Capital Reallocation and Private Firm Dynamics

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Disclaimer

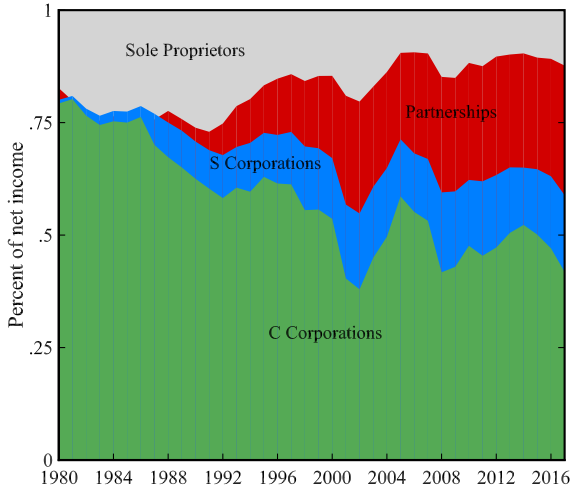
The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors and do not necessarily reflect the views or the official positions of the U.S. Department of the Treasury or the Internal Revenue Service. All results have been reviewed to ensure that no confidential information is disclosed.

Motivation

Privately-owned businesses

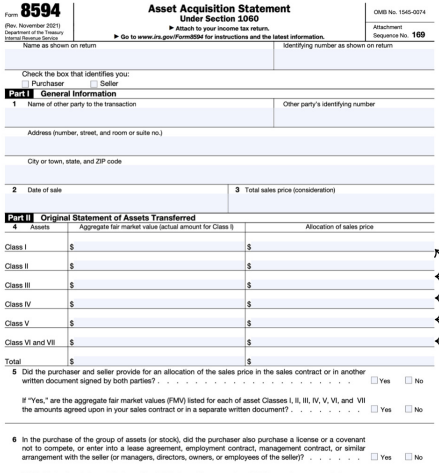
- Account for 1/2 of US business net income
- Dominate discussions on growth, wealth inequality, tax policy
- But pose challenge for
  - measurement: lack of reliable data on capital used in private businesses
  - theory: technology of capital accumulation and transfer

Significant Growth in Private Business



Motivating Evidence on Private Business Capital

details



Form **8594** (Rev. November 2021) Department of the Treasury Internal Revenue Service

Asset Acquisition Statement Under Section 1060

▶ Attach to your income tax return.

▶ Go to www.irs.gov/Form8594 for instructions and the latest information.

OMB No. 1545-0074

Attachment Sequence No. **169**

Name as shown on return | Identifying number as shown on return

Check the box that identifies you:
☐ Purchaser ☐ Seller

Part I General Information

1 Name of other party to the transaction | Other party's identifying number

Address (number, street, and room or suite no.)

City or town, state, and ZIP code

2 Date of sale | 3 Total sales price (consideration)

Part II Original Statement of Assets Transferred

4 Assets	Aggregate fair market value (actual amount for Class I)	Allocation of sales price
Class I	$	$
Class II	$	$
Class III	$	$
Class IV	$	$
Class V	$	$
Class VI and VII	$	$
Total	$	$

5 Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document signed by both parties? . . . . . ☐ Yes ☐ No

If "Yes," are the aggregate fair market values (FMV) listed for each of asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document? . . . . . ☐ Yes ☐ No

6 In the purchase of the group of assets (or stock), did the purchaser also purchase a license or a covenant not to compete, or enter into a lease agreement, employment contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)? . . . . . ☐ Yes ☐ No

If "Yes," attach a statement that specifies **(a)** the type of agreement and **(b)** the maximum amount of consideration (not including interest) paid or to be paid under the agreement. See instructions.

← Cash/securities
← Inventories
← Fixed assets
← Sec. 197 intangibles

Motivating Evidence on Private Business Capital

- Transferred assets are primarily **intangible** (from Form 8594 $\approx$ 70%) details
  - Customer bases and client lists, non-compete covenants
  - Licenses and permits, trademarks, tradenames
  - Workforce in place
  - Goodwill and on-going concern value
- Assets are **sold as a group**
- Sale **requires time** to find buyers/negotiate (from brokered data $\approx$ 290 days)

WANT OPERATOR: A Theory of Private Firm Dynamics

- "Traditional" Firm Dynamics (Lucas-Hopenhayn-Melitz)
  - Capital is divisible, rentable, and observable
- This paper
  - Capital is lumpy, nonrentable, and not observable until transacted

Related Literature

- Firm Dynamics
  - Hopenhayn (1992), Hsieh, Klenow (2009, 2014), David, Venkateswaran (2019), Sterk, Sedlacek, Pugsley (2021)
- Capital Reallocation
  - Holmes, Schmitz (1990), Ottonello (2014), Guntin, Kochen (2020), Gaillard, Kankanamge (2020), David (2021)
- Entrepreneurship and Private Wealth
  - Cagetti, De Nardi (2006), Saez, Zucman (2016), Smith, Yagan, Zidar, Zwick (2019)
- Capital Gain Taxes and Wealth Taxes
  - Chari, Golosov, Tsyvinski (2003), Scheuer, Slemrod (2020), Guvenen, Kambourov, Kuruscu, Ocampo (2021), Agersnap and Zidar (2021)

Theory

Model Environment

- Infinite horizon with continuous time
- Demographics
  - Unit mass of individuals
  - Birth/death at constant rate ψ
- Individuals choose
  - Paid-employment, supplying labor inelastically or
  - Self-employment, running a private business
- Preferences risk neutral, discount rate ρ

Technology

- Goods technology: $y(z, k, n, b) = zk^{\alpha} n^{\gamma} b^{\beta}$
- Non-transferable factors: productivity, z
  - evolves exogenously when running a business: $dz = \mu(z)dt + \sigma(z)\sqrt{dt}d\mathcal{W}$
- Transferable factors: business capital, k
  - built through investment: $dk = (\theta - \delta_k k)\, dt$, convex cost $C(\theta)$
  - bilaterally traded
- Rentable factors: labor n, other fixed assets and equipments b
  - spot markets
  - investment in other fixed assets: $db = (x - \delta_b b)\, dt$

Entry and Exit Technology

Let $s = (z, k)$ be the state of a business

- Entry:
  - draw $s \sim G(s)$, where $s = (z, k = 0)$
  - decide whether to run a firm or be a worker
- Exit:
  - option to exit at rate δ
  - exit due to death at rate ψ

Spot Markets for Labor and Buildings

- Spot market for labor n
  - supply: occupation choice
  - demand: incumbent firms
- Spot rental market for other fixed assets b
  - supply: competitive "mutual fund"
  - demand: incumbent firms

Market for Transferable Business Capital

- Businesses access market at rate η
- Bilateral trades: For a pair $s, \tilde{s}$
  - Feasible allocations: $k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\} \Rightarrow$ indivisibility (extension w/ costly divisibility)
  - Transfers (prices): $p^m(s, \tilde{s})$, negative if selling (extension w/ financing constraints: $p^m(s, \tilde{s}) \leq \xi y(s, n)$)
- Miniminal restrictions on outcomes
  - any can trade with anyone
  - any feasible allocation s.t. $p^m(s, \tilde{s}) + p^m(\tilde{s}, s) \geq 0$

Owner's Value Solves HJB

- Existing firms

$$
\begin{aligned}
(\rho+\psi)V(s) = & \underbrace{\max_{n,b} y(s,n,b) - wn - rb}_{\text{production}} \\
& + \underbrace{\max_{\theta} \partial_k V(s)(\theta - \delta_k k) - C(\theta)}_{\text{investment}} + \underbrace{\max_{\lambda} \eta W(s;\lambda)}_{\text{trade}} \\
& + \underbrace{\mu(z)\partial_z V(s) + \frac{1}{2}\sigma(z)^2 \partial_{zz} V(s)}_{\text{evolution of productivity}} + \underbrace{\delta \max\left\{\frac{w}{\rho+\psi} - V(s), 0\right\}}_{\text{endog exit}}
\end{aligned}
$$

where

$$
\begin{aligned}
& W(s;\lambda) = \int [V(z, k^m(s,\tilde{s})) - V(z,k) - p^m(s,\tilde{s})]\lambda(s,\tilde{s})ds \\
& \int \lambda(s,\tilde{s})d\tilde{s} + \lambda(s,0) = 1
\end{aligned}
$$

Law of Motion

- Let ϕ = distribution over states s and m be the mass of firms
  - ϕ evolves with exogenous shocks: $\mu(z), \sigma(z)$, δ, ψ and choices

    Investment: θ
    Trade: λ
    Entry/Exit: $\iota_{\text{entry}}(s), \iota_{\text{exit}}(s)$

  - Law of motion for ϕ and m given by:

$$d\phi = \Gamma(\phi; \theta, \lambda, \iota_{\text{entry}}, \iota_{\text{exit}})dt$$

$$dm = \psi \int \iota_{\text{entry}}(s) dG(s) - m \left(\psi + \delta \int \iota_{\text{exit}}(s)\phi(s) \right) dt$$

  - Stationarity
    - Distribution $d\phi = 0$
    - Mass $dm = 0$

Definition of Recursive Equilibrium

A (stationary) equilibrium is a set of value functions $V(s)$, policy functions for investment $\theta(s)$, entry $\iota_{entry}(s)$, exit $\iota_{exit}(s)$, and trade $\lambda(s, \tilde{s})$, terms of trade $(k^m(s, \tilde{s}), p^m(s, \tilde{s}))$, wage w, rental rates r, and distribution over the state space $\phi(s)$ and a mass of firms m that satisfy

- business owners' optimality
- indifference in occupational choice
- market clearing
- consistency and stationarity of measures and mass

Properties of the Equilibrium: A Warm Up

- To better understand properties
- First consider the simplest model with
  - Businesses that can be instantaneously traded
  - Capital that is perfectly divisible and rentable

Nested Limiting Model

- If:
  - Businesses can be instantaneously traded
  - Capital is perfectly divisible and rentable
- Theory predicts:
  - Capital perfectly allocated across firms
  - Valuation ratios, marginal product of capital, return on business investment same for all firms
    - Tobin's Q : $p = C'(\theta)$
    - Marginal product of capital (MPK): $\alpha y/k$
    - Returns: $\rho + \delta_k$

Our Model

- If:
  - Businesses cannot be instantaneously traded
  - Capital is not perfectly divisible

- Theory predicts:
  - Gradual capital reallocation toward higher MPK firms
  - Transferable share of capital below 100%
  - Dispersion in Tobin's Q, MPKs, net returns

An Intuitive Example

Who Trades with Whom?

- Intuitive example:
  - simple production function: $y = zk$
  - productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - capital pre-trade: all have $k = 1$

- Efficient allocation implies:
  - 10 low types sell to 10 of the high types

How are Terms of Trade Determined?

- Intuitive example:
  - simple production function: $y = zk$
  - productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - capital pre-trade: all have $k = 1$

- Price leaves high types indifferent between:
  - Trading, with $k = 2$ post-trade
  - Not trading, with $k = 1$ post-trade

Equilibrium Policy Functions

- Intuitive example:
  - simple production function: $y = zk$
  - productivity types: 20 with $z_H = 1$, 10 with $z_L = 0$
  - capital pre-trade: all have $k = 1$

- Equilibrium in this example:
  - Capital allocations: $k^m(s_H, s_L) = 2, k^m(s_L, s_H) = 0$
  - Prices: $p^m(s_H, s_L) = 1, p^m(s_L, s_H) = -1$
  - Choice probabilities: $\lambda(s_H|s_L) = 1,\ \lambda(s_L|s_H) = 1/2,\ \lambda_o(s_L) = 0,\ \lambda_o(s_H) = 1/2$

More Generally...

- Given (ϕ, V), solve linear programming problem:
  - maximize static social surplus
  - subject to adding up constraints
- Which delivers:
  - equilibrium allocations: λ, k^m
  - prices: p^m
  - gains from trade: $W(s)$
- Then update: $(\phi, V) \to (\lambda, p^m, k^m) \to (\phi', V')$
- Easy to extend if utility non-transferable

details

Properties of the Equilibrium

- **Pairwise stability**: $\nexists(s, \tilde{s})$ and feasible trade that makes the pair (strictly) better off
- **Competitive allocation** solves the planner's problem starting at $\phi(s, t = 0) = \phi^{ss}(s)$
- **Competitive prices** are independent of seller's z

$$p^m(s, \tilde{s}) = \mathcal{P}(k(\tilde{s}))$$

Intuition: competitive nature of the equilibrium $\Rightarrow$ same good sold at same price

Using the Model

- Calibration using data on
  - tax returns
  - business transfers
- Model deliverables
  - dispersion in marginal product of capital (MPK)
  - business price and value
- Tax policy analysis

Measurement

Sample

- Sample Period: 1996 to 2022
- An observation is firm-year pair with
  - Subchapter S corporation filing (Form 1120S)
  - Wage bill $>$ 10K
  - At least 3 years of data
- Two samples:
  - *Full* sample: all firm-year pairs meeting above criteria
  - *Trading* sample: seller-buyer pairs with
    - Seller: meeting above criteria and counterparty on Form 8594
    - Buyer: counterparty on Form 8594 with wage bill $>$ 10K

Variable Definitions

- Business Age: Number of years from Date of Incorporation (F1120S,Box E)
- Business Size (Wage Bill): Sum of all Wages, tips, other compensation (FW2, Box 1)
- Business Valuation: Total consideration (Form 8594, line 3)
- Relative Size: Wage bill of buyer t+1 / wage bill of seller t-1 (F8594, F1120S)

Data Samples

Table: IRS Samples

Business samples	Counts
S corporation population	3,167,266
S corporation sellers	105,162
Sales to S corporations	46,708
to Partnerships	33,462
to C corporations	35,792
Seller-buyer pairs	51,286
S Corporation–S Corporation	28,078
–Partnership	14,040
–C Corporation	9,168

Identification Strategy

- Life-cycle firm dynamics
  - productivity process
  - rentable input share
  - exit rates
  - size distributions
- Transaction data
  - trading rate
  - investment cost
  - output elasticity wrt business capital

Transaction Data and Key Parameters

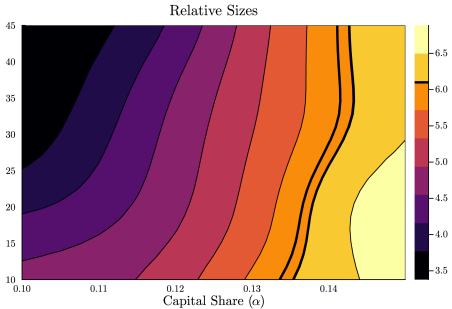
Key parameters

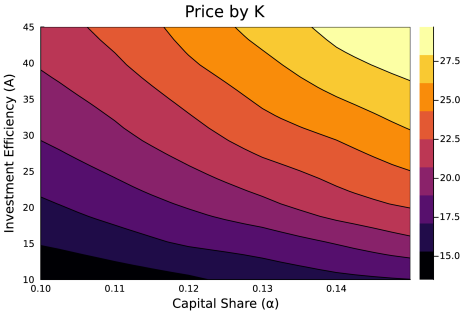
- trading rate η
- investment cost $C(\theta) = \frac{A}{1+\chi}\theta^{1+\chi}$
- output elasticity wrt k, $y(z,k,n,b) = zk^{\alpha}n^{\gamma}b^{\beta}$

Key moments from data

- brokered sales: time to sell
- IRS filings
  - relative size of buyer/seller
  - sale price/wage bill

Identification of A and α





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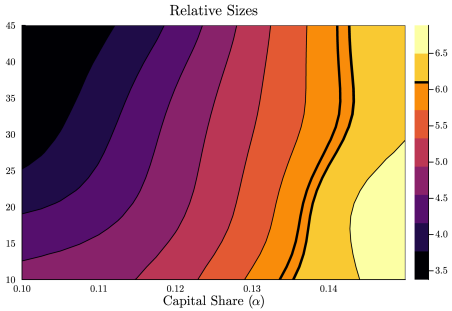
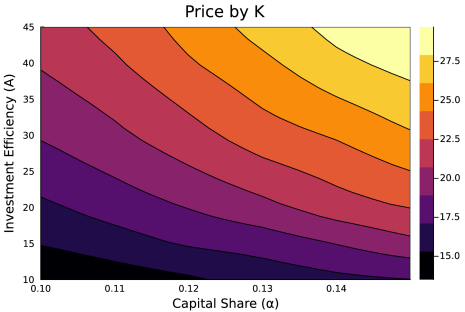
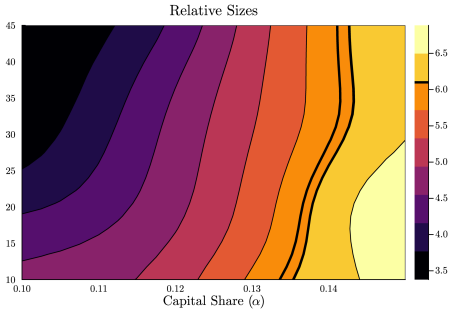
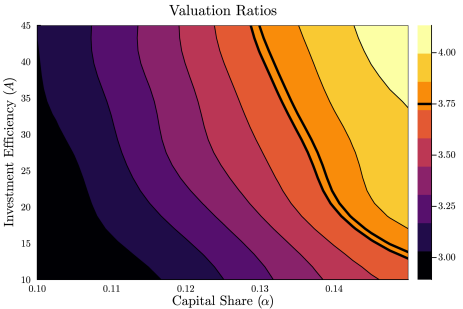




- Same relative size if $\alpha \uparrow$ (more linear technology) and $A \uparrow$ (more costly for buyer to scale up)
- Same $\mathcal{P}/k$ if $\alpha \uparrow$ (buyer willing to pay more) and $A \downarrow$ (buyer willing to pay less)
- But what is the empirical counterpart of $\mathcal{P}/k$??

Identification of A and α





- Same relative size if $\alpha \uparrow$ (more linear technology) and $A \uparrow$ (more costly for buyer to scale up)
- Same $\mathcal{P}/k$ if $\alpha \uparrow$ (buyer willing to pay more) and $A \downarrow$ (buyer willing to pay less)
- Proxy $\mathcal{P}/k$ with $\mathcal{P}/wn$. Similar intuition + selection on z

Functional Forms and Parameters

- Production

  $y(s, n, b) = z(s)k(s)^{\alpha} n^{\beta} b^{\gamma}, \quad \alpha = 0.15,\ \beta = \gamma = 0.35$

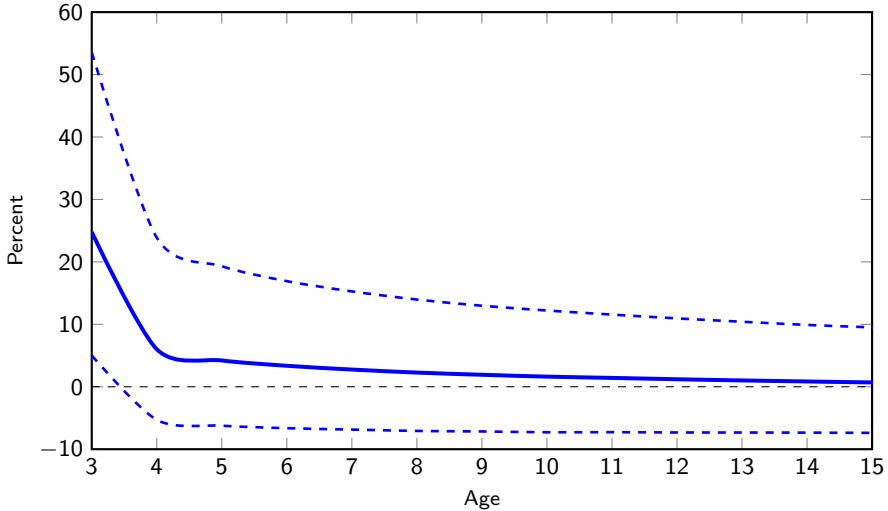
- Investment

  cost $C(\theta) = A\theta^{1+\chi}/(1+\chi), \quad A = 30,\ \chi = 2$

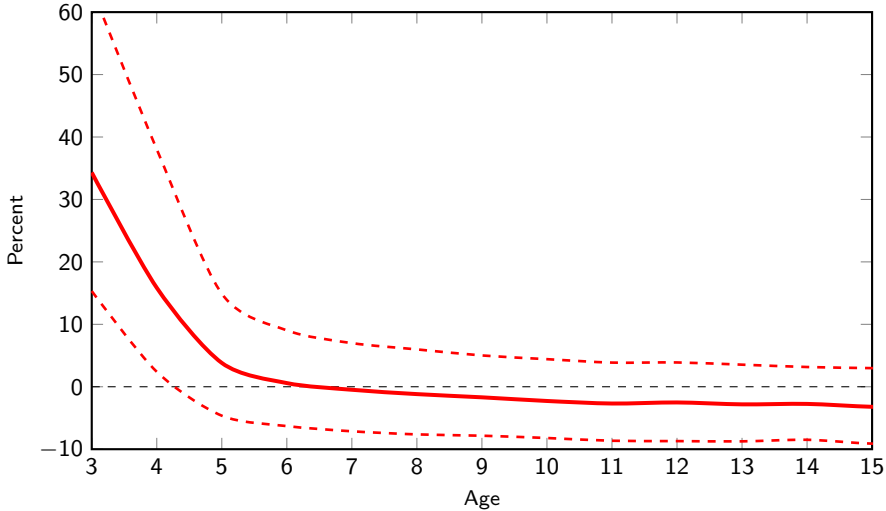
  depreciation, $\delta_k = \delta_m = 0.1$

- Productivity
  - entry distribution $G(z) \propto z^{-2}$
  - post entry process $d \log z = \mu dt + \sigma d\mathcal{W},\ \mu = -0.02, \sigma = 0.1$
- Rates
  - discounting, $\rho = 0.05$
  - trading, $\eta = 1.0$
  - exiting, $\psi = 1/40,\ \delta \to \infty$

Distribution of Annualized 3-Year Growth by Age: Sample of U.S. S Corporations



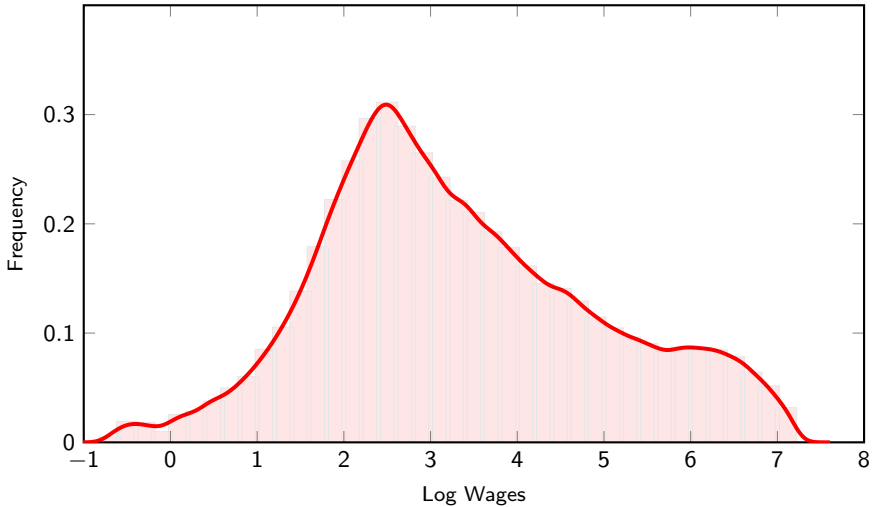
Distribution of Annualized 3-Year Growth by Age: Model



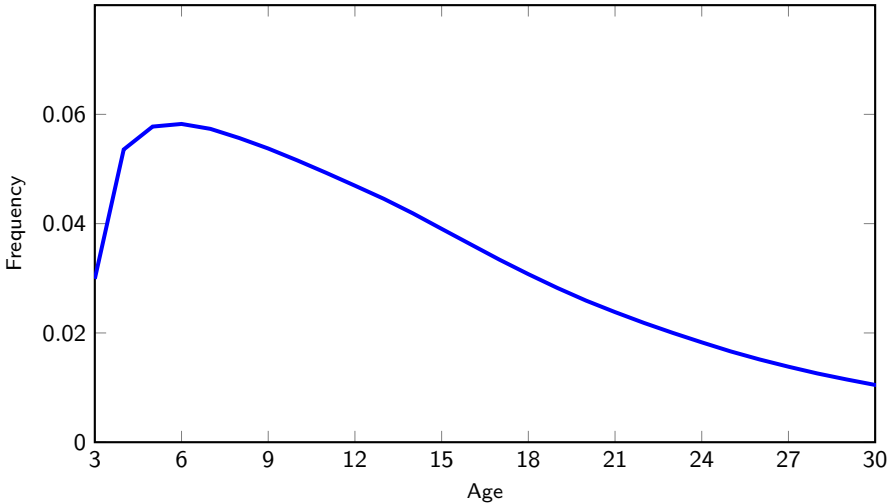
Size Distribution: Sample of U.S. S Corporations

Log Wage Bill	Percentiles		
	25^{th}	50^{th}	75^{th}
Entrants	11.0	11.7	12.5
Population	11.1	11.9	12.8

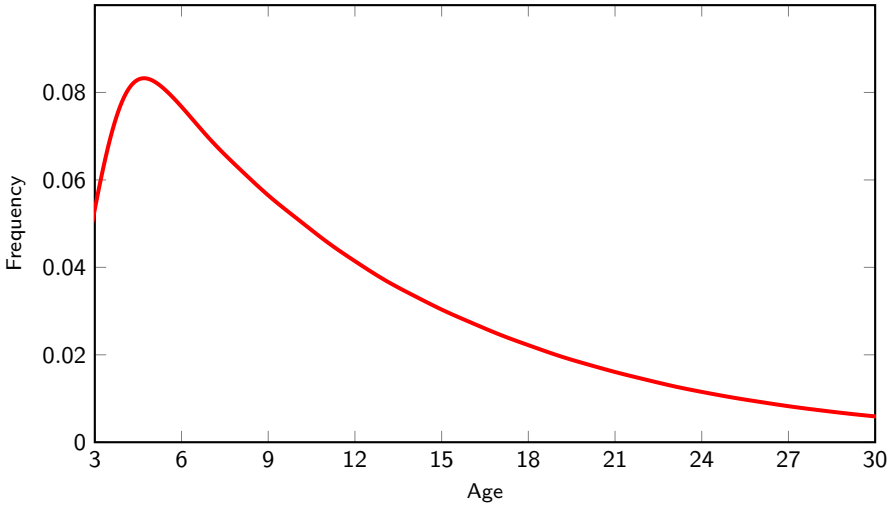
Size Distribution: Model



Age Distribution for US S Corporations



Age Distribution: Model



Valuation moment: Sample of U.S. S Corporation Sellers

Statistic	Percentiles 25th	50th	75th
Valuation Ratios			
Sales to S corporations	1.0	2.4	5.2
C corporations	1.4	3.5	8.6
Partnerships	1.5	4.0	9.9
All sales	1.2	2.9	6.7
Relative Wage Bill Sizes			
Sales to S corporations	0.7	1.4	5.6
C corporations	1.0	2.8	17.4
Partnerships	2.2	14.9	130.7
All sales	0.9	2.1	13.5

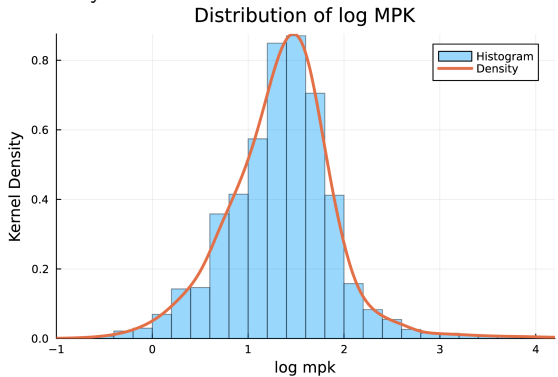
Valuation Moments: Model

Statistic	Percentiles		
	25^{th}	50^{th}	75^{th}
Valuation Ratios	3.1	3.7	4.8
Relative Wage Bill Sizes	2.3	6.0	18.5

Applications

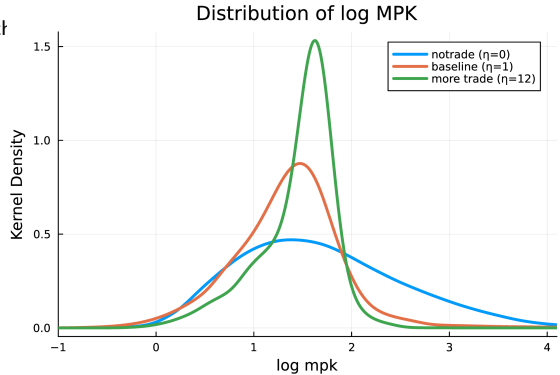
Dispersion in MPK

- Idiosyncratic change in productivity → input reallocation toward higher MPK
- Dispersion in marginal product of capital induced by
  - decentralized trading
  - indivisibility of asset sold
- Standard deviation of log-MPK: 57%



Dispersion in MPK: Role of η

- Consider 3 values of η
  - $\eta = 0$: No trade
  - $\eta = 1$: trading opportunity every year
  - $\eta = 12$: trading opportunity every month
- Standard deviation of log-MPK
  - Ranges from 40% to 90%



Business Wealth

- Finance textbook: Present value of owner's dividend
  - Model counterpart: $V(s)$
- SCF respondent: Answer to the survey question–"What could you sell it for?"
  - Model counterpart: $\mathcal{P}(k)$

$\Rightarrow$ Both are inputs to analyses of business capital and wealth

Model Predictions for Business Wealth (in %)

Statistic	Income Yield	Transferable Share
Percentiles		
5^{th}	2.6	9.8
10^{th}	3.1	11.0
25^{th}	4.6	12.3
50^{th}	7.5	21.3
75^{th}	12.7	30.3
90^{th}	16.5	38.1
95^{th}	17.4	43.6
Average	8.7	23.1
Aggregate	13.6	27.0

Business Taxation

- Recent debate on business taxation
- What to tax
  - flows: business income
  - stocks: business capital (Guvenen et al. 2022)
  - transfers: capital gains (Sarin et al 2022, Agersnap and Zidar 2021)
- Our model can speak to all three forms of taxation

Tax Instruments

- Compare instruments:
  - **capital gains**: $\tau_c \mathcal{P}(k)$
  - **business income**: $\tau_b(y - wn - rb)$
  - **business capital**: $\tau_k \mathcal{P}(k)$ [capital ownership]

- Compare several outcomes for a given level of Revenue R

Business Taxation

- Revenue raised 1.5% of baseline output
  - Business income tax: $\tau_b = 4.8\%$
  - Capital gains tax: $\tau_c = 50\%$
  - Capital tax: $\tau_k = 2.7\%$
- Outcomes % change from baseline

Business Taxation: Outcomes

Table: Predicted Tax Policy Changes

Statistic	Business Income	Capital Stock	Capital Gains
Mass of firms	0.8	−7.1	−32.3
Fraction traded	3.8	−5.8	−64.6
Average investment	0.3	−0.9	−1.6
Dispersion in MPK	3.5	−2.5	2.2
Wage	−0.5	−1.3	−6.0

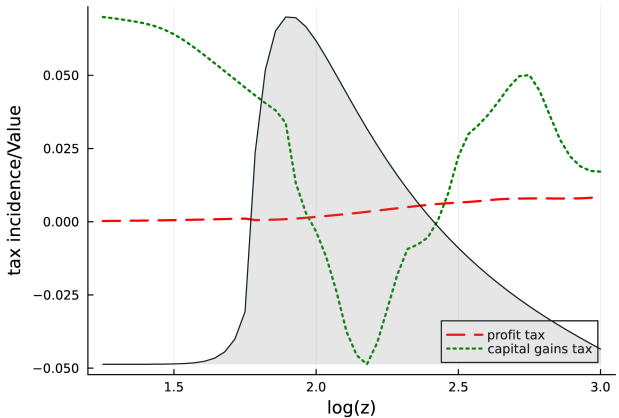
Main Result

- Taxing income dominates taxing gains
- Taxing capital gains
  - distorts capital reallocation across businesses
  - decreases entry
  - decreases investment by owners
  - lowers welfare
- Taxing business capital is between the two

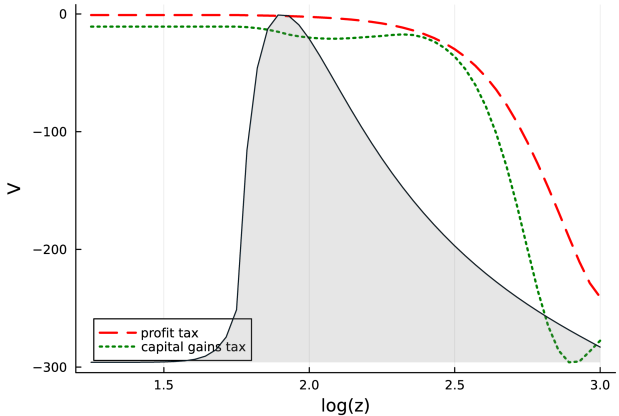
Next: Discuss intuition by comparing taxing income vs. gains

Taxing income vs. gains: Avg. Tax incidence by Productivity

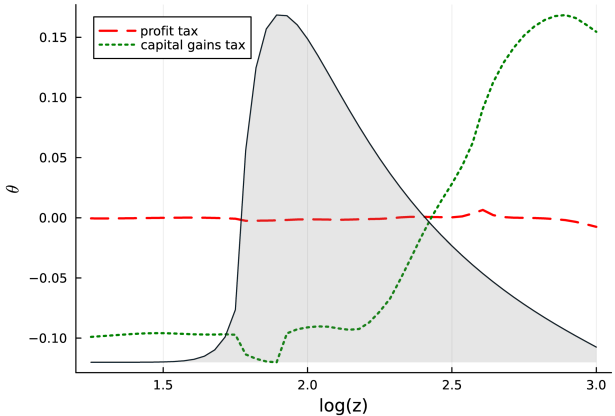
Capital gain tax incidence measured as the change in net price received/paid for selling/buying capital



Taxing income vs. gains: Entrant Values by Productivity



Taxing income vs. gains: Avg. Investment by Productivity



Main Result: Intuition

The main differences are driven by extensive margin of entry.

- Incidence of taxes
  - business income: high productivity firms earn profits
  - capital gains: low productivity firms typically sell
- Distortionary effects on entry
  - business income: small because high productivity firms enter anyways
  - capital gains: large because low productivity firms are marginal entrants
- Distortionary effects on investment
  - business income: small because tax base is wider
  - capital gains: small because higher prices make productive firms substitute toward investment

Conclusions

- Need for new theory and measurement to study private firms
- We make progress on both fronts using
  - model that captures features of market for private businesses
  - new data combining value of intangible capital with firm dynamics

Business Transfers are Taxable Events

back

- Buyers and sellers both report sale
  - seller has to pay capital gains
  - buyer has to report depreciable assets
- Price allocated across asset types
  - seller wants to allocate to long-term
  - buyer wants to allocate to short-term

$\Rightarrow$ Conflict of interest and thus consistent reporting

Auxiliary Problem: Static Planner

back

Define gains from trade between $s, \tilde{s}$:

$$X(s,\tilde{s}) = \max_{k^m \in \{k(s)+k(\tilde{s}),0\}} \{V(z(s),k^m) + V(z(\tilde{s}),k(s)+k(\tilde{s})-k^m)\} - (V(s)+V(\tilde{s}))$$

$$\begin{aligned} Q(\phi,V) &= \max_{\pi \geq 0} \Sigma_{s,\tilde{s}} X(s,\tilde{s})\pi(s,\tilde{s}) \\ s.t.\ \ \Sigma_{\tilde{s}}\pi(s,\tilde{s}) + \pi(s,0) &= \frac{\phi(s)}{2}\ \forall s \quad [\mu^a(s)] \\ \Sigma_{\tilde{s}}\pi(\tilde{s},s) + \pi(0,s) &= \frac{\phi(s)}{2}\ \forall s \quad [\mu^b(s)] \end{aligned}$$

Auxiliary Problem: Static Planner

Lemma

- $W(s) = \frac{\partial Q}{\partial \phi(s)} = \frac{\mu^a(s)+\mu^b(s)}{2} \equiv \mu(s)$
- $\lambda(s, \tilde{s}) = \frac{2\pi(s,\tilde{s})}{\phi(s)}$
- $k^m(s, \tilde{s}) = \arg\max X(s, \tilde{s}) \qquad p^m(s, \tilde{s}) = V(z, k^m(s, \tilde{s})) - V(z, k) - W(s)$

- Multipliers $\mu = \mu^a = \mu^b$ capture gains from trade

$$\mu = \nabla_\phi Q$$

- Prices implement gains from trade

$$p^m(s, \tilde{s}) = V(z(s), k^m(s, \tilde{s})) - \mu(s)$$

back

Dual Problem

back

- From the minimax thm, the solution of the primal problem is equal to the solution of the dual
- The multipliers in the primal are equal to the choice variable in the dual, and vice versa

$$Q(\phi) = \min_{\mu^a \geq 0, \mu^b \geq 0} \Sigma_s \left(\mu^a(s) + \mu^b(s) \right) \frac{\phi(s)}{2}$$
$$s.t. \;\; \mu^a(s) + \mu^b(\tilde{s}) \geq X(s, \tilde{s}) \;\; \forall s, \tilde{s} \quad [\pi(s, \tilde{s})]$$

Trade with Preference Shocks

back

- After-trade values for buyers (v_b) and sellers (v_s)
  - $v_b(s, \hat{k}; p)$: value from buying $\hat{k}$
  - $v_s(s, 0; p)$: value from selling $k(s)$
- Matching probability

$$\lambda(s, \hat{k}; p) = \exp\left(\frac{v_b(s, \hat{k}; p) - W(s)}{\sigma}\right)$$

$$\lambda(s, 0; p) = \exp\left(\frac{v_s(s, 0; p) - W(s)}{\sigma}\right)$$

where $W(s) = \mathbb{E}\max\{v_b(s, \hat{k}; p), v_s(s, 0; p)\}$

- Find $\{p(s)\}$ such that $\forall \hat{k}$

$$\underbrace{\int \lambda(s, \hat{k}; p)}_{\text{demand}} = \underbrace{\int \lambda(s, 0; p)\mathbb{I}\{k(s) = \hat{k}\}}_{\text{supply}}$$

Price Cap and Taxes

- Under capital gain tax τ,

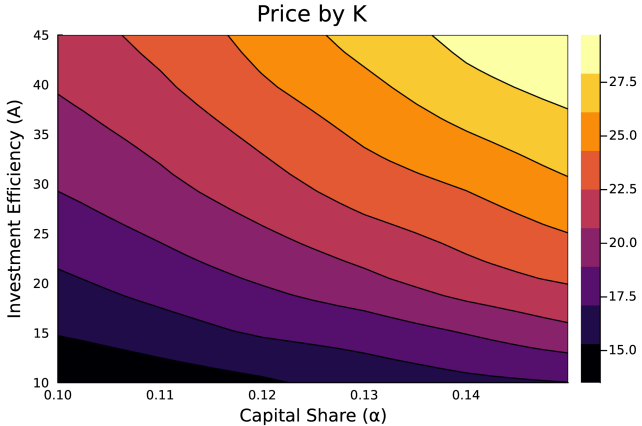
$$v_b(s;\hat{k}) = V(z, k(s)+\hat{k}) - p(\hat{k})$$
$$v_s(s) = V(\tilde{s}, 0) + (1-\tau)p(k(s))$$

- Under cap on paid price equal to $\xi y(s,n)$

$$v_b(s;\hat{k}) = \begin{cases} V(z, k(s)+\hat{k}) - p(\hat{k}) & \text{if } p(\hat{k}) \leq \xi y(s,n) \\ -\infty & \text{o/w} \end{cases}$$
$$v_s(s) = V(\tilde{s}, 0) + p(k(s))$$

Selection into Selling

back



$$\frac{P(k)}{w\ell} \propto \frac{P(k)}{zk^{\alpha}} \quad \overset{\text{A}\uparrow}{\Longrightarrow} \quad \frac{P(k)}{k^{\alpha}}\uparrow \quad \frac{1}{z}\downarrow$$