optimality
- no-arbitrage in occupational choice
- market clearing
- consistency of measures
Trade of multiple differentiated goods

• Standard approach:
  - CES demand/monopolistic competition
  - frictional market with fixed point on matching set

• Our model:
  - rich heterogeneity in market participants
  - friction less matching with a competitive forces
Trading Protocol: Implementation

- Who trades with whom?
  - Solve assignment problem maximizing total gains
- How are terms of trade determined?
  - Compute shadow prices from assignment problem
- Flexible block structure: \((\phi, V) \rightarrow (\lambda, p^m, k^m) \rightarrow (\phi', V')\)
  - Easy to extend to non-transferable utility environment
Properties of the Equilibrium

- **Competitive prices** are independent of seller's \( z \)

\[ p^m(s, \tilde{s}) = \mathcal{P}(\kappa(\tilde{s})) \]

Intuition: competitive nature of the equilibrium, same good sold at same price

- **Pairwise stability**: \( (s, \tilde{s}) \) and feasible trade that makes the pair (strictly) better off

- **Competitive allocation** solves the planner's problem

\[
\int \exp(-\rho t) \int [y(s) - C(\theta(s, t)) - m(t)c_e] \phi(s, t) dt ds
\]

given \( \phi(s, 0) = \phi^{ss}(s) \)
Using the Model

- Calibration using data on
  - firm dynamics
  - business transfers

- Model deliverables
  - dispersion in mpk
  - business price and value

- Tax Policy Analysis
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$r = 0.06$</td>
</tr>
<tr>
<td>Share of rentable input</td>
<td>$\gamma = 0.70$</td>
</tr>
<tr>
<td>Entry distribution, $G$</td>
<td>mass point at $z = z_0$, $k = 1$</td>
</tr>
<tr>
<td>Death rate, depreciation rate</td>
<td>$\delta = 0.1$, $\delta_k = 0.058$</td>
</tr>
<tr>
<td>Investment cost, $C(\theta) = A\theta^\rho$</td>
<td>$A = 13$, $\rho = 2$</td>
</tr>
<tr>
<td>Trading rate</td>
<td>$\eta = 1$</td>
</tr>
<tr>
<td>Returns to scale</td>
<td>$\alpha = 0.09$</td>
</tr>
<tr>
<td>Productivity process</td>
<td>$\mu_z = 0$, $\sigma_z = 0.075$</td>
</tr>
</tbody>
</table>
Identification Strategy

- Life-cycle firm dynamics ⇒ productivity process, rentable input share, exit rate
- Transaction data ⇒ production, investment, meeting technology
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- Transaction data ⇒ production, investment, meeting technology

Key parameters
- meeting rate $\eta$
- investment cost $C(\theta) = A\theta^\rho$
- output elasticity wrt $k, y(z, k, n) = z^\alpha n^\gamma$
- volatility of $\log(z), \sigma_z$

Key moments from data
- brokered sales: time to sell
- IRS filings
  - relative size of buyer/seller
  - sale price/wage bill
  - level and volatility of growth rates
Life-Cycle of the Firm

- Declining growth rates over the life cycle (from 5% to < 1%)
Trade Patterns

- Buyer’s size does not scale up with seller’s
- Lower price per unit for large sellers (less competition)
Dispersion in MPK

- Idiosyncratic change in productivity → input reallocation toward higher MPK
- Dispersion in marginal product of capital induced by
  - decentralized trading
  - indivisibility of asset sold
- Standard deviation of log-mpk: 55%
• Finance textbook: Present value of owner’s dividend
  - Model counterpart: $V(s)$

• SCF respondent: Answer to the survey question—"What could you sell it for?"
  - Model counterpart: $P(k)$
Model Predictions for Business Wealth

- Heterogeneity in transferable share and returns
- Inputs to analysis of capital and wealth taxation

<table>
<thead>
<tr>
<th>Distribution Pctile</th>
<th>Transferable Share</th>
<th>Income Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.00</td>
<td>-0.06</td>
</tr>
<tr>
<td>25</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>50</td>
<td>0.21</td>
<td>0.10</td>
</tr>
<tr>
<td>75</td>
<td>0.29</td>
<td>0.11</td>
</tr>
<tr>
<td>95</td>
<td>0.43</td>
<td>0.13</td>
</tr>
<tr>
<td>99</td>
<td>0.57</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Recent debate on business taxation

What to tax

- flows: business income
- stocks: business capital or wealth (Guvenen et al. 2022)
- transfers: capital gains (Sarin et al 2022, Agersnap and Zidar 2021)

Our model can speak to all three forms of taxation
Tax Instruments and Welfare

Comparison of

- **capital gains**: $\tau_c \mathcal{P}(k)$ [capital transaction]

- **business income**: $\tau_b (y - wn)$

- **business capital**: $\tau_k \mathcal{P}(k)$ [capital ownership]

- **wealth**: $\tau_v V$

Welfare measure: steady-state value at birth conditional on raising revenue $R$

- by indifference at entry, all agents' ex-ante value is proportional to $w$
Main Results: Welfare

- For most levels of $R$, exclusively use $\tau_b$
Main Results: Investment and MPK dispersion

![Graph showing the relationship between Revenue and Investment. The x-axis represents Revenue, and the y-axis represents Investment. The graph includes three curves, each representing different scenarios.]

![Graph showing the relationship between Revenue and std. ln MPK. The x-axis represents Revenue, and the y-axis represents std. ln MPK. The graph includes four curves, each representing different scenarios: Income, Value, Capital, and Capital Gain.]
Compared with tax on income,

- tax on capital gains
  - distorts capital reallocation across firms
  - decreases investment to sell
- tax on business capital
  - higher incidence on low and medium $z$ firms which are more elastic
- tax on wealth $\approx$ tax on income + tax on option value of selling capital

Practical implementation: $k$ and $V$ are not observed
Next Steps

- Add other salient features
  - undiversifiable risk
  - other motives (retirements, etc)
  - financing constraints

- Fuller study of tax policy
Business Transfers are Taxable Events

- Buyers and sellers both report sale
  - seller has to pay capital gains
  - buyer has to report depreciable assets

- Price allocated across asset types
  - seller wants to allocate to long-term
  - buyer wants to allocate to short-term

⇒ Conflict of interest and thus consistent reporting
Define gains from trade between $s, \tilde{s}$:

$$X(s, \tilde{s}) = \max_{k^m \in \{k(s) + k(\tilde{s}), 0\}} \{V(z(s), k^m) + V(z(\tilde{s}), k(s) + k(\tilde{s}) - k^m)\} - (V(s) + V(\tilde{s}))$$

$$Q(\phi, \pi) = \max_{\pi \geq 0} \sum_{s, \tilde{s}} X(s, \tilde{s}) \pi(s, \tilde{s})$$

s.t.

$$\sum_{\tilde{s}} \pi(s, \tilde{s}) + \pi(s, 0) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^a(s)]$$

$$\sum_{s} \pi(\tilde{s}, s) + \pi(0, s) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^b(s)]$$
Auxiliary Problem: Static Planner

**Lemma**

- \( W(s) = \frac{\partial Q}{\partial \phi(s)} = \frac{\mu^a(s) + \mu^b(s)}{2} \equiv \mu(s) \)
- \( \lambda(s, \tilde{s}) = \frac{2\pi(s, \tilde{s})}{\phi(s)} \)
- \( k^m(s, \tilde{s}) = \arg \max X(s, \tilde{s}) \quad p^m(s, \tilde{s}) = V(z, k^m(s, \tilde{s})) - V(z, k) - W(s) \)

- Multipliers \( \mu = \mu^a = \mu^b \) capture gains from trade
  \[ \mu = \nabla \phi Q \]

- Prices implement gains from trade
  \[ p^m(s, \tilde{s}) = V(z(s), k^m(s, \tilde{s})) - \mu(s) \]

- Post-trade values are intuitively connected
  \[ V(s) = \max y(s) - C(\theta) + (1 - \delta) \beta \mathbb{E}\mu(s') \]
• From the minimax thm, the solution of the primal problem is equal to the solution of the dual

• The multipliers in the primal are equal to the choice variable in the dual, and vice versa

\[ Q(φ) = \min_{μ^a ≥ 0, μ^b ≥ 0} \sum_s \left( μ^a(s) + μ^b(s) \right) \frac{φ(s)}{2} \]

s.t. \( μ^a(s) + μ^b(\tilde{s}) \geq X(s, \tilde{s}) \quad \forall s, \tilde{s} \quad [π(s, \tilde{s})] \]
Trade with Preference Shocks

- After-trade values for buyers ($v_b$) and sellers ($v_s$)
  - $v_b(s, \hat{k}; p)$: value from buying $\hat{k}$
  - $v_s(s, 0; p)$: value from selling $k(s)$

- Matching probability

$$
\lambda(s, \hat{k}; p) = \exp \left( \frac{v_b(s, \hat{k}; p) - W(s)}{\sigma} \right)
$$

$$
\lambda(s, 0; p) = \exp \left( \frac{v_s(s, 0; p) - W(s)}{\sigma} \right)
$$

where $W(s) = \mathbb{E} \max\{v_b(s, \hat{k}; p), v_s(s, 0; p)\}$

- Find $\{p(s)\}$ such that $\forall \hat{k}$

$$
\int_{\text{demand}} \lambda(s, \hat{k}; p) = \int_{\text{supply}} \lambda(s, 0; p) \mathbb{1}\{k(s) = \hat{k}\}
$$
• Under capital gain tax $\tau$,

$$v_b(s; \hat{k}) = V(z, k(s) + \hat{k}) - p(\hat{k})$$

$$v_s(s) = V(\bar{s}, 0) + (1 - \tau)p(k(s))$$

• Under cap on paid price equal to $\xi_y(s, n)$

$$v_b(s; \hat{k}) = \begin{cases} 
V(z, k(s) + \hat{k}) - p(\hat{k}) & \text{if } p(\hat{k}) \leq \xi_y(s, n) \\
-\infty & \text{otherwise}
\end{cases}$$

$$v_s(s) = V(\bar{s}, 0) + p(k(s))$$
Terms of trade \( \{p^m, k^m\} \) satisfy

- feasibility

\[
k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\} \\
k^m(s, \tilde{s}) + k^m(\tilde{s}, s) \leq k(s) + k(\tilde{s}) \\
p(s, \tilde{s}) + p(\tilde{s}, s) \geq 0
\]

- pair-wise stability: \( \not\exists (s, \tilde{s}) \) and feasible trade that makes the pair (strictly) better off
Price per Unit of Capital

(price/quantity) vs. (quantity sold)

(price per unit) decreases as (quantity sold) increases.
Understanding the Tax Results

Who pays more from taxing business income instead of business capital?

log-ratio of taxes paid: $\log\left(\frac{\tau_b (y - w_n)}{\tau_k P(k)}\right)$