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Appendix: The Impact of Brexit on Foreign Investment and Production*

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* The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

In this appendix, we provide modeling and computational details for the model in the main text. We also show the full mapping between the model variables and their empirical analogues in the national and international accounts. Finally, we write out the current accounts and financial accounts for a 2-country version of the model and show how they relate to household budget constraints.

1. Model Details

The model is a multicountry (or, more generally, a multi-union) dynamic general model designed to study the dissoluation of an economic union. Economic unions are indexed by i (unless otherwise noted) and multinational firms are indexed by ω. Firms incorporated in i have indices in the set Ω_i, that is, $\omega \in \Omega_i$. Economic unions are characterized by their productive capacity (N_i), their TFP (A_i), their policies governing traded goods ($\zeta_i(\omega)$), and their policies governing investments by foreigners ($\sigma_i(\omega)$). These characteristics are taken as given by multinational firms when making their production and investment decisions and are critical determinants of where production occurs and who the final consumers are.

1.1. Multinational Problem

Multinational with technology ω maximizes the present value of after-tax worldwide dividends, namely,

$$\max \left(1-\tau_d\right) \sum\nolimits_t p_t D_t\left(\omega\right),$$

where τ_d is the tax rate on shareholder dividends, p_t is the Arrow-Debreu price, and $D_t(\omega)$ is the total dividend payment given by

$$\begin{aligned} D_t\left(\omega\right) = \sum\nolimits_i \Big\{ & \left(1-\tau_{p,it}\right)\left(P_{it}\left(\omega\right)\left[Y_{it}\left(\omega\right) - \delta_T K_{T,it}\left(\omega\right) - X_{I,it}\left(\omega\right) - \chi_i\left(\omega\right) X_{Mt}\left(\omega\right)\right]\right. \\ & \left. - W_{it} L_{it}\left(\omega\right)\right) - P_{it}\left(\omega\right)\left(K_{T,i,t+1}\left(\omega\right) - K_{T,it}\left(\omega\right)\right) \Big\}. \end{aligned}$$

with $\chi_i(\omega) = 1$ if $\omega \in \Omega_i$. The dividends are summed across all countries and are equal to after-tax profits less retained earnings. Profits are equal to sales ($P_i(\omega)Y_i(\omega)$) net of depreciation ($\delta_T K_{T,i}(\omega)$), expensed intangible investments ($X_{I,i} + \chi_i(\omega)X_M$), and labor costs ($W_i L_i(\omega)$). Retained earnings is equal to net investment, which is the change in plant-specific tangible capital

$(K_{T,i}(\omega))$. The producer price of goods made by firm ω in i is $P_{it}(\omega)$ and total output is given by

$$Y_{it}(\omega) = \sigma_i t(\omega) A_{it} (N_{it} M_t(\omega))^{\phi} (Z_{it}(\omega))^{1-\phi}$$

$$Z_{it}(\omega) = (K_{T,it}(\omega))^{\alpha_T} (K_{I,it}(\omega))^{\alpha_I} (L_{it}(\omega))^{1-\alpha_T-\alpha_I}.$$

The firm's factor inputs are plant-specific tangible capital $(K_{T,i}(\omega))$, plant-specific intangible capital $(K_{I,i}(\omega))$, technology capital (M), and labor $(L_i(\omega))$. The technology capital can be used nonrivalrously at all possible production locations. Investments in capital are denoted by X and the capital accumulation equations are given by:

$$K_{T,i,t+1}(\omega) = (1-\delta_T) K_{T,it}(\omega) + X_{T,it}(\omega)$$

$$K_{I,i,t+1}(\omega) = (1-\delta_I) K_{T,it}(\omega) + X_{I,it}(\omega)$$

$$M_{t+1}(\omega) = (1-\delta_M) M_t(\omega) + X_{M,t}(\omega).$$

The first-order conditions for multinational ω producing in i with respect to $L_i(\omega)$, $K_{T,i}(\omega)$, $K_{I,i}(\omega)$, and $M(\omega)$ are as follows:

$$\begin{aligned}
W_{it} &= (1-\phi)(1-\alpha_T-\alpha_I) P_{it}(\omega) Y_{it}(\omega) / L_{it}(\omega) \\
\frac{p_t P_{it}(\omega)}{p_{t+1} P_{it+1}(\omega)} &= 1 + (1-\tau_{p,i,t+1}) \left((1-\phi)\alpha_T Y_{i,t+1}(\omega) / K_{T,i,t+1}(\omega) - \delta_T\right) \\
&\equiv 1 + (1-\tau_{p,i,t+1}) (r_{T,i,t+1}(\omega) - \delta_T) \\
\frac{p_t P_{it(\omega)}}{p_{t+1} P_{it+1}(\omega)} &= \frac{(1-\tau_{p,i,t+1})}{(1-\tau_{p,it})} \left((1-\phi)\alpha_I Y_{i,t+1}(\omega) / K_{I,i,t+1}(\omega) + 1 - \delta_I\right) \\
&\equiv \frac{(1-\tau_{p,i,t+1})}{(1-\tau_{p,it})} (r_{I,i,t+1}(\omega) + 1 - \delta_I) \\
\frac{p_t}{p_{t+1}} &= \frac{\sum_i (1-\tau_{p,i,t+1}) P_{it+1}(\omega) \left(\phi Y_{i,t+1}(\omega) / M_{t+1}(\omega) + \chi_i(\omega)(1-\delta_M)\right)}{\sum_i (1-\tau_{p,it}) \chi_i(\omega) P_{it}(\omega)} \\
&\equiv \frac{\sum_i (1-\tau_{p,i,t+1}) P_{it+1}(\omega) \left(r_{M,i,t+1}(\omega) + \chi_i(\omega)(1-\delta_M)\right)}{\sum_i (1-\tau_{p,it}) \chi_i(\omega) P_{it}(\omega)}.
\end{aligned}$$

If there are costs to adjusting capital, then the capital accumulation constraints are replaced by:

$$K_{T,i,t+1} = (1-\delta_T) K_{T,it} + X_{T,it} - \varphi\left(X_{T,it}(\omega) / K_{T,it}(\omega)\right) K_{T,it}(\omega),$$

$$K_{I,i,t+1}(\omega) = (1-\delta_I) K_{T,it}(\omega) + X_{I,it}(\omega) - \varphi\left(X_{I,it}(\omega) / K_{I,it}(\omega)\right) K_{I,it}(\omega),$$

$$M_{t+1}(\omega) = (1-\delta_M) M_t(\omega) + X_{M,t}(\omega) - \varphi\left(X_{M,t}(\omega) / M_t(\omega)\right) M_t(\omega).$$

We use the following functional form:

$$\varphi(X/K) = \frac{\varphi_0}{2}(X/K - \delta - \gamma_Y)^{\varphi_1},$$

where δ is the depreciation rate of the relevant investment series and γ_Y is trend growth in the global output. Below, we'll use the following short-cut notation:

$$\varphi_{T,it} = \varphi(X_{T,it}(\omega)/K_{T,it}(\omega))$$
$$\partial\varphi_{T,it} = \varphi'(X_{T,it}(\omega)/K_{T,it}(\omega)).$$

The new first-order conditions with respect to capital stocks $K_{T,i,t+1}(\omega)$, $K_{I,i,t+1}(\omega)$, and $M_{t+1}(\omega)$ are given by

$$\begin{aligned}&\frac{p_t P_{it}(\omega)}{p_{t+1}P_{it+1}(\omega)(1-\partial\varphi_{T,it})}\\ &\quad= (1-\tau_{p,i,t+1})(1-\phi)\alpha_T Y_{i,t+1}(\omega)/K_{T,i,t+1} + \tau_{p,i,t+1}\delta_T\\ &\quad+\frac{(1-\delta_T-\varphi_{T,i,t+1}+\partial\varphi_{T,i,t+1}X_{T,i,t+1}(\omega)/K_{T,i,t+1}(\omega))}{(1-\partial\varphi_{T,i,t+1})}\end{aligned}$$

$$\begin{aligned}&\frac{p_t P_{it}(\omega)}{p_{t+1}P_{it+1}(\omega)(1-\partial\varphi_{I,it})}\\ &\quad= \frac{(1-\tau_{p,i,t+1})}{(1-\tau_{p,it})}\Big[(1-\phi)\alpha_I Y_{i,t+1}(\omega)/K_{T,i,t+1}\\ &\quad+\frac{(1-\delta_I-\varphi_{I,i,t+1}+\partial\varphi_{I,i,t+1}X_{I,i,t+1}(\omega)/K_{I,i,t+1}(\omega))}{(1-\partial\varphi_{I,i,t+1})}\Big]\end{aligned}$$

$$\begin{aligned}&\frac{p_t}{p_{t+1}(1-\varphi_{Mt})}\\ &\quad= \frac{\sum_i(1-\tau_{p,i,t+1})P_{i,t+1}(\omega)\phi Y_{i,t+1}(\omega)/M_{t+1}(\omega)}{\sum_i(1-\tau_{p,it})\chi_i(\omega)P_{it}(\omega)}\\ &\quad+\frac{(1-\delta_M-\varphi_{Mt+1}+\partial\varphi_{Mt}X_{M,t+1}(\omega)/M_{t+1})}{(1-\partial\varphi_{M,t+1})}\frac{\sum_i(1-\tau_{p,i,t+1})\chi_i(\omega)P_{it+1}(\omega)}{\sum_i(1-\tau_{p,it})\chi_i(\omega)P_{it}(\omega)}\end{aligned}$$

1.2. Household Problem

Households in country i choose sequences of consumption goods C_{it}, labor L_{it}, shares in companies $S_{it}(\omega)$, and bonds B_{it} to solve the following problem:

$$\begin{aligned}&\max\sum\nolimits_t \beta^t\left[log(C_{it}/N_{it}) + \psi log(1-L_{it}/N_{it})\right]N_{it}\\ &\text{subj. to}\ \ \sum\nolimits_t p_t\left[P_{it}C_{it}+\sum\nolimits_\omega V_t(\omega)(S_{i,t+1}(\omega)-S_{it}(\omega))+B_{i,t+1}-B_{it}\right]\\ &\qquad\le\sum\nolimits_t p_t\left[(1-\tau_{li})W_{it}L_{it}+(1-\tau_{dt})\sum\nolimits_\omega S_{it}(\omega)D_t(\omega)+r_{bt}B_{it}+\kappa_{it}\right],\end{aligned}$$

where

$$C_{it} = \left(\sum\nolimits_{\omega} C_{it}(\omega)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

$$L_{it} = \sum\nolimits_{\omega} L_{it}(\omega) + \bar{L}_{nb,it},$$

and

$$C_{it}(\omega) = \left(C_{it}^{F}(\omega)^{\frac{\varrho-1}{\varrho}} + C_{it}^{T}(\omega)^{\frac{\varrho-1}{\varrho}}\right)^{\frac{\varrho}{\varrho-1}}, \omega \notin \Omega_i$$

with $\rho > 0$, $\varrho > 0$, and $\varrho \gg \rho$, where recall that Ω_i are technologies that have been developed in i. Also, total consumption expenditures are given by

$$P_{it}C_{it} = \sum\nolimits_{\omega} P_{it}(\omega)\, C_{it}(\omega)$$
$$P_{it}(\omega)\, C_{it}(\omega) = P_{it}^{F}(\omega)\, C_{it}^{F}(\omega) + P_{it}^{T}(\omega)\, C_{it}^{T}(\omega), \omega \notin \Omega_i.$$

We assume that $\bar{L}_{nb,it}$ is exogenously determined labor in the nonbusiness sector, τ_{li}, and τ_d are tax rates on labor, and company distributions, $V_t(\omega)$ is the price of a share of a firm producing using technology ω, W_{it} is the wage rate in country i, and r_{bt} is the after-tax return on lending/borrowing. We assume that country i has a population of size $N_{it} = n_{it}(1+\gamma_N)^t$, with common growth rate γ_N and a country-specific shifter n_{it}. Note that the measure of a country's production locations is proportional to its population. Hence, we use the same notation for both variables and set the constant of proportionality equal to one (without loss of generality).

We have included nonbusiness hours (exogenously) in total hours and will include nonbusiness income less investment in κ_i. The nonbusiness sector is added in order to ensure that the national income and product accounts (NIPA) aggregates are of the right order of magnitude. Because our focus is on returns to capital, we also assume that taxes on labor are constant over time while technology parameters and tax rates on dividends and profits vary over time.

The first-order conditions with respect to $C_{it}(\omega)$, $C_{it}^F(\omega)$, $C_{it}^T(\omega)$, L_{it}, B_{it}, and $S_{i,t+1}(\omega)$ for the household in country i are

$$\lambda p_t P_{it}(\omega) = \beta^t U_{c,it} \frac{\partial C_{it}}{\partial C_{it}(\omega)}$$

$$= \beta^t \frac{N_{it}}{C_{it}} \left(\frac{C_{it}}{C_{it}(\omega)} \right)^{1/\rho}, \omega \in \Omega_i$$

$$\begin{aligned}
\lambda p_t P_{it}^F(\omega) &= \beta^t U_{c,it} \frac{\partial C_{it}}{\partial C_{it}(\omega)} \frac{\partial C_{it}(\omega)}{\partial C_{it}^F(\omega)} \\
&= \beta^t \frac{N_{it}}{C_{it}} \left(\frac{C_{it}}{C_{it}(\omega)} \right)^{1/\rho} \left(\frac{C_{it}(\omega)}{C_{it}^F(\omega)} \right)^{1/\varrho}, \omega \notin \Omega_i \\
\lambda p_t P_{it}^T(\omega) &= \beta^t U_{c,it} \frac{\partial C_{it}}{\partial C_{it}(\omega)} \frac{\partial C_{it}(\omega)}{\partial C_{it}^T(\omega)} \\
&= \beta^t \frac{N_{it}}{C_{it}} \left(\frac{C_{it}}{C_{it}(\omega)} \right)^{1/\rho} \left(\frac{C_{it}}{C_{it}^T(\omega)} \right)^{1/\varrho}, \omega \notin \Omega_i \\
\lambda (1-\tau_{li}) p_t W_{it} &= \beta^t U_{l,it} = \psi \beta^t / (1 - L_{it}/N_{it}) \\
\frac{p_t}{p_{t+1}} &= (1 + r_{b,t+1}) \\
\frac{p_t}{p_{t+1}} &= \left(\frac{V_{t+1}(\omega) + (1-\tau_d) D_{t+1}^j(\omega)}{V_t(\omega)} \right), \quad \forall \omega,
\end{aligned}$$

where λ is the Lagrange multiplier associated with the household budget constraint.

Using the first-order conditions, it is easy to show that

$$\begin{aligned}
\frac{p_t}{p_{t+1}} &= \frac{N_{it} P_{it+1} C_{it+1}}{\beta N_{it+1} P_{it} C_{it}} \\
C_{it}^F(\omega) &= \left(\frac{P_{it}^F(\omega)}{P_{it}(\omega)} \right)^{-\varrho} C_{it}(\omega), \omega \notin \Omega_i \\
C_{it}^T(\omega) &= \left(\frac{P_{it}^T}{P_{it}(\omega)} \right)^{-\varrho} C_{it}(\omega), \omega \notin \Omega_i \\
C_{it}(\omega) &= \left(\frac{P_{it}(\omega)}{P_{it}} \right)^{-\rho} C_{it}, \omega \in \Omega_i
\end{aligned}$$

and that the prices are given by:

$$\begin{aligned}
P_{it}^F(\omega) &= P_{it}(\omega) \\
P_{it}^T(\omega) &= P_{jt}(\omega)(1 + \zeta_{it}(\omega)) \\
P_{it}(\omega) &= \left(\left(P_{it}^F(\omega)\right)^{1-\varrho} + \left(P_{it}^T(\omega)\right)^{1-\varrho} \right)^{\frac{1}{1-\varrho}}, \omega \notin \Omega_i \\
P_{it} &= \left(\sum\nolimits_{\omega \in \Omega_i} P_{it}(\omega)^{1-\rho} + \sum\nolimits_{\omega \notin \Omega_i} P_{it}(\omega)^{1-\rho} \right)^{\frac{1}{1-\rho}}
\end{aligned}$$

1.3. Resource Constraints

The resource constraints for each technology ω are given as follows:

$$Y_{jt}(\omega) = C_{jt}^{F}(\omega) + X_{T,jt}(\omega) + X_{I,jt}(\omega), j \neq i$$

$$Y_{it}(\omega) = C_{it}(\omega) + \sum_{j \neq i} (1 + \zeta_{jt}(\omega)) C_{jt}^{T}(\omega) + X_{I,it}(\omega)$$

$$+ XiIt(\omega) + X_{M,t}(\omega) + \bar{X}_{nb,it} - \bar{Y}_{nb,it},$$

where i is home for the multinational firm with this technology, that is, $\omega \in \Omega_i$, and j are the economic unions that host the firm's foreign affiliates. Here, we have explicitly included (exogenous) nonbusiness investment $\bar{X}_{nb,j}$ and output $\bar{Y}_{nb,j}$.

2. Computational Details

2.1. Detrended First-Order Conditions

We will use small letters for growth-detrended variables. Specifically, let

$$c_{it}(\omega) = \frac{C_{it}(\omega)}{N_{it}(1+\gamma_y)^t} = \frac{C_{it}(\omega)}{n_{it}(1+\gamma_Y)^t}$$

$$c_{it}^{F}(\omega) = \frac{C_{it}^{F}(\omega)}{N_{it}(1+\gamma_y)^t} = \frac{C_{it}^{F}(\omega)}{n_{it}(1+\gamma_Y)^t}$$

$$c_{it}^{T}(\omega) = \frac{C_{it}^{T}(\omega)}{N_{it}(1+\gamma_y)^t} = \frac{C_{it}^{T}(\omega)}{n_{it}(1+\gamma_Y)^t}$$

$$y_{it}(\omega) = \frac{Y_{it}(\omega)}{N_{it}(1+\gamma_y)^t} = \frac{Y_{it}(\omega)}{n_{it}(1+\gamma_Y)^t}$$

$$l_{it} = \frac{L_{it}}{N_{it}}, \; l_{it}(\omega) = \frac{L_{it}(\omega)}{N_{it}}$$

$$w_{it} = \frac{W_{it}}{(1+\gamma_y)^t}$$

$$k_{\cdot,it}(\omega) = \frac{K_{\cdot,it}(\omega)}{N_{it}(1+\gamma_y)^t} = \frac{K_{\cdot,it}(\omega)}{n_{it}(1+\gamma_Y)^t}$$

$$x_{\cdot,it} = \frac{X_{\cdot,it}(\omega)}{N_{it}(1+\gamma_y)^t} = \frac{X_{\cdot,it}(\omega)}{n_{it}(1+\gamma_Y)^t}$$

$$x_{M,t}(\omega) = \frac{X_{M,t}(\omega)}{(1+\gamma_Y)^t}$$

$$m_t(\omega) = \frac{M_t(\omega)}{(1+\gamma_Y)^t}$$

$$d_t(\omega) = \frac{D_t(\omega)}{(1+\gamma_Y)^t}$$

$$a_{it} = \frac{A_{it}}{(1+\gamma_A)^t},$$

where γ_Y is the growth rate of output, γ_y is the growth rate of per capita output, and γ_A is the growth rate of TFP. Using the production technology, we can determine the growth rate of total output on the balanced growth trend:

$$\begin{aligned}(1+\gamma_Y) &= (1+\gamma_A)(1+\gamma_N)^{\phi}(1+\gamma_Y)^{\phi}(1+\gamma_Y)^{\alpha_T(1-\phi)} \\ &\quad \cdot (1+\gamma_Y)^{\alpha_I(1-\phi)}(1+\gamma_N)^{(1-\alpha_T-\alpha_I)(1-\phi)} \\ &= (1+\gamma_A)^{\frac{1}{(1-\alpha_T-\alpha_I)(1-\phi)}}(1+\gamma_N)^{\frac{1-(\alpha_T+\alpha_I)(1-\phi)}{(1-\alpha_T-\alpha_I)(1-\phi)}},\end{aligned}$$

where recall that γ_N is the growth rate of the population (and locations).

2.2. Equilibrium Paths

Substituting detrended variables into first-order conditions (and assuming that the only time-varying tax rates are on profits) implies:

$$k_{T,i,t+1}(\omega) = [(1-\delta_T)k_{T,it}(\omega) + x_{T,it}(\omega)]\,n_{it}/\left(n_{i,t+1}(1+\gamma_Y)\right)$$

$$k_{I,i,t+1}(\omega) = [(1-\delta_I)k_{I,it}(\omega) + x_{I,it}(\omega)]\,n_{it}/\left(n_{i,t+1}(1+\gamma_Y)\right)$$

$$m_{t+1}(\omega) = [(1-\delta_M)m_t(\omega) + x_{M,t}(\omega)]/(1+\gamma_Y)$$

$$\begin{aligned}d_t(\omega) &= \sum\nolimits_i \left\{(1-\tau_{p,it})\,n_{it}\left(p_{it}(\omega)\left[y_{it}(\omega) - \delta_T k_{T,it}(\omega) - x_{I,it}(\omega)\right] - w_{it}l_{it}(\omega)\right)\right\} \\ &\quad - \sum\nolimits_i (1-\tau_{p,it})\,\chi_i(\omega)\,p_{it}(\omega)\,x_{M,t}(\omega) \\ &\quad - \sum\nolimits_i p_{it}(\omega)\left\{n_{i,t+1}(1+\gamma_Y)\,k_{T,i,t+1}(\omega) - n_{it}k_{T,it}(\omega)\right\}\end{aligned}$$

$$y_{it}(\omega) = \sigma_i(\omega)\sum\nolimits_i a_{it}m_t(\omega)^{\phi}\left((k_{T,it}(\omega))^{\alpha_T}(k_{I,it}(\omega))^{\alpha_I}(l_{it}(\omega))^{1-\alpha_T-\alpha_I}\right)^{1-\phi}$$

$$y_{it}(\omega) = c_{it}(\omega) + \sum\nolimits_{j\neq i}(1+\zeta_{jt}(\omega))\,n_{jt}c^T_{jt}(\omega)/n_{it} + x_{T,it}(\omega)$$

$$+ x_{I,it}(\omega) + x_{Mt}(\omega)/n_{it} + \bar{x}_{nb,it} - \bar{y}_{nb,it}$$

$$y_{jt}(\omega) = c^F_{jt}(\omega) + x_{T,jt}(\omega) + x_{I,jt}(\omega), \; j \neq i$$

$$p_t/p_{t+1} = (1+\gamma_y)\, p_{i,t+1} c_{i,t+1}/(\beta p_{it} c_{it})$$

$$p_t/p_{t+1} = 1 + r_{b,t+1}$$

$$(1-\tau_{li})\, w_{it} = \psi p_{it} c_{it}/(1 - l_{it} - l_{nb,it})$$

$$w_{it} = (1-\phi)(1-\alpha_T-\alpha_I)\, p_{it}(\omega)\, y_{it}(\omega)/l_{it}(\omega), \quad \text{all } \omega$$

$$r_{T,it}(\omega) = (1-\phi)\,\alpha_T y_{it}/k_{T,it}(\omega)$$

$$r_{I,it}(\omega) = (1-\phi)\,\alpha_I y_{it}(\omega)/k_{I,it}(\omega)$$

$$r_{M,it}(\omega) = \phi n_{it} y_{it}(\omega)/m_t(\omega).$$

In equilibrium, $\sum_i S_{it}(\omega) = 1$ for all ω and $\sum_i B_{it} = 0$.

2.3. Steady State

We do not linearize around a steady state when computing equilibria, but the steady state is useful for gaining intuition about our solutions.

The steady state in this economy, with $\chi_i(\omega) = 1$ if $\omega \in \Omega_i$ and 0 otherwise, can be computed as follows. Given parameters, guess c_i, l_i, $m(\omega)$, and $p_i(\omega)$ (with $p_1(\omega) = 1, \omega \in \Omega_1$) and compute

$$r_b = (1+\gamma_y)/\beta - 1$$

$$p^F_i(\omega) = p_i(\omega)$$

$$p^T_i(\omega) = p_j(\omega)(1+\zeta_i(\omega))$$

$$p_i(\omega) = \left(p^F_i(\omega)^{1-\varrho} + p^T_i(\omega)^{1-\varrho}\right)^{\frac{1}{1-\varrho}}, \; \omega \notin \Omega_i$$

$$p_i = \left(\sum\nolimits_\omega p_i(\omega)^{1-\rho}\right)^{\frac{1}{1-\rho}}$$

$$c_i(\omega) = \left(\frac{p_i(\omega)}{p_i}\right)^{-\rho} c_i$$

$$c^F_i(\omega) = \left(\frac{p^F_i(\omega)}{p_i}(\omega)\right)^{-\varrho} c_i(\omega), \; \omega \notin \Omega_i$$

$$c^T_i(\omega) = \left(\frac{p^T_i(\omega)}{p_i}(\omega)\right)^{-\varrho} c_i(\omega), \; \omega \notin \Omega_i$$

$$w_i = \psi p_i c_i / \left[\left(1-\tau_{li}\right)\left(1-l_i\right)\right]$$

$$y_i\left(\omega\right)/l_i\left(\omega\right) = w_i / \left(p_i\left(\omega\right)\left(1-\phi\right)\left(1-\alpha_T-\alpha_I\right)\right),\ \omega \in \Omega_1$$

$$y_i\left(\omega'\right)/l_i\left(\omega'\right) = p_i\left(\omega\right) y_i\left(\omega\right)/l_i\left(\omega\right)/p\left(\omega'\right),\ \omega \in \Omega_1, \omega' \notin \Omega_1$$

$$k_{T,i}\left(\omega\right)/y_i\left(\omega\right) = \left(\left(1-\phi\right)\alpha_T\right)/\left(r_b/\left(1-\tau_{pi}\right)+\delta_T\right)$$

$$k_{I,i}/y_i\left(\omega\right) = \left(\left(1-\phi\right)\alpha_I\right)/\left(r_b+\delta_I\right)$$

$$y_i\left(\omega\right) = \left[a_i\left(\omega\right)\left(k_{T,i}\left(\omega\right)/y_i\left(\omega\right)\right)^{(1-\phi)\alpha_T}\left(k_{I,i}\left(\omega\right)/y_i\left(\omega\right)\right)^{(1-\phi)\alpha_I}\right.$$
$$\left.\cdot\left(l_i\left(\omega\right)/y_i\left(\omega\right)\right)^{(1-\phi)(1-\alpha_T-\alpha_I)}\right]^{1/\phi} m\left(\omega\right)$$

$\Rightarrow$ values for $k_{T,i}\left(\omega\right), k_{I,i}\left(\omega\right), l_i\left(\omega\right)$ given ratios above

$$d\left(\omega\right) = \sum_i n_i\{\left(1-\tau_{pi}\right)\left(p_i\left(\omega\right)y_i\left(\omega\right) - w_i l_i\left(\omega\right) - \delta_T p_i\left(\omega\right)k_{T,i}\left(\omega\right) - \left(\gamma_Y+\delta_I\right)p_i\left(\omega\right)k_{I,i}\left(\omega\right)\right)$$
$$-\gamma_Y p_i\left(\omega\right)k_{T,i}\left(\omega\right)\} - \left(\gamma_Y+\delta_M\right)\left(1-\tau_{pi}\right)\chi_i\left(\omega\right)p_i\left(\omega\right)m\left(\omega\right)$$

$$\kappa_i = \tau_{ci}p_i c_i + \tau_{li}w_i l_i + \tau_d \sum_\omega s_i\left(\omega\right)d\left(\omega\right)/n_i$$
$$+\tau_{pi}\sum_\omega\left(p_i\left(\omega\right)y\left(\omega\right)_i - w_i l_i\left(\omega\right) - \delta_T p_i\left(\omega\right)k_{T,i}\left(\omega\right)\right.$$
$$\left.-\left(\gamma_Y+\delta_I\right)p_i\left(\omega\right)k_{I,i}\left(\omega\right)\right) - \tau_{pi}\left(\gamma_Y+\delta_M\right)p_i\left(\omega\right)m\left(\omega\right)/n_i + \bar{y}_{nb,i} - \bar{x}_{nb,i}.$$

With these intermediate results, we check that

$$l_i = \sum_\omega l_i\left(\omega\right) + \bar{l}_{nb,i},\ \text{all } i$$

$$\left(r_b-\gamma_Y\right)b_i = p_i c_i - \left(1-\tau_{li}\right)w_i l_i - \left(1-\tau_d\right)\sum_\omega s_i\left(\omega\right)d\left(\omega\right)/n_i - \kappa_i,\ \text{all } i$$

$$r_b - \delta_M = \sum_i\left(1-\tau_{pi}\right)p_i\left(\omega\right)\left(\phi n_i y_i\left(\omega\right)/m\left(\omega\right)\right)/\sum_i\left(1-\tau_{pi}\right)\chi_i\left(\omega\right)p_i\left(\omega\right),\ \text{all } \omega$$

$$0 = y_i\left(\omega\right) + \bar{y}_{nb,i} - c_i\left(\omega\right) - \sum_{j\neq i}\left(1+\zeta_j\left(\omega\right)\right)n_j c_j^T\left(\omega\right)/n_i - x_{T,i}\left(\omega\right) - x_{I,i}\left(\omega\right) - x_M\left(\omega\right)/n_i - \bar{x}_{nb,i}, i>1, \omega\in\Omega_i$$

$$0 = y_j\left(\omega\right) - c_j^F\left(\omega\right) - x_{T,j}\left(\omega\right) - x_{I,j}\left(\omega\right), j\neq i, \omega\notin\Omega_i.$$

If it does not, we update the guess and continue.

To make sure that everything is adding up properly, we also double-check that the first resource constraint holds:

$$0 = y_i\left(\omega\right) + \bar{y}_{nb,i} - c_i\left(\omega\right) - \sum_{j\neq i}\left(1+\zeta_j\left(\omega\right)\right)n_j c_i^T\left(\omega\right)/n_i - x_{T,i}\left(\omega\right) - x_{I,i}\left(\omega\right) - x_M\left(\omega\right)/n_i - \bar{x}_{nb,i}$$

for $i=1$ and $\omega\in\Omega_i$.

3. BEA Accounts

3.1. Multi-country Case

We now apply the BEA's procedure to set up the national and international accounts for our economy. This implies the following for GDP and GNP and their components:

- Nominal $\text{GDP}_{it} = \sum_\omega P_{it}(\omega)(Y_{it}(\omega) - X_{I,it}(\omega) - \chi_i(\omega)X_{M,t}(\omega)) + P_{it}\bar{Y}_{nb,it}$

  Income:

  Depreciation: $\delta_T \sum_\omega P_{it}(\omega)K_{T,it}(\omega)$

  Compensation: $W_{it}\sum_\omega L_{it}(\omega) = W_{it}L_{it}$

  Profits:

  Tax: $\tau_{p,it}\sum_\omega\{P_{it}(\omega)(Y_{it}(\omega) - \delta_T K_{T,it}(\omega) - X_{I,it}(\omega) - \chi_i(\omega)X_{M,t}(\omega))$
  $-W_{it}L_{it}(\omega)\}$

  Dividends: $\sum_\omega\{(1-\tau_{p,it})\{P_{it}(\omega)(Y_{it}(\omega) - \delta_T K_{T,it}(\omega) - X_{I,it}(\omega) - \chi_i(\omega)X_{M,t}(\omega))$
  $-W_{it}L_{it}(\omega)\} - P_{it}(\omega)(K_{T,i,t+1}(\omega) - K_{T,it}(\omega))\}$

  Retained earnings: $\sum_\omega P_{it}(\omega)(K_{T,i,t+1}(\omega) - K_{T,it}(\omega))$

  Nonbusiness income: $\bar{Y}_{nb,it}$

  Product:

  Consumption: $P_{it}C_{it}$

  Measured investment: $\sum_\omega P_{it}(\omega)(X_{T,it}(\omega) + \bar{X}_{nb,it})$

  Exports: $\sum_{j\neq i} P_{it}(\omega)(1+\zeta_{jt}(\omega))C^T_{jt}(\omega), \omega \in \Omega_i$

  Less Imports: $\sum_{j\neq i} P_{jt}(\omega)(1+\zeta_{it}(\omega))C^T_{it}(\omega), \omega \notin \Omega_i$

  Net Exports: $\sum_{\omega\in\Omega_i} EX_{it}(\omega) - \sum_{\omega\notin\Omega_i} IM_{it}(\omega)$

- $\text{GNP}_{it} = \text{GDP}_{it}$ + Net factor receipts less payments

  Net factor receipts (from j to i)

  Direct investment: $\sum_{j\neq i}\sum_{\omega\in\Omega_i} D_{jt}(\omega) + P_{jt}(\omega)\left[K_{T,j,t+1}(\omega) - K_{T,jt}(\omega)\right]$

  Portfolio equity: $\sum_{j\neq i}\sum_{\omega\in\Omega_j} S_{it}(\omega)D_t(\omega)$

  Portfolio interest: $r_{bt}B_{it}$ if $B_{it} \geq 0$

  Net factor payments (to all j from i)

  Direct investment: $\sum_{j\neq i}\sum_{\omega\in\Omega_j} D_{it}(\omega) + P_{it}(\omega)\left[K_{T,i,t+1}(\omega) - K_{T,it}(\omega)\right]$

  Portfolio equity: $\sum_{j\neq i}\sum_{\omega\in\Omega_i} S_{jt}(\omega)D_t(\omega)$

  Portfolio interest: $r_{bt}B_{it}$ if $B_{it} \leq 0$

- Balance of Payments: Current account = Financial account

  Current account:

  Net exports (see above)

  Net factor receipts less payments (see above)

  Financial account

  Direct investment: $\sum_{j \neq i} \sum_{\omega \in \Omega_j} D_{it}(\omega) + P_{it}(\omega) \left[K_{T,i,t+1}(\omega) - K_{T,it}(\omega)\right]$

  Portfolio equity: $\sum_{\omega \notin \Omega_i} V_t(\omega)(S_{i,t+1}(\omega) - S_{it}) - \sum_{j \neq i} \sum_{\omega \in \Omega_i} V_t(\omega)(S_{j,t+1}(\omega) - S_{jt}(\omega))$

  Portfolio debt: $B_{i,t+1} - B_{it}$

3.2. Two-country Case

It is useful to examine the current account and financial account for a two-country case, since we can relate it to the household budget constraints. Let u be the United Kingdom and e be the remaining EU countries. We will index companies in the United Kingdom by ω_d, which we will refer to as "domestic." We will index EU companies by ω_f, which we will refer to as "foreign." We will assume full expensing at home.

In this case, the current account can be written as net exports (NX) plus net factor receipts (NFR) less net factor payments (NFP):

$$\begin{aligned}
\mathrm{CA}_{ut} &= NX_{ut} + \mathrm{NFR}_{ut} - \mathrm{NFP}_{ut} \\
&= [P_{ut}(\omega_d)(Y_{ut}(\omega_d) - X_{I,ut}(\omega_d) - X_{T,ut}(\omega_d) - X_{M,t}(\omega_d)) + P_{ut}(\omega_f)(Y_{ut}(\omega_f) - X_{I,ut}(\omega_f) - X_{T,ut}(\omega_f)) \\
&+ P_{ut}(\omega_d)\left(\bar{Y}_{nb,ut} - \bar{X}_{nb,ut}\right) - P_{ut}C_{ut}] \\
&+ [(1-\tau_{p,et})\{P_{et}(\omega_d)(Y_{et}(\omega_d) - \delta_T K_{T,et}(\omega_d) - X_{I,et}(\omega_d)) - W_{et}L_{et}(\omega_d)\} + S_{ut}(\omega_f)D_t(\omega_f)] \\
&- \left[(1-\tau_{p,ut})\{P_{ut}(\omega_f)(Y_{ut}(\omega_f) - \delta_T K_{T,ut}(\omega_f) - X_{I,ut}(\omega_f)) - W_{ut}L^f_{ut}\} + S_{et}(\omega_d)D_t(\omega_d) - r_b B_{ut}\right] \\
&= (1-\tau_{lu})W_{ut}L_{ut} + (1-\tau_d)\sum\nolimits_j S_{ut}(\omega_j)D_t(\omega_j) + r_{bt}B_{ut} + \kappa_{ut} - (1-\tau_{cu})P_{ut}C_{ut} \\
&\qquad + P_{et}(\omega_d)(K_{T,e,t+1}(\omega_d) - K_{T,et}(\omega_d)) - P_{ut}(\omega_f)(K_{T,u,t+1}(\omega_f) - K_{T,ut}(\omega_f)), \qquad (3.1)
\end{aligned}$$

where

$$\begin{aligned}
\kappa_{ut} &= \tau_{cu}P_{ut}C_{ut} + \tau_{lu}W_{ut}L_{ut} + \tau_d(S_{ut}(\omega_d)D_t(\omega_d) + S_{ut}(\omega_f)D_t(\omega_f)) \\
&+ \tau_{p,ut}\{P_{ut}(\omega_d)(Y_{ut}(\omega_d) - \delta_T K_{T,ut}(\omega_d) - X_{I,ut}(\omega_d) - X_{M,t}(\omega_d))
\end{aligned}$$

$$+ P_{ut}(\omega_f)(Y_{ut}(\omega_f) - \delta_T K_{T,ut}(\omega_f) - X_{I,ut}(\omega_f)) - W_{ut}L_{ut}\} + P_{ut}(\omega_d)\left(\bar{Y}_{nb,ut} - \bar{X}_{nb,ut}\right).$$

In writing net factor payments, we assume that $B_{ut} < 0$ and therefore net factor interest is paid by the United States to rest of world.

When considering trade costs, we want to distinguish exports and imports. In this case, we can write out net exports, say for the United Kingdom, as follows:

$$\begin{aligned}
NX_{ut} &= P_{ut}(\omega_d)\left(Y_{ut}(\omega_d) - X_{I,ut}(\omega_d) - X_{T,ut}(\omega_d) - X_{M,t}(\omega_d) + \bar{Y}_{nb,ut} - \bar{X}_{nb,ut}\right) \\
&\quad + P_{ut}(\omega_f)(Y_{ut}(\omega_f) - X_{I,ut}(\omega_f) - X_{T,ut}(\omega_f)) - P_{ut}C_{ut} \\
&= P_{ut}(\omega_f)\left(Y_{ut}(\omega_d) - X_{I,ut}(\omega_d) - X_{T,ut}(\omega_d) - X_{M,t}(\omega_d) + \bar{Y}_{nb,ut} - \bar{X}_{nb,ut}\right) \\
&\quad + P_{ut}(\omega_f)(Y_{ut}(\omega_f) - X_{I,ut}(\omega_f) - X_{T,ut}(\omega_f)) \\
&\quad - P_{ut}(\omega_d)C_{ut}(\omega_d) - P^F_{ut}(\omega_f)C^F_{ut}(\omega_f) - P^T_{ut}(\omega_f)CT_{ut}(\omega_f) \\
&= P_{ut}(\omega_d)(1 + \zeta_e(\omega_d))C^T_{et}(\omega_d) - P_{et}(\omega_f)(1 + \zeta_u(\omega_f))C^T_{ut}(\omega_f)
\end{aligned}$$

where the first term is total exports from UK to EU and the second term is total imports from EU to UK. If there were more countries, these terms would be sums.

Next, consider the financial account (FA), which is the change in assets and given by

$$\begin{aligned}
\mathrm{FA}_{ut} &= [P_{et}(\omega_d)(K_{T,e,t+1}(\omega_d) - K_{T,et}(\omega_d)) - V_t(\omega_d)(S_{e,t+1}(\omega_d) - S_{et}(\omega_d))] \\
&\quad - [P_{ut}(\omega_f)(K_{T,u,t+1}(\omega_f) - K_{T,ut}(\omega_f)) - V_t(\omega_f)(S_{u,t+1}(\omega_f) - S_{ut}(\omega_f))] + B_{u,t+1} - B_{ut} \\
&\quad = \sum\nolimits_\omega V_t(\omega)(S_{u,t+1}(\omega) - S_{ut}(\omega)) + B_{u,t+1} - B_{ut} \\
&\quad + P_{et}(\omega_d)(K_{T,e,t+1}(\omega_d) - K_{T,et}(\omega_d)) - P_{ut}(\omega_f)(K_{T,u,t+1}(\omega_f) - K_{T,ut}(\omega_f)),
\end{aligned} \tag{3.2}$$

where we use the fact that $\sum_i S_{it}(\omega) = 1$ for all t and all ω. By the balance of payments, FA less CA is equal to zero and therefore

$$\begin{aligned}
(1 - \tau_{cu})P_{ut}C_{ut} &+ \sum\nolimits_\omega V_t(\omega)(S_{u,t+1}(\omega) - S_{ut}(\omega)) + B_{u,t+1} - B_{ut} \\
&= (1 - \tau_{lu})W_{ut}L_{ut} + (1 - \tau_d)\sum\nolimits_\omega S_{ut}(\omega)D_t(\omega) + r_{bt}B_{ut} + \kappa_{ut},
\end{aligned}$$

which in turn implies that the household period t budget holds each period.